COLLEGE PHYSICS

(FOR DEGREE STUDENTS)

VOLUME III

(LIGHT)

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MODERN BOOK AGENCY PRIVATE LTD.
10, BANKIM CHATTERJEE STREET,
CALCUTTA-700 073

Published by :

RABINDRANARAYAN BHATTACHARYYA, B.A.
for **Modern Book Agency Private Ltd.**
10, Bankim Chatterjee Street,
Calcutta-700 073

First Edition—September, 1963
Second Edition—July, 1964
Third Edition--November, 1966

Price : Rupees Fortyfive only.

Printed by :

Pran Kumar Mukherjee
S. ANTOOL & CO. PV. LTD.
91, Acharya P. C. Roy Road,
Calcutta-700 009.

&

Pradeep Kumar Banerjee
MANASI PRESS
73, Manicktalla Street,
Calcutta-700 006.

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LIGHT

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GEOMETRICAL OPTICS

CHAPTER I FUNDAMENTAL IDEAS

1. Light :—Our knowledge of the various objects in this world and of the different phenomena that occur in nature is derived from the impressions they produce upon our *senses*. The eye is the *sense-organ* which enables us to see the objects around us. Though there is no visible link between our eyes and the objects, still from the objects something reaches our eyes which creates the sensation of vision. This *something* which enables us to see the objects around us is known as *light*. *Light is, therefore, the external cause of the sensation of sight*. It is a form of **radiant energy** to which our eyes are sensitive.

The branch of Physics which deals with the phenomena associated with light is known as **Optics**. Light is a form of energy. Various theories have been suggested regarding the emission of light by a source. Light may be supposed to be ejected as a continuous stream of energy along a ray, or as a wave motion, or as a stream of particles (as photons). So far as the propagation of light and the other related phenomena are concerned, the emission of light in the form of a periodic wave motion becomes an almost essential concept. In certain other cases, the conception of rays or energy streams provides us with an important method of arriving at the approximate effects of the waves (e.g. in rectilinear propagation), while the concept of discrete energy packets (or photons) possessing definite momentum and obeying principles of conservation of energy and momentum, is essential for the proper interpretation of the origin of spectra and other phenomena involving interaction of light with matter (e.g. photo-electric effect, Raman effect etc.). Depending on these different modes of approach for interpretation of the different optical problems, the subject of Optics has been, for convenience, subdivided into the following three parts, though light which is an absolute entity should have, in the ultimate analysis, only one method of emission and propagation.

(a) Geometrical Optics (Ray Optics).—Here the formation of images is explained in accordance with certain fundamental principles and observed laws purely by geometrical methods without entering into any theory regarding the nature of light. A ray of light is defined as the direction of energy flow or as the normal to the wave-front of the propagated light wave. According to this definition, *i.e.*

the rays travel in straight lines, some of the optical problems lend themselves to simple treatment when dealt with from the stand-point of ray-optics. But in many cases it yields only approximate results while in others quite erroneous results too. In the case of the passage of light through a narrow slit as also in the case of converging spherical waves, the ray-optics cannot interpret the observed facts. The wave-theory, however, can explain these facts in an elegant way. But in view of the simplicity involved in the calculations of ray-optics, the theory of the ordinary optical experiments particularly related to the optical instruments and the designs of such instruments are mainly based on the ray-optics.

(b) **Physical Optics.**—It deals with the theories regarding the nature and propagation of light and explains the experimental facts with the help of those theories. The phenomena of interference, diffraction, polarisation, etc. come under this head. The wave theory, the electromagnetic theory, the quantum theory and the dualistic theory of wave-mechanics should be included under this head.

(c) **Physiological Optics.**—This branch of Optics deals with the sensation caused by light in the human brain and also with the other physiological effects which light produces on the various living organisms.

2. (i) **Light is Energy** :—*Light is a form of energy and it obeys the law of conservation of energy.* Like all other forms of energy, it is also indestructible and can be transformed into different forms of energy, such as mechanical, chemical, electrical, etc. It can be also derived from any other form of energy.

(*a*) *Light energy and Mechanical energy.*—The production of fire by the rubbing of two pieces of stone, or by pressing a piece of iron against a rotating stone wheel, is an example of how mechanical energy can be first converted into heat energy which is then transformed into light energy. The reverse process, where light is transformed into mechanical energy, can be found in the fact that when light falls on a surface it exerts pressure on the latter. In 1900, Lebedew actually made a thin vane rotate by this mechanical pressure.

(*b*) *Light energy and Chemical energy* —The burning of a candle, of a gas in the case of an incandescent gas lamp, etc. are examples where the substances combine chemically with the oxygen of the air, and the energy of the chemical combination is transformed into heat and light.

The reverse phenomenon, when light energy is transformed into chemical energy, occurs in ordinary photographic plates where chemical decomposition of certain silver salts takes place when

exposed to light. The plates, in which the changes have been so produced, are then *developed* by chemical treatment to reveal the picture.

(*c*) *Light energy and Electrical energy.*—Modern electrical glow lamps are examples which illustrate how electrical energy is transformed first into heat and then into light energy.

Light energy can also be converted into electrical energy. It has been found that electrons are readily emitted by certain metals, *e.g.* potassium, caesium, etc. when light falls on them. This property of the above metals has been utilised to produce currents by photocells, which are extensively used in modern 'talkies' and 'television'.

(ii) **Light is Invisible.**—Light is itself invisible. But it makes things visible to us. Light as such is not seen, but the objects around are seen with the help of light. The track of a beam of sunlight entering through any slit into a dust-free room cannot be visualised, but when dust particles are scattered in the path of the beam, the track becomes noticeable. The light falls on the particles which are then seen by an observing eye by the light scattered by the particles.

3. Some Definitions :—

Ray.—A *ray* of light is the path along which the light travels. Rays are invisible. They travel straight in the same homogeneous medium and can thus be represented by straight lines with arrow heads indicating the direction of travel of the light. According to the wave theory, the rays are merely the normals to the wave-front and represent the approximate directions of the energy-flow.

Beam : Pencil.—A collection of adjacent rays along a definite direction is called a *beam.* A narrow beam is called a *pencil* of light. A ray is single and a beam is a bundle of rays.

A beam of light may be (*a*) parallel, (*b*) convergent, or (*c*) divergent (Fig. 1).

Parallel Beam.—A beam is said to be parallel when the rays making the beam are parallel to each other, *i.e.* when the rays do not meet each other if produced backward or forward. When light comes from a source placed at a very large distance, the adjacent rays are sensibly parallel. For this reason, the sun's rays are taken to be parallel. In Fig. 1, the beam of rays to the left-hand side of the lens *L* are parallel.

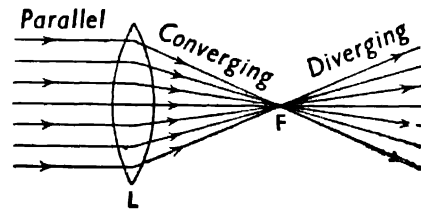


Fig. 1

Convergent Beam.—When the rays making a beam are directed towards a point, *i.e.* when in the forward direction the mutual distance between the rays decreases gradually and progressively, the beam is said to be convergent.

