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GEOMETRY BU

BUATSAMHUTE :—

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THUHMAHRUAI

He Geometry bu hi Middle School naupangte zir atana syllabus thar zuia siam a ni a Hall and Stevens siam School Geometry leh Euclid a Geometry bua mi a ni ber a Problem erawh chu a zirna bu hran Geometrical Drawing bu a awm tawh avangin kan telh ve lo va Hmanhmawh deuh taka siam a ni a Science a nih avangin Zo tawnga lehlin that hi a harsa lehzuai nghal a Chhut leh huna tha zawka kan siam theih nan hmang tuten remchang tha zawk minrawn hrilh zel theih chuan a lawm awm hle ang

Naupangten an Geometry zir hmanga anmahnia thil chhut ohbuah an zir nan Exercise tiem kan telh a Hetiang hi tihtir a tulzia entirna ang lek chauh a ni a Zirtirtuten a dang pawh tihtir thin sela a tha viau ang

He lehkhabu hi duh thusam ni rih kher mah suh sela a zir tirtu leh naupangten sawtpui nan an lo hman theih chuan a lawmawm hle ang

LUNGLEH
June 15 1951

C S Z & L C N

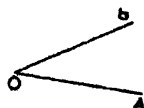
A CHHÔNGA THU AWM

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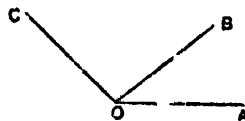
DEFINITIONS

- 1 **Point** chu awm hmun chauh nei lenze nei lo a ni
- 2 **Rin** chu dung lam chauh nei vâng lam nei lo a ni
- 3 **Rin ngil** chu point leh point inkâra mar taka rin hi a ni
- 4 **Surface** chu dung leh vâng nei chhah lam nei lo a ni
- 5 **Plane surface** chu a chungah point engpawh pahnh chhin chhiab ila chu chu rin zawmin a kar chu rual takin awh ziah sela chu chu a ni

- 6 **Angle** chu rin ngil pahnh insuhfinna kil hi a ni a Rin ngil pahnh chu **arms** (bân) an vuah a an insuhfinna kil tak chu **vertex** an vuah



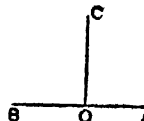
- Rin ngil pakhat sir tuaka angle awnte chu **adjacent angles** an vuah



- Tin rin ngil pahnh inkawkalh sela angle in zâwnte chu **vertically opposite angles** an ti

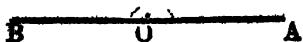


- 7 Rin ngil pakhat chungah rin ngil dang dangin angle thiang intia rial siam sela chûng angle te chu **right angle** ve ve a ni a tin rin ngil dang chu rin ngil pakhat tân chuan **perpendicular** a ni

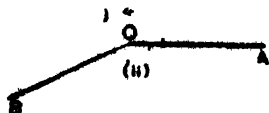


- Angle zauzia tehna chu **degree** ($^{\circ}$) a ni a Right angle pakhat hi degree 90 a zau a ni Degree khat $\frac{1}{60}$ chu minute (') an vuah a minute khat $\frac{1}{60}$ chu second (") an vuah

8 **Straight angle** chu rin ngil pahnih in angle pakhat right angle pahnih tia chiah a siam hi a m



9 **Reflex angle** chu right angle pa hnih sia zau zawk right angle pah sia zim zawk m hi a ni



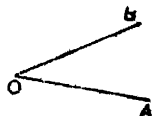
10 Angle pahnih finkhawm hi right angle pahnih tia a nih chuan **supplementary angles** an vuah a a angle eng ve ve pawh chu a pakhat tan supplement a m

11 Angle pahnih finkhawm hi right angle pakhat tia a nih chuan **Complementary angles** an vuah a a angle eng ve ve pawh chu a pakhat tan complement a m

12 **Obtuse angle** chu right angle sia angle zau zawk hi a m



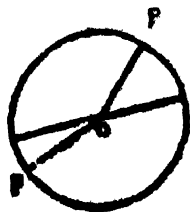
13 **Acute angle** chu right angle sia angle zim zawk hi a m



14 **Plane figure** rin pakhatin emaw a sia tamin emaw a kual khung chhung plane surface hi a m. A kual khungtu rin zawm khawm chu a **perimeter** a ni a, a kual khung chhung zauzau chu a **area** a ni.