Divergent Beam.—When the rays making a beam originate from a point and diverge out, *i.e.* when the mutual distance between the rays increases gradually and progressively as the light travels forward, the beam is divergent. The rays from any point of a source of light spread out in all directions in the form of a divergent beam.

The point F (Fig. 1) of convergence or divergence of a parallel pencil of rays is called its **focus**, and the central ray of the pencil is usually referred to as the **axial ray.**

A substance, or any portion of space through which light can pass, is called an **optical medium**, or simply **medium.**

A medium is called **isotropic** or **optically homogeneous** when it has the same optical property throughout ; and a medium having different optical properties at different points is called **anisotropic.**

Luminous and Non-luminous Bodies.—All bodies around us are either luminous or non-luminous. A body is to be *self-luminous* or simply luminous, if it itself emits light. It may be used as a source of light. The sun, a burning candle, burning coal, etc. are luminous. A *non-luminous* body is that which cannot emit any light by itself. It becomes visible by means of light which it receives from other luminous bodies. Most bodies around us are non-luminous *e.g.* wood, stone, paper, water, etc. The moon is also non-luminous.

Non-luminous bodies may be transparent, translucent, or opaque.

Transparent bodies are those through which light can pass with only negligible absorption and, as such, an object can be seen distinctly through them, *e.g.* glass, water, air, etc.

Translucent bodies are those which allow a part of the light incident on them to pass through and scatter or absorb the rest. Through them other bodies can be seen but not so distinctly as in the case of transparent bodies. Greased or oiled paper, ground glass, waxed paper, etc. are translucent.

Opaque bodies are those which do not allow any light to pass through them and so no object can be seen through them, *e.g.* stone, wood, etc.

The difference between transparent and opaque bodies is often a question of thickness. Thus a thin leaf of silver can transmit some light, but a thick slab of silver is opaque. Though water is transparent, an object at a great depth of water cannot be seen.

Visibility of an object.—To see any point of an object, rays from the point must reach the eye. In the case of a luminous body, rays diverge out from each point of it in all possible directions and some of them reching the eye makes the point visible. But a non-luminous body, in order to be visible, must receive light from other sources of light, some of the incident light being scattered by it. Any point of the body will be seen, if some of the rays scattered from the point reach the observing eye.

4. **Fundamental Principles and Laws of Geometrical Optics :—** The subject of Geometrical Optics has been developed on the basis of a few fundamental principles and laws as have been found to be obeyed by the rays of light. These fundamental principles and laws may be given as follows :—

(i) **The principle of Rectilinear propagations of light.**—In an optically homogeneous medium a ray of light travels in a straight line. The formation of shadows and the occurrences of solar and lunar eclipses are the results of such rectillinear propagation. The principle is, however, found to be only approximately true. The wave theory of light gives an explanation of this rectilinear propagation which is found to be true to a first order of approximation.

(ii) **The principle of Physical independence of the path of rays.**—When two rays of light travelling in an optically homogeneous medium intersect, they will follow their individual paths even after intersection.

(iii) **Laws of Reflection.**—(*a*) The incident ray, the reflected ray and the normal to the reflecting surface at the point of incidence lie in the same plane. This plane is known as the *plane of incidence.*

(*b*) The angle of incidence is equal to the angle of reflection.

These laws are valid only for regular reflection. In the cases of irregular reflection from a rough surface and the scattering of light from particles of various sizes etc., the laws of the reflection are not generally found to hold good.

(iv) **Laws of Refraction.**—When a ray passes from one medium to another it suffers refraction *i.e.* bending either towards or away from the normal at the point of incidence on the separating surface according to the relative optical density of the two media. Refraction occurs according to the two following laws :—

(*a*) The incident ray, the refracted ray and the normal at the point of incidence lie in the same plane.

(*b*) The ratio of the sine of the angle of incidence to the sine of the angle of refraction for any two media is constant for the light of a particular colour. This law is known as *Snell's law.*

(v) The principle of reversibility of the path of a ray.—The principle states that if a ray of light starts from a point P and reaches a second point Q by a particular path after a number of reflections or refractions, then if the source be placed at Q, the ray would retrace the same former path point to point and travel back to P, all the conditions of reflection and refraction remaining the same as in the previous journey from P to Q. These points P and Q are thus interchangeable and are said to be *conjugate* to each other.

(vi) Fermat's Principle.—This principle states that the path actually followed by light has a stationary value. If a ray of light travels from one point to another suffering a number of reflections or refractions, the actual path differs from a neighbouring path by an infinitesimal quantity—both the paths beginning at a particular point and ending also at another particular point.

In its wider aspects, it actually includes both the laws of reflection and refraction, inasmuch as the validity of this law can also be established from the above laws. So in a sense, Fermat's principles of *stationary path* is considered to be a fundamental law in ray optics. This law has its analogues in other branches of physics as well. The law itself is directly deducible from the *wave theory of light* and thus serves as a stepping-stone linking up the two important branches of optics.

Questions (Chapter I)

1. How do you establish the fact that light is a kind of energy ?
2. State the fundamental principles on which the subject of geometrical optics has been built up.
3. Define.—Parallel beam, convergent beam, transparent and opaque bodies, isotropic and anisotropic bodies.

CHAPTER II || REFLECTION AT PLANE AND CURVED SURFACES

5. The Laws of Reflection :—Regular reflection takes place according to the following two laws—

(*1*) *The incident ray, the normal to the reflecting surface at the point of incidence, and the reflected ray lie in one plane,* which is known as the *plane of incidence.*

(*2*) *The angle of incidence is equal to the angle of reflection.*

If i be the angle subtended by the incident ray with the normal and r the angle between the reflected ray and the normal, then these angles are respectively called the angle of incidence and the angle of reflection.

So, from the second law, we have, $\angle i = \angle r$.

Normal Incidence.—In the particular case when a ray is incident normally on the surface of a reflector, *i.e.* when the angle of incidence, $\angle i = 0$, the reflected ray returns along the same path, because, in this case, the angle of reflection, $\angle r = 0$ according to the second law of reflection. So *a normal ray on reflection retraces its own path.*

Hartle's Optical Disc Method of Verifying Laws of Reflection : Hartle's disc (Fig. 2) is essentially a plane circular disc (D) vertically mounted on a heavy stand and is capable of rotation in the vertical plane about a horizontal axis passing through the centre of the disc.

The disc is divided into four quadrants by a horizontal and a vertical diameter (not shown in the figure). Each quadrant is divided into 90 equal parts, each being a degree. The vertical diameter is marked $0°-0°$, and the horizontal $90°-90°$. Around the edge of the vertical disc is a curved metallic screen which is provided with an adjustable aperture. The aperture can be closed by means of a slide (not shown in the figure) which has one or more slits through which light from a source can be made to pass tracing its path along the disc.

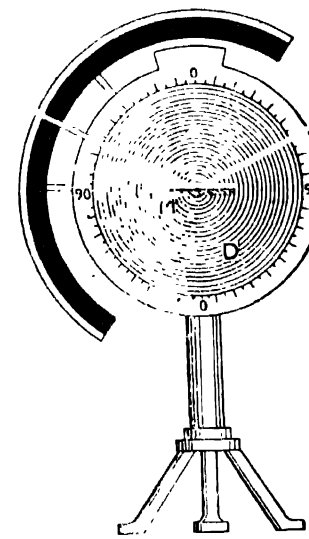


Fig. 2 —Hartle's Disc.

An optical element, say a piece of plane mirror, M, may be held at the centre of the disc, with the plane normal to the disc, along the $90°-90°$ line by means of a suitable clamp attached to the disc.

An adjusted thin pencil of light is made to pass through the slit in the screen and trace its path along the disc and fall on the mirror at the centre of the disc so that the $0°-0°$ line may be taken as the normal at the point of incidence. The trace of the reflected pencil is seen on the other side of the normal. Because the incident ray and the reflected ray lie in the

same plane in which the normal lies, *i.e.*, in the plane of the disc, *the first law is verified.*

The angle of incidence, as also the angle of reflection, can be read off from the circular scale along the edge of the disc and they are found to be equal. On rotating the disc (the screen containing the slit remaining fixed), the angle of incidence is changed and the corresponding angle of reflection found as above. It is seen that in every case the angle of incidence equals the angle of reflection whatever may be the angle of incidence. *This verifies the second law.*

6. Real and virtual objects:—If rays of light originate from a point or appear to converge to or to diverge from a point, that point is called a *point object* in optics. An extended object

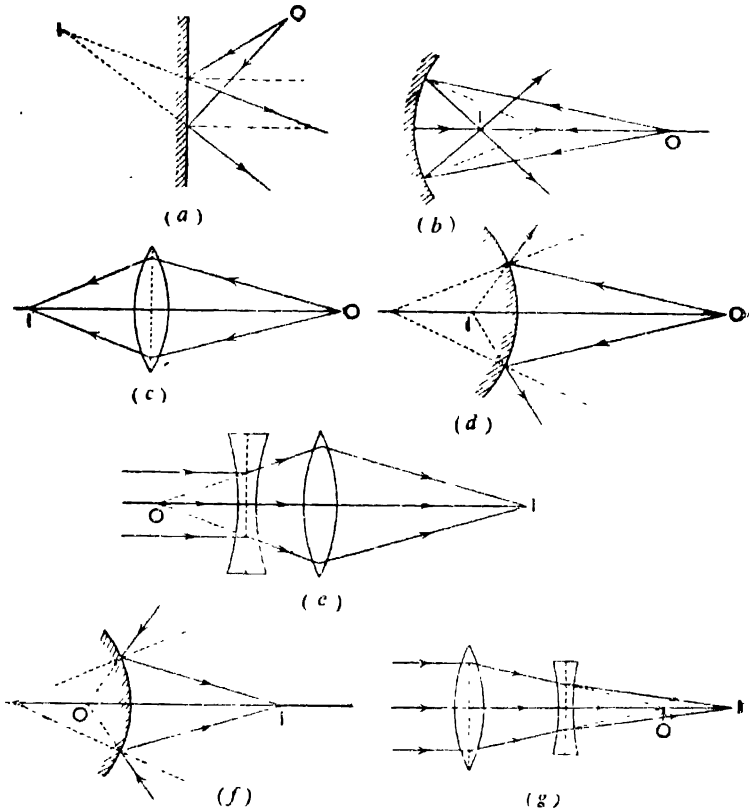


Fig. 3

consists of a number of such point objects. The term object is associated with the incident beam which originates from it.

An *object* is said to be *real* when an incident beam of rays actually originates from it. The rays starting from a point object actually diverge in all directions. Real point objects are indicated by the letter *O* in Fig. 3 (*a*), (*b*), (*c*) and (*d*).

An *object* (*O*) is said to be *virtual* when an incident beam of rays either appears to converge to it [Fig. 3 (*f*) & (*g*)] or seems to diverge from it. [Fig. 3 (*e*)]

7. **Optical Image** :—If rays diverging from a point source suffer changes of direction due to reflection (or refraction) such that the reflected (or refracted) pencil either actually converges to, or appears to diverge from, a second point, the second point is called the *optical image* of the first point. The term image is associated with reflected or refracted rays.

Real and Virtual Images.—When a pencil of rays diverging from a point *appears*, after reflection, or refraction, to diverge from a second point, the second point is called the *virtual image* of the first point. This is represented by the letter *I* in Fig. 3 (*a*) and (*d*).

When the pencil of rays diverging from a point, after reflection or refraction, actually converges to a second point, the second point is called the *real image* of the first point. This is represented by *I* in Fig. 3 (*b*) (*c*), (*e*) (*f*) and (*g*).

A real image is formed by the *actual intersection* of the rays and can always be received on a screen. But a virtual image is formed by the *imaginary intersection* of the rays. It cannot be received on a screen as it has no real existence.

Note :—According to ray-optics, a point image of a point object is always produced by a perfect optical system, *i e.*, the incident rays all converge to the image point in the image space after reflection or refraction through an optical system. But as the rays do not truly represent the direction of energy flow, the whole of the energy associated with the incident beam is not concentrated at the image point. The wave theory shows that the image formed by a perfect optical system is a diffraction pattern formed in a plane perpendicular to the axis of the system at the position of the geometrical image point. In the cases of optical systems having circular apertures this pattern consists of a central bright circular disc (Airy's disc) surrounded by an infinite number of alternately dark and bright rings. But as the intensity of the central disc is much more pronounced, that of the next bright fringe being only 1·7 percent of

the central one, for all practical purposes, the central disc may be identified with the actual image of the point object. Hence the calculations based on Ray-optics yield most of the important results concerning the different optical systems to a quite good order of approximation.

8. Image formed by a Plane Mirror :—If MM' (Fig. 4) be the trace of a plane mirror and O, a point object placed in front of it, due to reflection of the rays coming from O at the reflecting surface of the mirror, a virtual image I of O is formed. Suppose two rays OC and OE fall on the mirror and are reflected along CD and EF respectively. These two reflected rays would appear to diverge from a point I behind the mirror. So to an eye placed in the direction of these reflected rays a point image I will seem to be formed behind the mirror, which is *virtual* in nature as none of the actually reflected rays intersect at I.

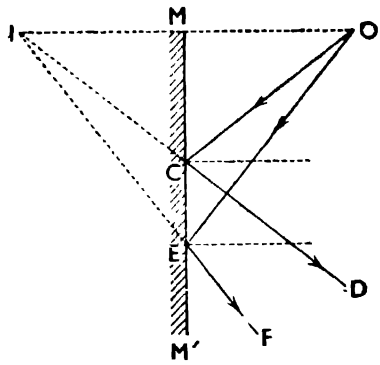


Fig. 4.

But if a convergent pencil of rays, O_1 O_2 (Fig. 5) strikes the mirror, the rays will, after reflection, intersect at a point I. So to an eye placed near I, a real image will be visible. It should be remembered that this is the only case of real image formation by a plane mirror, otherwise, a plane mirror, always gives a virtual image.