15 **Circle** chu rin pakhat **circumference** an tihm a kual khung plane figure hi a ni a chu mi figure lai tak point atanga circumference thlenga rin zawng zawng chu a inchen vek tûr a ni Chu figure lai tak point chu **circle centre** an vuah



16 **Radius** chu circle centre atanga circumference thlenga ngil taka rin hi a ni

17 **Diameter** chu circumference thlenga centre rin tlanga circumference leh lam thlenga ngil taka rin hi a ni

18 **Semi Circle** chu circle diameter leh circumference diameter in a tan bun rin a kual khung figure hi a ni



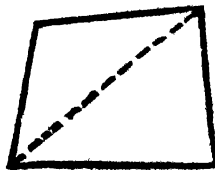
19 Circumference bung tawh phawt chu **arc** an vuah

20 **Rectilinear figure** chu rin ngil hlirin a kual khung figure hi a ni

21 **Triangle** chu rin ngil pathumin a kual khung plane figure hi a ni



22 **Quadrilateral** chu rin ngil palun a kual khung plane figure hi a ni a Tin a angle inepate vertex zawmtu rin ngil chu **diagonal** an vuah



DEFINITIONS

23 **Polygon** chu rin ngil pak aia tamun a kual khung plane figure hi a ni



TRIANGLES

24 **Equilateral triangle** chu triangle a sir rin pathumte inchen vek hi a ni



25 **Isosceles triangle** chu triangle a sir rin hnihte chu inchen hi a ni

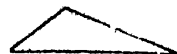
26 **Scalene triangle** chu triangle a sir rin pathumte inchen lo vek hi a ni



27 **Right-angled triangle** chu triangle, right angle pakhat nei hi a ni Tin chu ni right angle ep taka rin chu **hypotenuse** an vuah



28 **Obtuse-angled triangle** chu triangle obtuse angle pakhat nei hi a ni



29 **Acute angled triangle** chu triangle a angle pathumte acute angle ni vek hi a ni.



0 Triangle a a vertex engpawh a ep taka rin lai tak nêna
awmtu rin ngil chu **median** an vuah

QUADRILATERALS

31 **Square** chu rin ngil paha figure siam a rin
zawng zawng inchen vek a kil pawh right angle
ni vek lu a ni



32 **Oblong** chu rin ngil paha figure siam a kil zawng zawng
right angle ni vek a rin zawng zawng erawh chu inchen vek lo
hi a ni

33 **Rhombus** chu rin ngil paha figure
siam a rin zawng zawng inchen vek a kil
erawh chu pakhat mah right angle ni lo
hi a ni

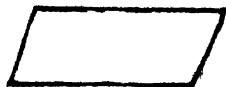


34 **Rhomboid** chu rin ngil paha figure siam a rin inepte inchen
a rin zawng zawng erawh chu inchen vek lo a kil pawh pakhat
mah right angle ni lo hi a ni

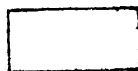
35 Rin ngil awm tlar a rin hmawrte a khaw
lam lam pawh eng chen pawha tihseia insuth
fin tawp lote chu an in **parallel** a ni



we hi a ni



37 **Rectangle** chu parallelogram hi a kil khat
right angle ni sela chu chu a ni

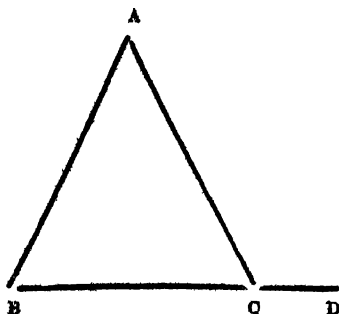


Rhomboid = Parallelogram

38 Trapezium chu rin ngil paha figure siam
a rin pahnhite chauh in parallel hi a ni



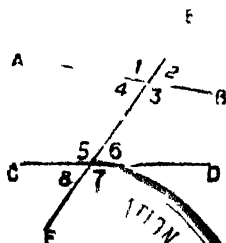
39 Triangle a rin pakhat tihsei a chu mi rin tihsei leh triangle a
rin pakhat nena a pawn lama angle an siam chu **exterior angle**
an vuah Tin triangle ohhng lama angl pahnh exterior angle
dep lotute chu **interior opposite angles** an vuah He mi a lema
ACD hi exterior angle a ni ABC angle leh BAC angle te hi in
terior opposite angle an ni



40 Rin ngil pakhat hF rinu r n ngil dang pahnh AB leh CD
tan tiangun angle pariat siam sela chung angle to chu hetiang
hian a hming an vuah A lema angle 1 na te 2 na te 7 na te
8 na te hi **exterior angles** an vuah a chu chu pawn lam angle-
te tihna a ni Tin angle 3 na te 4 na te 5 na te 6 na te hi
interior angles an vuah a chu chu ohhng lam angle te tihna a ni

Tin, angle 4 na leh angle 6-na hi **alternate angles** an vuah
a, angle 3 na leh angle 5-na pawh hi an vuah bawka

Tin EF rin leh lama angle 2 na leh angle 6-na hi sawi dawn
ila angle 2 na chu exterior angle a ni a angle 6 na chu angle
2 na interior opposite angle a ni Chutiang chiah chuan angle 7 na
leh angle 3 na te angle 8 na leh angle 4 na te angle 1 na leh
angle 5 na te hi an hming vuah a ni



POSTULATES

Geometry ah bian a lem siam nan instrument (hmanrua) then
khat kan mamawh a. Chung chu he leh khat atan hi chuan rin
igilna leh compass a ni ber a. A thlang lan sawi Postulates te
lu ti turin chung instrument te chu hman sawi a ni a tin chung
hming chuan tih tara sawite hi dik trah tih theih tura ng ih a ni