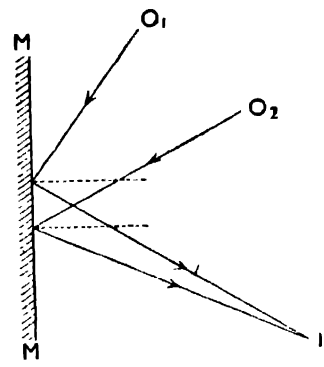


Fig 5.

If an extended object is placed in front of a plane mirror every point of it may be regarded as a source of light. The image of the object will be the resultant impression of the point images of all its parts.

Conditions of Seeing an Image in a plane mirror :—In order that an eye at E (Fig. 6) can see the image I of a point object O

formed by a plane mirror MM', it must receive rays of light reflected from some part of the mirror. Hence in order to see the image (*i*) the source O is to be placed anywhere in front of the mirror, and (*ii*) the eye E must be placed within the cone formed by the rays reflected from the edges of the mirror. The region within this cone is included between AM and BM'. As all the rays appearing to diverge from I are limited within the cone AIB, the image I will be visible only when the eye is placed in front of the mirror in the region limited by this cone.

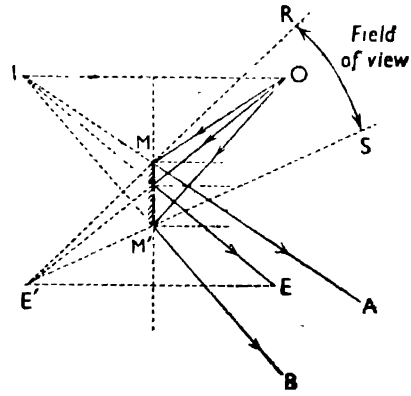


Fig. 6

9. Field of View in a Plane Mirror :—The ray which reaches E after reflection must have been incident on the mirror MM' in a direction OE' where E' represents the image of E. Similarly if rays are incident in the direction RE' or SE' so as to be reflected from the very boundary of the mirror, they would also reach E after reflection. So if lines are drawn from E' to the boundary of mirror M and produced, these would enclose another conical region between R and S in front of the mirror, within which if an object is placed, its image will be visible to the eye at E. This conical region is called the **field of view** in the plane mirror for the given position of the eye (Fig. 6).

10. Rotation of a Beam of Light by a Plane Mirror :—*When a plane mirror is rotated through an angle, a ray of light reflected from it is rotated through twice that angle.*

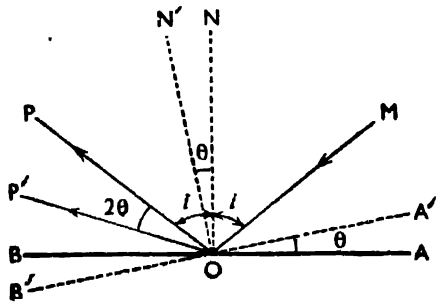


Fig. 7

Let a ray MO be incident on the mirror AB at an angle i to the normal ON (Fig. 7). The angle of reflection NOP is equal to $\angle i$.

Hence $\angle MOP = 2i$.

When the mirror is rotated through an angle θ into the new position $A'B'$, the normal ON is also rotated

through θ to the position ON', so the angle of incidence for the ray MO, in this case, is $i+\theta$. The new angle of reflection $N'OP'$ is also equal to $i+\theta$.

$\therefore$ The $\angle MOP' = 2(i+\theta)$. So the $\angle POP = \angle MOP' - \angle MOP = 2(i+\theta) - 2i = 2\theta$ = the angle through which the reflected ray is rotated. Thus if the mirror is rotated through θ, the reflected ray is rotated through 2θ.

This principle is utilized, in practice, in measuring small deflections in the case of reflecting galvanometers, sextants. etc.

11. Measurement of small Angular Deflection :—To measure a small angular deflection of the moving part of an instrument, a small mirror is attached to this moving part. A spot of light from a source after reflection from the mirror (M) is made to fall on a scale. This initial position of the light spot in the undisturbed position of the mirror is noted. Now if the mirror undergoes a small rotation α, the reflected spot of light is shifted on to a new position which is again noted. This shift is evidently due to the rotation of the reflected ray through 2α, since the incident ray is fixed in position. Let this shift be d cms. on the scale, which is at a considerable distances D cms. from the mirror. Then as the angles are small, we can write approximately,

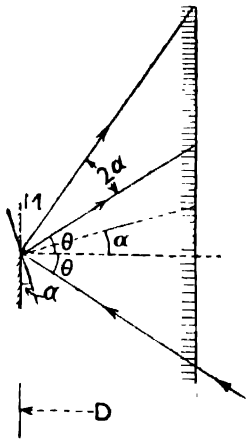


Fig. 8

$$\tan 2\alpha = \frac{d}{D}\ ;\ \text{or,}\quad 2\alpha = \frac{d}{D}\ \text{radians}\ ;$$

$$\text{or,}\quad \alpha = \frac{d}{2D}\ \text{radians} = \left(\frac{d}{2D} \times \frac{180}{\pi}\right)^{\circ}.$$

12. The Sextant :—This instrument, the working of which is based on the principle of double rotation of the reflected ray when a mirror is rotated, is used for measuring the angular separation of two distant objects. Its important use is in measuring the

altitude of the sun and other heavenly bodies from positions on the sea, as the instrument is not affected by the motion of the ship.

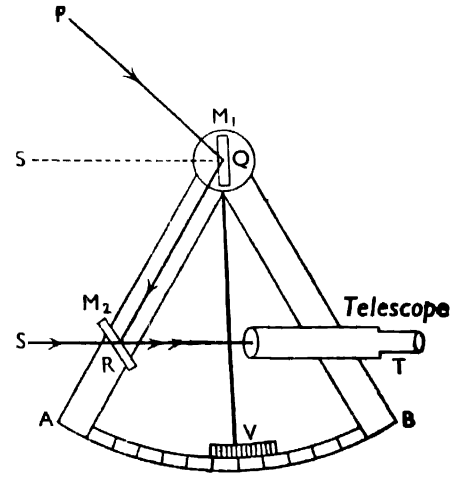


Fig. 9—The Sextant

It consists of a rigid metal frame to which is attached at right angles a mirror M_2 of which the upper half is transparent and the lower half is silvered (Fig. 9). This is called the *horizon mirror*. Another mirror M_1, known as the *index mirror*, also fixed at right angles to the frame, is attached to a radial index arm QV, fitted with a vernier V which moves over a circular scale AB, the arc being 60°, *i.e.* one-sixth part (hence the name, *Lat.*, sextant) of the complete circle. This 60°-scale is graduated as 120° in a sextant such that a half-degree is graduated as one degree in the sextant scale. A telescope T is also attached to the frame directed towards M_2 as shown in the figure. If the index arm QV is moved to the zero position (on the side of B) of the scale, the mirror M_1 becomes parallel to the mirror M_2.