- 1 Point engpawh point dang nen rin ngila zawm theih a ni
- 2 Rin ngil tih tawp tawh pawh eng chen pawha uloi theih a ni
- 3 Point engpawh hmanin radius eng chen pawh hmanin circle
siam theih a ni

AXIOMS

Geometry hi thu dik thenkhat mawl te te an dik tih a
chian sa em avangz finfish kher pawh ngala hriat lohte rin chhana
siam a ni a. Chung thu dik chiang sa rengte chu **Axioms** an
vuah a ni

Entirna — Thilte an tluk a thuhman chuan anmahni pawh an
intluk a ni

Heng a thlang sawi Axiom te hi Geometry a hman lâz berte chu an ni —

Behh — Intluakhte intluk vék kan behh chuan a belhkháwm chu a intluk a ni

Pahh — Intluakhte intluk kan lâk chuan a la bang chu a intluk a ni

Puntir — Thil intlukhte intluk véka kan puntir chuan a chhuak chu a intluk a ni

Entirna — Thil intluk leh hnh ve ve chu a intluk a ni

Sem — Thil intlukhte intluk véka kan sem chuan a chhuak chu a intluk a ni

Entirna — Thil intluk chhve ve ve chu a intluk a ni

Kan sawi tâk axiom te hi axiom awmdân kawhhmuhna ang lek chauh a ni a Geometry a axiom hman zawng zawngte chu kan sawi kim vek kher ngah lo va A tûlna apiangah sawi chawp zel a ni zâwk ang

PROPOSITION

Tûna kan zir tur hi Plane Geometry a ni a Plane Geometry ah chuan plane surface chung a rin leh figure sımte chungchâng thu ngahluah a ni a

Kan thil zir tûr hi thu hláwm hrang hranga then darh a ni a Chutiang then hrang hrangte chu **Propositions** an vuah a

Propositions chu chu hnh a awm a **Theorem** leh **Problem** **Theorem** chuan Geometry a thu sawite a dukzia finfiah a tum a **Problem** erawh chuan Geometry a thil mam tur rin te emaw figure te emaw siam a tum a ni

Proposition tin chu bung hrang hrang paliah a then darh leh chhawn theih a Hetiang huan —

(i) **General Enunciation** — **Proposition** in a thil tum han sawi fiah lâwkna satliah mai a ni.

(i) **Particular Enunciation** — Proposition in a thil tum diagram
to (a lem) nēna fiah leh/uala hminghmerh nei meuhva sawi nawn
lehna hi a ni

(ii) **Construction** — Proposition a kan thutthi tum, Problem a
nih chuan kan thil siam tūr Theorem a nih chuan kan thil fin
fiah tūr chu kan thutthi nāna tūl rin emaw circle te emaw
siam dân kawhhmuhna hi a ni

(iv) **Proof** — Problem in a tum ang tak chu kan siam putling
ta tih entirna leh Theorem a finfiah tūra ruat chu a dik ngei a
ni tih entirna hi a ni

SYMBOLS AND ABBREVIATIONS

(Chhinchhuah nate leh kahtawmate)

Heng chhinchhuah nate leh kahtawite hi an hmang fo thin —

chutichuan tih uan emaw chuvangin tih nān emaw an
hmang tlin

• intluk tihna a ni

∠ angle tihna a ni

△ triang'le tihna a ni

pt point tihna

st line straight line tihna

rt ∠ right angle tihna

parl emaw (||) parallel tihna

sq square tihna

perp perpendicular tihna

perm parallelogram tihna

rectal rectilineal tihna

○ circle tihna

Cce. circumference tihna

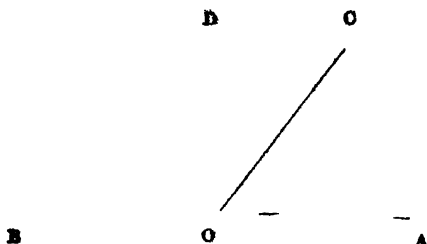
foot tihna

inches tihna

Inches 5 a sei inches 2 a zau tia ziaak aiin 5 × 2 tun an ziaak
thin 5 2 hi feet nga leh inches hnih tihna a ni

THEOREM 1 [EUCLID I 13]

*Rin ngil pakhat chungah rin ngil dang dungin angle thiang nam
seh chhang angle thiang finkhawm chu right angle pahnh tua a ni*



CO rin ngilin AB rin ngil tawkin angle thiang AOC leh COB
siam rawh se

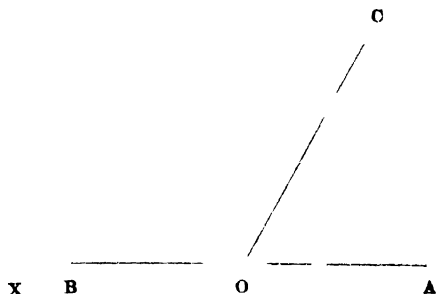
Tichuan AOC angle leh COB angle finkhawm chu right angle
pahnh tua a ni tih finfiah tûr a ni

OD rin ngil hi BA rin ngil chungah right angle thiang neia dung
ni rawh se

Finfiahna Tichuan AOC leh COB angle pahnh finkhawm chu
AOC leh COD len DOB angle pathum finkhawm tua a ni AOD
leh DOB angle pahnh finkhawm pawhun AOC leh COD leh DOB
angle pathum finkhawm a tat bawk a Chutichuan AOC leh COB
angle pahnh chu AOD leh DOB angle pahnh tua a ni AOD leh
DOB angle pahnh te hi chu right angle ve ve an ni si a
Chutichuan AOC leh COB angle pahnh finkhawm chu right angle
pahnh tua a ni

THEOREM 2 [EUCLID I 14]

Rin ngil pakhat point pakhat a sir tawnah rin ngil dang pahnhin angle thiang right angle p hnh tia siam sela chúng rin ngil pahnhie chu rin ngil pakhat a lo n



CO rin ngila O point ah a sir tawnah OA leh OB rin ngilin angle thiang finkhâwm right angle pahnh tia siam sela chúng rin ngil pahnhie chu rin ngil pakhat a lo n

Tichuan OB leh OA chu rin ngil pakhat a lo n tih finfiah tûr a n

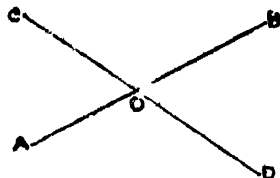
AO rin h O piah hmah X point thlengin tisei ila tichuan OX leh OB chu rin ngil pakhat a n tih entir a n ang

Finfiahna AOX chu rin ngil pakhat siam rêng a n a Chuti n...an COX angle chu COA angle supplement a n (Theor 1) Amah rawhchu COB angle chu COA angle supplement n tûra ruat n a Chuvangin COX angle leh COB angle chu a intia a n Chutichuan OX leh OB chu rin khat a n Nimahsela OX leh OA hu rin ngil pakhat siam a n si a Chuvângin OB leh OA w n rin ngil pakhat a n baw k a n

Q E D

THEOREM 3 [EUCLID I 15]

Rin ngil pahnh a inkawkalh chuan vertically opposite angle te chu a intia a ni



AB leh CD rin ngilte O point ah inkawkalh rawh se

Tichuan (1) AOC angle chu DOB angle tia a ni tih leh

(u) COB angle chu AOD angle tia a ni tih finfiah tûr a ni

Finfiahna AO in CD rin ngil a sut avângin AOC leh AOD angle thiange chu right angle pahnh tia a ni Tichuan AOC angle chu AOD angle supplement a ni

Tin DO in AB rin ngil a sut avângin DOB leh AOD angle thiange chu right angle pahnh tia a ni

Tichuan DOB angle chu AOD angle supplement a ni AOC leh DOB angle te chu AOD angle supplement ve ve an ni a Chuvângin AOC angle leh DOB angle chu a intia a ni

Chutiang bawkin COB angle chu AOD angle tia bawk a ni

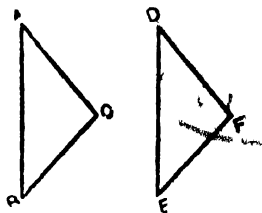
Q.E.D

EXERCISES

- 1 Rn ngil pahnh inkawkalhn angle pali a samte chu belh awmun right angle pali tia a ni tih finfiah rawh
- 2 ABC triangle ah ABC leh ACB angle chu a intia BC rin leh lam leh lama tihseun pawn lam angle pahnhite chu a a ni tih finfiah rawh
- 3 Rn ngil pahnh AB leh CD O point ah inkawkalh sela COD angle chu OX rinin hmun hniah then rualsela AOC angle pawn OY rinin hmun hnih bawkah then rual sela OX leh OY a chu rin ngil pakhat a ni tih finfiah rawh

THEOREM 4 [EUCLID I 4]

Triangle pahnhah a pakhatu rin hniah a dangu rin hnih en inchen ve ve sela chung rin inkâr anglete chu intia bawh in chu triangle-te chu engkimah a intluk a ni.



ABC leh DEF hi triangle pahnhite chu ni sela AB chu DE a ni sela AC chu DF chen ni sela tin rin inkâr angle BAC EDF chu intia ni bawh rawh se

lichuan ABC triangle leh DEF triangle te chu engkimah a a ni tih finfiah tûr a ni

anisa A point D point chungu awm tûr leh AB rin DE rin ga awm zân tûrin ABC triangle chu DEF triangle chungah na AB chu DE chen a nih avângun B point chu E point

chungah a awm ngei ang Tin, AB chu DF chungah a awm avâng leh BAC angle chu EDF angle tia a nih bawk avângin AC chu DF chungah a awm ngei tûr a ni

Tin AC chu DF chen a nih avângin C point chu F point chungah a awm tûr a ni

Tin B chu E chungah a awma C chu F chungah a awm avângin BC rîn pawh EF rîn chungah a awm ngei tûr a ni