(a) **Measurement of the angular distance between two stars.**—In order to measure the angular distance between two stars S and P (Fig. 9) the instrument is to be so placed that the telescope directly casts the image of S in its focal plane after refraction through the transparent part of M_2. The movable arm QV is now rotated till M_1 is parallel to M_2 when the image of S formed in the focal plane of the telescope after two successive reflections from M_1 and M_2 coincides with the image of S there as received directly. The vernier reading is now noted. It should be normally zero or very near zero. The arm QV is then rotated so as to cast the image of P after reflections from M_1 and M_2 on to exactly the same position in the focal plane of T, where the directly received image of S lies. In this position the vernier reading is again recorded. The difference of these two readings gives the angle of rotation of M_1. But as the reflected beam in both the positions of M_1 remains fixed in direction, the incident beam must have shifted through twice the angle of rotation of M_1. Thus the angle SM_1P i.e. the angular distance between S and P is equal to twice the

actual angle recorded by the sextant. This is why each half degree of the circular scale of the sextant is always labelled as one degree. Thus the angular distance is directly obtained in degrees from the difference of the above two readings taken on the sextant scale.

(b) Measurement of the altitude of the Sun.—Here mercury is placed in a flat dish such that an image S' of the sun is seen in the mercury pool when viewed through dark glasses (Fig. 10). The reflected beam OM_1 is made to fall on the mirrors M_1 and M_2 of the sextant so that two coincident images are received in the telescope. Let the vernier attached to M_1 read θ_1. The mirror M_1 is now rotated till the image of the sun reflected from M_1 and M_2 coincides with the image of S'. If the new vernier reading be θ_2, then the sun's altitude is given by θ, where $2\theta = \theta_2 - \theta_1$.

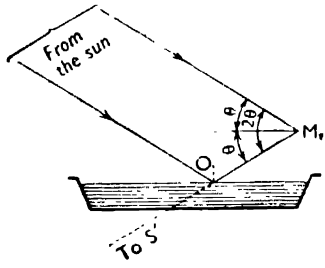


Fig. 10

13. Deviation of a ray by Reflection :—

(a) At a single plane mirror.—Let PO be the ray incident on the mirror MN (Fig. 11), the angle of incidence being i. The corresponding reflected ray is OR. If the mirror were absent PO would have proceeded in the direction PQ. But due to the interception of the mirror, the ray has been diverted from its original direction OQ to the direction OR. So it has been deviated through $\angle QOR = \angle\delta$ as a result of reflection. $\therefore \angle\delta = \pi - 2i$.

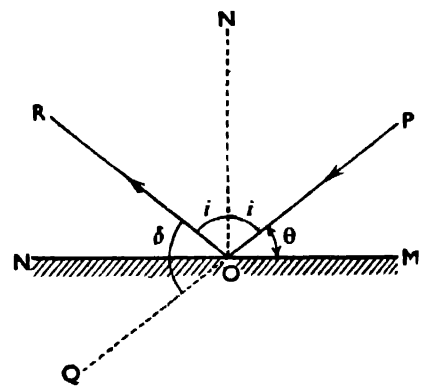


Fig. 11

If θ be the inclination of the incident ray with the mirror,

$$\theta = \left(\frac{\pi}{2} - i\right) ; \text{ or, } i = \frac{\pi}{2} - \theta, \quad \therefore \quad \angle\delta = \pi - 2\left(\frac{\pi}{2} - \theta\right) = 2\theta.$$

(b) At two inclined plane mirrors.—Let two plane mirrors be inclined at an angle α (Fig. 12). A ray of light PA suffers successive reflections at the points A, B, C etc.

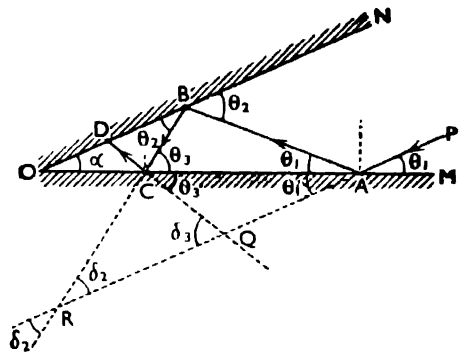


Fig. 12

If the angles of inclination of the ray with the mirrors at the different points of incidence be θ_1, θ_2, θ_3 etc. then,

in $\triangle ABO$, $\theta_2 - \theta_1 = \alpha$;
in $\triangle BCO$, $\theta_3 - \theta_2 = \alpha$;
... ...
similarly, $\theta_{n+1} - \theta_n = \alpha$.
So by addition $\theta_{n+1} - \theta_1 = n\alpha$.

(i) When n is even.—When two reflections are considered the incident ray appearing to come along $\overrightarrow{PAR}$ is diverted along the path $\overrightarrow{BCR}$ after the second reflection. Hence from $\triangle ACB$ the deviation (anti-clockwise) is $\delta_2 = \theta_3 - \theta_1 = (\theta_3 - \theta_2) + (\theta_2 - \theta_1) = 2\alpha$, anti-clockwise rotation being considered positive. Similarly considering four reflections, $\delta_4 = 4\alpha$,
and for n reflections, $\delta_n = n\alpha$.

(ii) When n is odd.—The incident ray coming along the $\overrightarrow{PAQ}$ is diverted in the direction $\overrightarrow{QCD}$ after three reflections. Hence from $\triangle AQC$ the clockwise $(-ve)$ deviation is $\delta_3 = -(\theta_1 + \theta_3) = \theta_3 - \theta_1 - 2\theta_3 = 2\alpha - 2\theta_3$. Similarly for an odd value of n, $\delta_n = (n-1)\alpha - 2\theta_n$.

If the incident ray falls on the first mirror at such an angle of incidence that the successively reflected rays gradually recede from the line of intersection of the mirrors (*i.e.* O), the deviation will be simply $2\pi - \delta_n$ where δ_n is the deviation already derived.

14. (i) Image Formation by Two Mirrors inclined at an Angle :—Let OA and OB represent two plane mirrors facing each other and inclined at an angle AOB (Fig. 13). Let P be a source of light. The image of P formed by reflection at the mirror OA is at P_1 on the normal PNP_1 to the mirror. This virtual image is formed as a result of reflection of the rays diverging from the point P and incident on the mirror OA. It can be shown with the

help of geometry that $OP = OP_1$. Therefore, P_1 lies on a circle drawn with O as centre and radius OP. It will be seen that other images formed will also lie on this circle. Now, P_1 serves as an object in front of the mirror OB. Draw a normal P_1P_2 on OB so that P_1P_2 is bisected by OB. Then P_2 is the image of P_1 formed by the mirror OB, and OP_2 will be equal to OP_1. Similarly, P_2 serves as an object in front of the mirror OA, and an image P_3 is formed, on the other side of OA so that OP_2 is equal to OP_3. P_3 falls behind both the mirrors and no further image is formed.

In the same way, another series of images P, P'', P''', etc. will be formed, the first image being formed by the mirror OB. The process

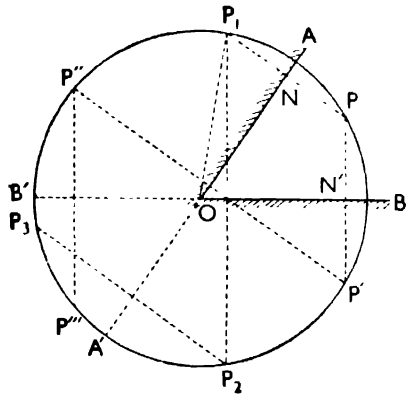


Fig. 13—Two mirrors inclined at an angle.

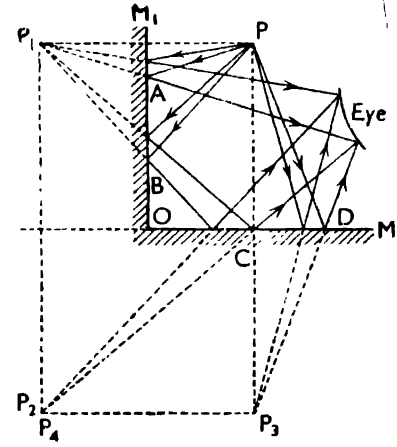


Fig 14—Two mirrors at right angles.

of formation of images by successive reflections at the surfaces of the mirrors goes on until an image is formed behind both the mirrors, *i.e.* within the angle $A'OB'$.

The number of images formed depends on the inclination of the mirrors, and can be proved to be equal to $\left(\frac{2\pi}{\theta} - 1\right)$, where θ is the angle between the mirrors. Thus, when the angle is 90°, the number of images is 3 (Fig. 14) when θ is 60°, the number of images is 5, and so on.

If θ is not a submultiple of 2π, *i.e.* if $2\pi/\theta$ is not a whole number, the total number of images is the integer next greater than $\left(\frac{2\pi}{\theta} - 1\right)$.

(ii) **Two Mirrors at Right Angles.**—In Fig. 14, two mirrors M_1 and M_2 are placed at right angles to each other and facing the object P. Rays of light diverging from P have been drawn showing how by successive reflections images produced by M_1 and M_2 are seen by an observer whose eye is suitably placed. P_1 and P_3 are the first images produced by M_1 and M_2 respectively. Each of P_1 and P_3 acts as a virtual object for the other mirror and produces images P_2 and P_4 respectively. In the case of two mirrors at right angles to each other, as shown in the figure, P_2 and P_4 merge into one image. So there are three images altogether seen by the eye.

15. Reflection at Two Parallel Mirrors :—Place two plane mirrors parallel to and facing each other (Fig. 15). Let P be a luminous point situated between them. Through P draw a straight line perpendicular to both the mirrors and produce the line both ways, meeting the mirrors LK and MN at A and B respectively. The image of P formed by the mirror LK will be at P_1 such that $AP_1 = AP$. Rays diverging from P will appear to come from P_1 after being reflected by the mirror LK. Some of those reflected rays will fall on MN and will appear to diverge from P_2, being the image of P_1, which serves as an object in front of the mirror MN, so that $BP_2 = BP_1$. Again in the same way, the image of P_2 is formed at P_3 behind LK. Starting with the rays first reflected from the mirror MN, another series of images will be produced as in the above case.

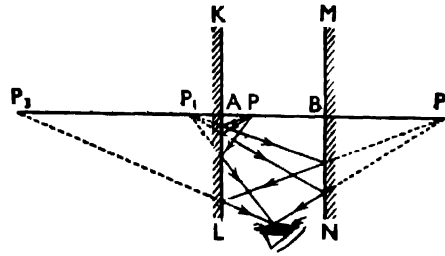


Fig. 15

Theoretically, the number of images thus formed in the case of

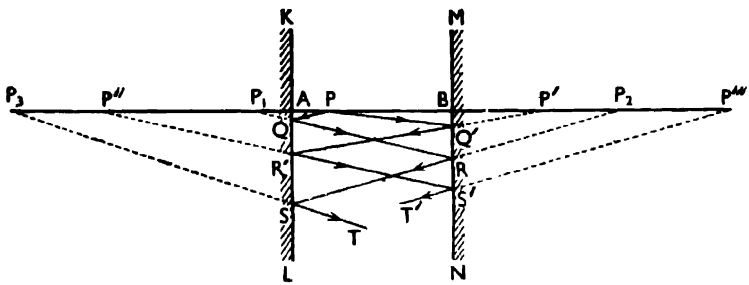


Fig. 16

parallel mirrors will be infinite, but as the images after each reflection

get fainter and fainter due to loss of light by absorption, the number of visible images is only limited, but quite large. In Fig. 16, the positions of the images formed by successive reflections from the two parallel mirrors have been shown. They extend, theoretically, to infinite numbers on either side.

16. Multiple Images formed by a Thick Glass mirror :—When a candle flame at S is viewed *obliquely* in a mirror or thick glass silvered at the back, a train of images S_1, S_2, S_3, etc. is seen in the mirror (Fig. 17). Of these images the first is faint, the second S_2 is usually the brightest and the others gradually diminish in brightness.

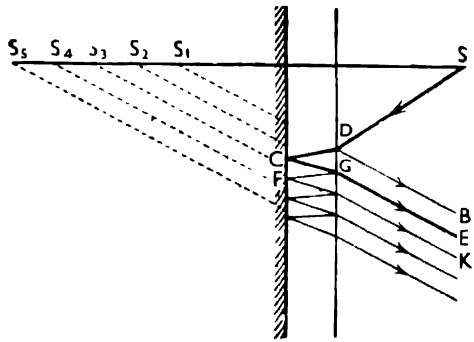


Fig 17

When an incident ray SD from a source, suppose a candle flame, falls on the mirror at D, only a small portion of the light is reflected at the glass surface in the direction DB forming a faint image S_1, while a larger portion enters into the mirror and is reflected back from the silvered surface C ; and, of this again a large part emerges out of the glass in the direction GE forming the second and the brightest image S_2. The rest is reflected to the back surface again, and the process is repeated giving other faint images. All the images lie on the normal from the object to the mirror.

When incidence is nearly normal, by far the greater part of the rays emerge along GE making the second image the brightest. When we look to our image in a thick mirror, there is no confusion arising out of multiplicity of images, because the images formed by the two surfaces overlap on each other almost exactly, the incidence of rays being almost normal. With the obliquity of incidence increasing, more and more light is reflected at the first surface and, *with very oblique rays, the first image becomes the brightest.*

Example

Show by means of a diagram that a man can see the whole of his person in a mirror the length of which is half his own height.

Ans. Let MN represent the man, and E be the position of his eye (Fig 18). Let mn be the image of the man formed by the mirror PQ. Both Mm and Nn are bisected by the surface of the mirror, or by the mirror produced

The man sees the image m by means of rays coming from M, which, after reflection at P appear to diverge from m. Similarly, the image n is seen on account of rays coming from N, which, after reflection at Q, appear to diverge from n.