Tichuan ABC triangle chuan DEF triangle a thua thlarh avângin chu triangle pahnh te chu engkimah a intluk vek a ni

Q E D

THEOREM 5 [EUCLID I 5]

Isosceles triangle a a bul thut rîn hnâw tawna a gle te chu a in tia a ni



ABC hi Isosceles triangle chu ni sela AB rîn chu AC rîn chen ni rawh se

Tichuan ABC angle chu ACB angle tia a ni tih finfiah tûr a ni

AD hi BAC angle hmun hniha then rualtu rîn ni sela a bul thut rîn BC chu D point-ah tawh rawh se

Fliaflahna BAD leh CAD triangle ahte hian BA chu CA chen a ni a AD rîn hi triangle pahnihte hian an intawm a tin rîn in kâr angle BAD chu rîn inkâr angle CAD tia a ni a Chuvângin triangle pahnihte chu engkimah a intluk vek a ni (Theor 4) Chutichuan ABD angle chu ACD angle nêh a intia a ni chu chu ABC angle chu ACB angle nêh a intia tihna ang a ni

Q E D

THEOREM 6 [EUCLID I 6]

Triangle a angle pahnihte intia sela chung angle intia epa rinte chu a inchen a ni



ABC hi triangle chu ni sela ABC angle chu ACB angle tia ni rawh se

Tiohuan AC rin leh AB rin chu a inchen a ni tih finfiah tûr a ni

AC leh AB hi inchen lo a nih chuan AB hi a sei zawk ni ang sela AC chenun BA rin atangin BD tan la D leh C rin zawn rawh

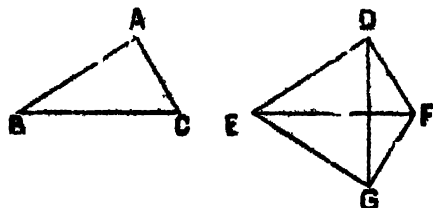
Finfiahna DBC leh ACB triangle-ah hian DB leh AC rin a inchen a BC rin an intawm a rin inkâr angle DBC leh ACB a inta hawk a Chuvângin DBC triangle leh ACB triangle chu a inta a ni (Theor 4) A phelin a pum a tia a lo ni ang Mahse chu zawn a ni thei lo (Axiom)

Chuvângin AB leh AC chu a inchen lo a ni lo va AB leh AC chu a inchen a ni

Q E D

THEOREM 7 [EUCLID I 8]

Triangle pabrish, a pakhatu rin ikumma, a danga rin ikum nân inchen sheuk sela chûng triangle-to chu engkimah a intluk a ni



ABC leh DEF hi triangle te chu ni sela AB chu DE chen ni sela AC chu DF chen ni sela, BC chu EF chen ni bawh rawh se

Ti huan triangle pahmhte chu engkimah a intluk a ni tih fufliah, tûr a ni

Finflahna B hi E chung a awm tûrin BC chu EF chung a awm bawh tûr leh A chu EF sirâ D awm lohna lama awm turin ABC triangle leh DEF triangle chu chuktuah ila BC chu EF chen a nih avângin C pawh chu F chungah a awm tûr a ni GEF hi ABC triangle awmna thar chu ni rawh se D leh G rin zawm rawh

ED chu EG chen a nih avângin EDG angle chu EGD angle tia a ni, (Theor 5) FD chu FG chen a nih avângin FDG angle chu FGD angle tia a ni bawh Chutih avâng chuan EDF angle pum leh EGF angle pum chu a intia a ni Chu chu EDF angle leh BAC angle chu a intia tahna a ni a Tin BAC leh EDF triangle-ah BA chu ED chen a ni a AC chu DF chen a ni a rin inktar angle BAC chu rin inktar angle EDF tia a ni bawh Chu vângin triangle te chu engkimah a intluk a ni (Theor 4)

Q E D

EXERCISES

1 Isosceles triangle-a a bul thut rin hmawr tawn atanga an ep rin lai tak point ve ve thlenga rinte chu a inchen a ni tih fin fiah rawh

2 ABC Isosceles triangle a a sir rin AC leh AB chu a inchen a tin, ABC angle leh ACB angle chu BO leh CO rinten hmun hnibah an then rual ve ve a

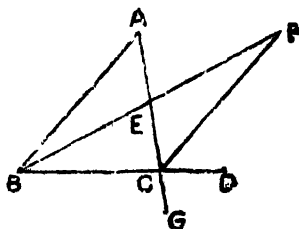
(i) BO rin leh CO rin a inchen a

(ii) AO rinin BAO angle chu hmun hnibah a then rual a ni tih finfiah rawh

ABCD quadrilateral a a rin inep AB leh CD a inchen a AD leh CB a inchen bawh a ADC angle leh ABC angle chu a intia a ni tih finfiah rawh

THEOREM 8 [EUCLID I 16]

Triangle a rin pahhat t hsei ni sela a exterior angle chu a interior opposite angle eng zawk ai pawhin a zau zawk a ni



ABC hi triangle chu ni sela BC chu D thlenga tihsei ni rawhse Tichuan exterior angle ACD chu a interior opposite angle ABC emaw BAC emaw aun a zau zawk a ni tih finfiah tur a ni

E chu AC rin lai tak point ni sela B leh E zawm la BF leh EF inchen turin F thlengin tisei rawh F leh C rin zawm rawh

Finfiahna AEB leh CEF triangle ah AE leh CE a inchen a EB leh EF a inchen a, AEB leh CEF vertically opposite angle te chu a inia a Chuvângin hêng triangle-te hi engkimah a iptluk a ni (Theor 4) Chutichuan BAE angle chu ECF angle tia a ni Nimaahsela ECD angle chu ECF angle aum a zau zâwk a Chu chu ACD angle chu BAC angle aini a zau zâwk tih ang a ni

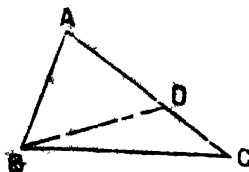
Chutiang bawkin AC rin chu G thlengin tasei ila A chu BC rin lai tak nen rin zawmah ngai ila BCG angle chu ABC angle aum a zau zâwk tin a finiah thesh ang Nimaahsela BCG angle chu a vertically opposite angle ACD tia a ni

Chuvângin ACD angle chu ABC angle aum a zau zâwk a ni

Q E D

THEOREM 9 [EUCLID I 16]

Triangle a rin pakhat a rin dang pakhat ava a sei zâwk chuan rin sei zâwk epa angle chu a tawr zâwk epa angle aum a zau zâwk a ni



ABC hi triangle chu ni sela AC rin chu AB aum sei zâwk rawh se Thohuan ABC angle chu ACB angle aini a zau zâwk a ni tih finiah tûr a ni

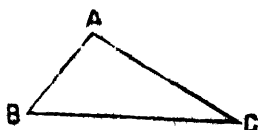
AC rin atangan AB chenun AD tan la B leh D rin zawm rawh

Finfiahna AB leh AD a inchen avângin ABD angle chu ADB angle tia a ni (Theor 5) Nimaahsela BDC triangle a exterior angle ADB chu a interior opposite angle DCB aini a zau zâwk chu chu ACB angle aum a zau zâwk tih ang a ni Chuvângin ABD angle chu ACB angle aini a zau zâwk a ni ABC angle phel chu ACB angle ai chuan a zau lehzuál a ni

Q E D.

THEOREM 10 [EUCLID I 19]

Triangle a angle pakhat a angle dang pakhat oio a zau zâwk chuan angle zau zâwk epa rin chu angle zîm zâwk epa rin aun a a sei zâwk a ni



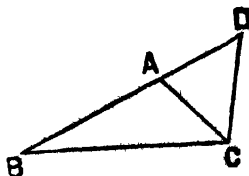
ABC hi triangle chu ni sela ABC angle chu ACB angle aun zau zâwk riwh se Tichuan AC rin chu AB rin aun a sei zâwk a ni tuh finfiah tûr a ni

Finfiahna AC chu AB aun a sei zâwk loh chuan AB nêh a in chen emaw AB aun a tâwî zâwk emaw a ni ngei ang AC chu AB chen a nih chuan ABC angle ohu ACB angle tia a ni ang (Theor 5) Nimahsela chutiang zawng ni tûra ruat a ni si lo AC chu AB aun tawî zâwk ang sela ABC angle chu ACB angle aun a zîm zâwk ang (Theor 9) Nimahsela chutiang pawh chu ni tura ruat a ni bik si lo Chutichuan AC chu AB chen a ni lo va tawî lah a tawî zâwk hek lo Chuvângin AC chu AB aun a sei zâwk a ni

Q E D

THEOREM 11 [EUCLID I 20]

Triangle rin hnih engpawh zawm chu a rin thumna aun a sei zâwk a ni



ABC hi triangle chu ni sela a rin pahnh engpawh zawm chu a rin thumna aun a sei zâwk a ni tuh finfiah tûr a ni

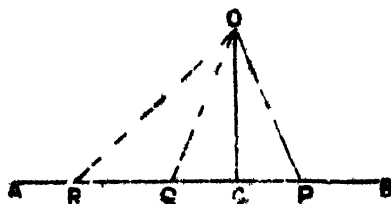
BC hi a rin sai ber n̄ sela BA leh AC zawm chu BC ain a sei zawk a ni t̄h kan ent̄ir chuan a tawk ang AD leh AC inchen t̄urin BA chu D thlengin tisei rawh D leh C rin zawm rawh

Finlahna ADC triangle ah chuan AD leh AC a inchen avāngin ACD angle chu ADC angle t̄iat a m̄. (Theor 5) Amaherawhchu BCD angle chu ACD angle ain a zau zawk a Chuvāngin BCD angle chu ADC angle ai p̄whn a zau zawk a m̄ Chu chu BDC angle ain a zau zawk t̄ihna a n̄ mai Tichuan BDC triangle-ah hian BD chu BC aun a sei zawk (Theor 10) Nimahsela BD chu BA leh AC inzawm chen a ni a Chuvāngin BA leh AC zawm chu BC ain a sei zawk a m̄