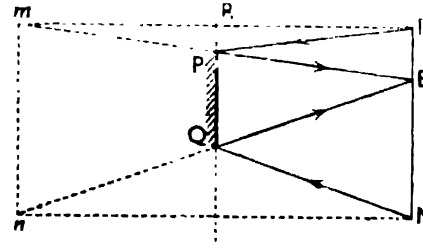


Fig. 18

So, PQ is the minimum length of the mirror required to see the complete image mn equal to MN.

From the similar triangles EPQ and Emn, $\frac{PQ}{mn}=\frac{PE}{mE}$ ... ... (1)

Again, from the similar triangles mPR and mEM, $\frac{mP}{mE}=\frac{mR}{mM}$;

or, $\frac{mE-mP}{mE}=\frac{mM-mR}{mM}$ or, $\frac{PE}{mE}=\frac{RM}{mM}=\frac{1}{2}$.

$\therefore$ From (1), $\frac{PQ}{mn}=\frac{PE}{mE}=\frac{1}{2}$. or, $PQ=\frac{1}{2}mn=\frac{1}{2}MN$ ($\because$ $mn=MN$), i.e., the minimum size of the mirror required is half the height of the observer.

17. Spherical Mirror :—A spherical mirror is a reflecting surface so curved that it forms part of a regular hollow sphere. It may be *concave* or *convex*.

Concave Spherical Mirror.—A spherical mirror is concave when the reflection takes place at the hollow side of the sphere of which the mirror is a part.

Convex Spherical Mirror.—A spherical mirror is convex when the reflection takes place at the bulging or the raised side. Obviously, when such a mirror is placed facing the eye, its middle point will be nearest to the eye.

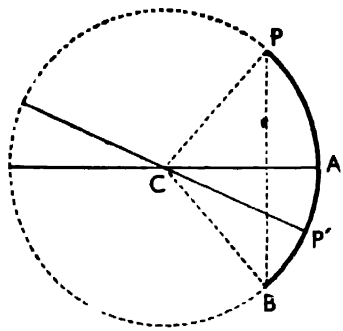


Fig. 19

The pole of a mirror PAB is the middle point (A) of its reflecting surface (Fig. 19).

The **Centre of Curvature** of a mirror PAB, Fig. 19, is the centre (C) of the sphere of which the mirror forms a part, and the radius (CP) of the sphere of which the mirror is a part is called the **radius of curvature** of the mirror.

The **Principal Axis** of a mirror PAB (**Fig. 19**) is a direction which may be obtained by the line

joining the centre of curvature (C) and the pole (A) of the mirror.

Secondary Axis.—Any direction obtained by a line passing through the centre of curvature (C) and any point (P') of the reflecting surface of the mirror other than the pole (A) is called a secondary axis of the mirror.

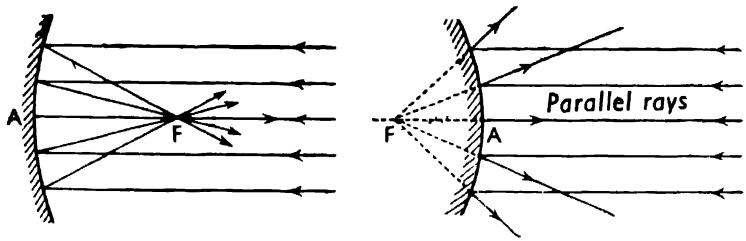
Any axis, principal or secondary, is perpendicular to the mirror.

Principal Section.—When a spherical mirror is intersected by a plane passing through its principal axis the cut-away section is a circular arc (PAB) called the principal section of the mirror (Fig. 19).

A mirror is ordinarily represented by its principal section.

Aperture.—If P and B, Fig. 19 are the extreme points of the principal section PAB of a mirror whose centre of curvature is C, the line joining PB is called the diameter of the mirror and is often referred to as the aperture of the mirror. The **angular aperture** is given by angle PCB which is subtended at the centre of curvature (C) by the diameter (PB) of the mirror. That is, angular aperture $= \frac{\text{arc } PAB}{\text{radius } CP}$.

Spherical mirrors of very small apertures (not exceeding 10° of arc) only will be considered in this chapter.



(*a*) Concave Mirror. (*b*) Convex Mirror.

Fig 20

Principal Focus and Focal Length.—A pencil of rays parallel to the principal axis of a spherical mirror will, after reflection either converge to [*as in the case of a concave mirror*, Fig. 20 (*a*)] or appear to diverge from [*as in the case of a convex mirror*, Fig. 20(*b*)] a fixed point F on the axis. This fixed point F is called the **principal focus** of the mirror, and the distance of this focus F measured from the pole of the mirror is called the **focal length** (AF in Fig. 20).

Focal Plane.—The plane passing through the principal focus of spherical mirror at right angles to its principal axis is called the focal plane of the mirror.

18. Convention of Signs :—(a) In all measurements of distances and angles in Geometrical optics and in the deduction of the various relations therein, the following sign conventions have been used in this book.

(*i*) *A suitable origin is to be selected for all measurements.* This origin may be different for different cases under consideration. For a *spherical reflecting* or *refracting surface, the pole* of the surface is generally chosen *as the origin* while for *a thin lens* the *optical centre* is taken as *the origin.*

(*ii*) *Distances measured against the direction in which the incident light travels are* positive, *and those measured in the same direction as the incident light are* negative. *In other words, the measurements* towards *the source of light are* positive, *and measurements away from the source of light are* negative.

(*iii*) All distances measured in a direction perpendicular to the axis are taken as *positive* when they lie *above* the axis and *negative* when *below* the axis.

(*iv*) The angle subtended between a ray and the axis is *positive* when the axis is to be rotated in an *anti-clockwise* direction in order to approach the ray and the angle is *negative* for a corresponding clockwise rotation.

***(b) New Convention of Signs** —Formerly it had been the practice (which is still followed in this book) to take the direction , along the incident light as negative and that against it as positive according to this, *a concave mirror* has a positive focal length and a positive radius of curvature and a *convex mirror* a negative focal length and a negative radius of curvature

In the case of **lenses**, however, the distances on the object side measured from the optical centre, *i e* the direction against which the incident light traverses, are taken as positive, while the distances on the other side *i.e.* the direction along the incident light are taken as negative, which means that all real image distances are negative and virtual image distances positive ; and thus this has no logical agreement with mirrors. Besides this, there is another difficulty namely, that the opticians regard the convex lenses as positive and concave lenses as negative.

To remedy all these difficulties, a new convention of signs was recommended by the Physical Society (London) in 1934 ; and in many text books this new convention of signs is being used, according to which **all real distances** (*i.e , distances actually traversed by light either in coming from a real object, or in going to a real image*) **are taken as positive and all virtual distances** (*i.e. distances traversed by light only virtually either in appearing to go towards a virtual object or in appearing to come from a virtual image*) **are taken as negative.**

According to this convention, the *focal length of a convex lens*, which brings parallel rays of light to a real focus, *is positive* and that of *a concave lens is negative*. Hence a **convex lens** *has got a positive power* and is called a **positive lens** while **a concave lens** *has got a negative power* and is called **a negative lens.** An object and its image may be on the same side of the lens, and yet the object distance may be positive and the image distance negative.