Q E D

THEOREM 12

Point ruat sa pokhat atanga rin nḡl ruat sa tawka rin nḡl rin sawng sawngah perpendicular chu a t̄h ber a ni



OC leh OP hi point ruat sa O atanga rin nḡl ruat sa AB tawka rin ve ve n̄ sela OC chu perpendicular n̄ sela OP erawh chu rin awn engpawh ni rawh se Tichuan OC hi OP ain a t̄awi zawk a n̄ t̄h finlah t̄ur a ni

Finlahna OCP triangle-ah OCP angle chu right angle a ni a, chuvāngin OPC angle chu right angle aia z̄im zawk a n̄ a (Theor 8) Chu chu OPC angle chu OCP angle aun a z̄im zawk t̄ihna a ni a Chuvāngin OC chu OP ain a t̄awi zawk a m̄ (Theor 10)

Q E D

EXERCISES

- 1 Triangle-a a angle engpawh pahnh belkhawm chu right angle pahnh ajin a zim zawk tih finfiah rawh
- 2 Triangle a a rin hnih engpawh insehlehna chu a rin thumna jin a tawi zawk tih finfiah rawh
- 3 Right angled triangle ah chuan a hypotenuse hi a sir rin sei ber a ni tih finfiah rawh
- 4 Triangle a a sir rin pakhat hmawr tawn atangin triangle phbunga point pakhat thlengin rin ngil pahnh rin ila heng rin pahnh zawm hi a sir rin dang pahnh zawm ayn a tawi zawk a ni tih finfiah rawh

THEOREM 13 [EUCLID I 27 & 28]

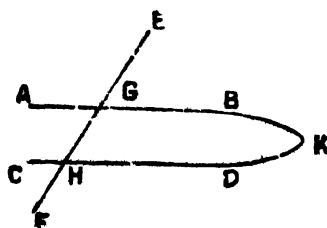
Rin ngil pakhatin rin ngil dang pahnh tan ilangin

(i) *alternate angles intia renga samon emaw*

(ii) *str thumna exterior angle chu a interior opposite angle ha a nahin emaw*

(iii) *str thumna interior angle te chu right angle pahnh hat a nahin emaw chuan*

chung rin ngil pahnh e chu a in-parallel a ni

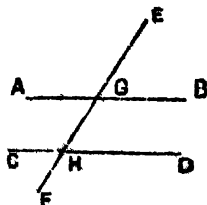


- (i) EGHF rin ngilin AB leh CD rin ngilte G leh H ah tanin AGH leh GHD alternate angle te chu intia rengin sam rawh se
 Tohuan AB leh CD hi a in parallel a ni tih finfiah tur a ni
 Finfiahna AB leh CD hi in parallel lo a nih chuan tisei ila B leh D lamah emaw A leh C lamah emaw a intawk ang A theih

chuan AB leh CD chu B leh D lamah tihseun intawk rawh se Tichuan, KGH chu triangle a lo ni ang a a rin pakhat KG chu a talanga tihsei a ni a

Chuvangin exterior angle AGH chu a interior opposite angle GHK ain a sau sawk a ni Nimahsela kan ruat tawh dân kha a sau sawk si lo Chuvangin AB leh CD chu B leh D lama tihsein a intawk thei lo a ni Chutiang bawkin A leh C lama pawh tihseun a intawk thei lo tih a entir theih a ni

Chuvangin AB leh CD chu a in parallel a ni



(ii) Exterior angle FGB chu a interior opposite angle GHD tia ni rawh se Tichuan AB leh CD chu an in parallel a ni tih finfiah tûr a ni

Finfiahna EGB angle chu GHD angle tia a ni a tin EGB angle chu vertically opposite angle AGH tia a nih baw si avangin AGH angle chu GHD angle tia a ni Hêngte hi alternate angle a ni, Chuvangin AB leh CD chu a in parallel a ni

(iii) Interior angle pahnih BGH leh GHD belkhhawm chu rigl angle pahnih tia ni rawh se

Tichuan AB leh CD chu a in parallel a ni tih finfiah tûr a ni

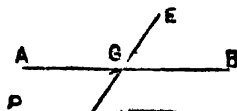
Finfiahna BGH leh GHD angle belkhhawm chu right angle pahni tia a ni a BGH leh AGH angle thiang belkhhawm pawh rigl angle pahnih tia a ni baw a Chuvangin BGH leh AGH angle belkhhawm chu BGH leh GHD angle belkhhawm tia a ni Hér intanah man BGH angle la bo ila Tichuan a la bang ve ve AGH angle leh GHD angle chu a intia a ni Tin hêngte hi alternate angle an ni a Chuvangin AB leh CD chu a in parallel a ni

Q E I

THEOREM 14 [EUCLID I 29]

Rin pahnh in parallel rin ngil pokhatin a tan tiang chuan

- (i) *Alternate angle te chu a nia a*
- (ii) *a str thuhmuna exterior angle chu a interior opposite angle tia a ni a*
- (iii) *a str thuhmuna interior angle pahnh belhkhawm chu right angle pahnh tiat a ni*



AB leh CD rin ngilte hi in-parallel sela EGF rin ngiln tan tiang rawh se Tichuan

- (i) AGH angle chu a alternate angle GHD tia a ni a
- (ii) Exterior angle EGB chu a interior opposite angle GHD tia a ni a
- (iii) Interior angle pahnh BGH leh GHD belhkhawm chu right angle pahnh tia a ni tih finfiah tar a ni

Finfiahna (i) AGH angle leh GHD angle tia a nih loh chuan PGH hi GHD angle tiat a alternate angle ang ni rawh se Tichuan PG leh CD an in parallel ang (Theor 13)

Nimahsela AB leh CD chu in parallel a ruat tawh a ni si a Chuvangin rin ngil pahnh inkawkalh AG leh PG chuan CD chu an parallel ve ve a lo ni ang Chu zawng a ni thei lo Chuvangin AGH angle chuan GHD angle chu a tia lo a ni lo va AGH leh GHD alternate angle te chu a intia a ni

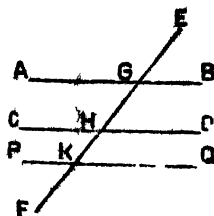
(ii) Tin EGB angle chu a vertically opposite angle AGH tia a ni a AGH angle chu a alternate angle GHD tia a ni bawk a chuvangin exterior angle EGB chu a interior opposite angle GHD tia a ni

(iii) Tin, EGB angle chu GHD angle tia a ni a lêngahte hian BGH angle belh ve rawh Tichuan EGB leh BGH angle belh khâwm chu BGH leh GHD angle belhkhâwm tia a ni Amah erawhote EGB leh BGH angle thiang belhkhâwm chu right angle pahnh tia a ni Chuvângin interior angle pahnh BGH leh GHD belhkhâwm chu right angle pahnh tia a ni.

Q E D

THEOREM 15 [EUCLID I 30]

Rin ngli thuhmun nê:n in-parallel rin ngliu chu anmahni pawh a in parallel a ni



AB leh CD rin ngliu chu PQ rin ngli nê:n in parallel ve ve rawh se

Tichuan AB leh CD an in parallel a ni tih finfiah tûr a ni

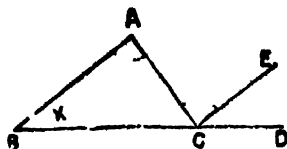
AB leh CD leh PQ te chu G te H te K point-tea tan tûrn EF rin ngli rin rawh

Finfiahna AB leh PQ an in parallel a EF rinin a suhfîn avâng in AGK angle chu a alternate angle GKQ tia a ni a Tin CI leh PQ an in parallel a EF rinin a suhfîn avângin exterior angle GHD chu a interior opposite angle GKQ tia a ni Chuvângin AGH angle leh GHD angle chu a intia a ni Hêngte hi a alternate angle an ni Chuvângin AB leh CD chu a in parallel a ni

Q. E D

THEOREM 16 [EUCLID I 3^o]

Triangle engpawh a angle pathum belhkhâwm chu right angle pahnh tia a ni



ABC hi triangle chu ni sela Tichuan ABC leh BCA leh CAB angle pathum belhkhâwm chu right angle pahnh tia a ni tih finfiah tûr a ni

BC rin chu D point thlengin tisei la tin CE chu BA nêna in parallel tûrin C aţanga rin ni bawh rawh se

Finfiahna BA leh CE an in parallel a AC in a suhfin avângin ACE angle chu a alternate angle CAB tia a ni

Tin BA leh CE an in parallel a BD in a suhfin avângin exterior angle ECD chu a interior opposite angle ABC tia a ni Chuvângin exterior angle pum pui ACD chu a interior opposite angle pahnh CAB leh ABC belhkhâwm tia a ni

Hêng intiahte hian BCA angle belh ve ve rawh Tichuan BCA leh ACD angle belhkhâwm chu BCA leh CAB leh ABC angle pathum belhkhâwm tia a ni

Nimahsela BCA leh ACD angle thiàng belhkhâwm chu right angle pahnh tia a ni Chuvângin BCA leh CAB leh ABC angle belhkhâwm chu right angle pahnh tia a ni

GEOMETRY

EXERCISES

1. The right triangles AB and CD share O point as common vertex in-
terior to the vertex A and C respectively. A and C are collinear. B and D
are collinear. AC and BD are parallel and the triangles are
right.

2. Quadrilateral figure with angles A and C are right angles. B and D
are right angles. AB and CD are parallel and the triangles are
right.

3. Rectilinear figure with angles A and C are right angles. B and D
are right angles. AB and CD are parallel and the triangles are
right.