In the case of **spherical mirrors**, a *concave mirror* with *real* focus has *positive focal length* and positive radius of curvature, while *a convex mirror* with *virtual* focus has a *negative focal length* and negative radius of curvature.

19. Conjugate Foci relation for spherical mirrors with pole as the origin :—

(1) Concave Mirror—Let O represent the position of a luminous point on the principal axis of concave mirror (Fig. 21). The ray OA coming along the axis will be reflected back along AO as it is normal to the surface. The ray OP incident at P will be reflected along PI making the angle of reflection CPI equal to the angle of incidence CPO, because PC is normal at P, C being the centre of curvature of the mirror. The intersection of these two rays determines the position I of the image of the point O.

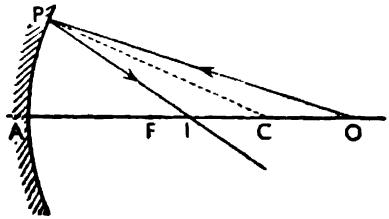


Fig. 21—Concave Mirror.

Since PC bisects the angle IPO, we have $\frac{IP}{PO}=\frac{IC}{OC}$. Considering AP to be small, we may write, $IA=IP$, and $OA=OP$ (approx.).

$$\therefore \quad \frac{IA}{OA}=\frac{IC}{OC} \qquad \ldots \qquad \ldots \qquad (1)$$

Let $OA=u$; $IA=v$, and $CA=r$. Then, we have, from (1),

$$\frac{v}{u}=\frac{r-v}{u-r} \text{ ; or, } v(u-r)=u(r-v) \text{ ; or, } ur+vr=2uv.$$

Dividing by uvr throughout, we have, $\frac{1}{v}+\frac{1}{u}=\frac{2}{r}$.

Now if $u=\infty$ (*i.e.* the incident ray OP is parallel to the principal axis AC), $v=f$, from the very definition of the principal focus.

$$\therefore \quad \frac{1}{f}+\frac{1}{\infty}=\frac{2}{r} \text{ ; or, } \frac{1}{f}=\frac{2}{r}.$$

$$\text{Hence, } \quad \frac{1}{v}+\frac{1}{u}=\frac{2}{r}=\frac{1}{f} \qquad \ldots \qquad \ldots \qquad (2)$$

(2) **Convex Mirror.**—In this case, every ray from a point O on the principal axis appears, after reflection from the mirror, to diverge from a point on the principal axis, and so the image is virtual. In Fig. 22, the ray OA reflects back along AO, since OC is the principal axis and the incidence is normal. Any other ray OP reflects along PB. The two reflected rays seem to intersect as I, which gives the position of the virtual image formed in this case. PN is the bisector of the exterior angle OPB of the triangle IPO. So, $\frac{IP}{OP}=\frac{IC}{OC}$.

Considering AP to be small, we may write IA IP, $OA=OP$ (approx.). $\frac{IA}{OA}=\frac{IC}{OC}$. Substituting with proper signs, we have,

$$\frac{-v}{u}=\frac{-r-(-v)}{u+(-r)}=\frac{-r+v}{u-r};$$

or, $\frac{v}{u}=\frac{r-v}{u-r}$ or, $ur+vr=2uv$.

Dividing by uvr throughout, we have,

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{r}=\frac{1}{f}$$

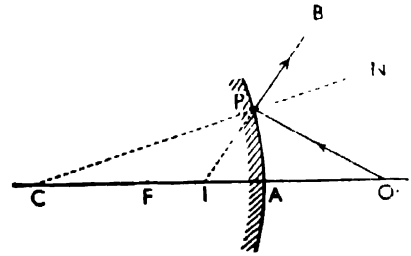


Fig. 22—Convex Mirror.

(3) **For non-paraxial rays.**—If the incident ray from a point object, O, on the axis of a concave mirror is reflected from a point P sufficiently away from the pole, an image I will be formed on the principal axis as shown in Fig. 23.

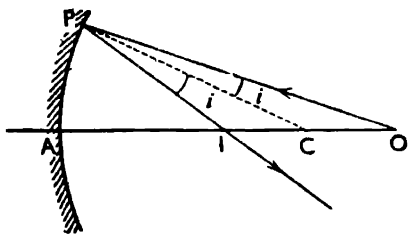


Fig. 23

Here also a definite relation between u and v can be established though in the case of non-paraxial rays they are measured with reference to the point of incidence. In the figure,

$u=PO$, $v=PI$

and $r=AC=PC$.

From figure we get,

$\triangle IPO=\triangle IPC+\triangle CPO$;

or, $\frac{1}{2}\,PO.\,PI.\sin 2i=\frac{1}{2}\,PC.\,PI.\sin i+\frac{1}{2}\,PO.\,PC\sin i$;

or, $2\,uv\sin i\cos i=rv\sin i+ur\sin i$;

or, $2\,uv\cos i=vr+ur$.

Dividing both sides by uvr, we get,

$$\frac{2\cos i}{r}=\frac{1}{v}+\frac{1}{u} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

Conjugate Points:—Two points on the axis of a spherical mirror are said to be **Conjugate** with respect to one another when an object placed at one produces the image at the other. In Fig. 21, a real object at O produces a real image at I and the positions are interchangeable. In Fig. 22, a real object at O produces a virtual image at I. A virtual object at I will produce a real image at O. Such points O and I are called **Conjugate Foci**. Pairs of such points occur on the principal axis of a spherical mirror, their distances from the mirror being respectively u and v, satisfying the relation, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, where the values of v and u may be interchanged between them without violating the equation. So this relation is often called the conjugate foci relation.

20. Rules for Tracing the Image in the case of Spherical Mirrors:—

(1) *Draw a ray from the top of the object parallel to the principal axis. This ray after reflection will pass through, or appear to pass from, the focus.*

(2) *Draw a ray from the top passing, or appearing to pass, through the centre of curvature of the mirror. This ray being normal to the mirror will be reflected back along the same path. If the bottom of the object be not on the axis, the position of its image may be determined exactly as above.*

(3) *A ray passing through the principal focus, and incident on the mirror, will be reflected in a direction parallel to the principal axis.*

(4) *The point to which any two of these reflected rays converge or from which they appear to diverge, is the* **required image.**

21. Determination of the Position of Image of an Extended Object formed by a Spherical Mirror:—An extended object may be supposed to consist of an assemblage of points only and when the image corresponding to each point is found, the image of the whole object is obtained. *When the point source is on the principal axis, the image is also on it,* and when it is not on the axis (as P in Fig. 24), the image is obtained by the rules given in Art. 20.

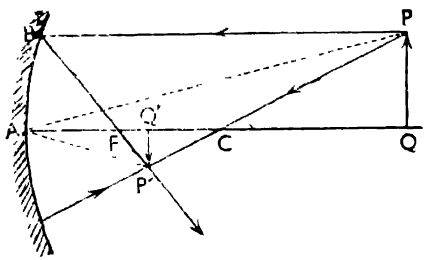


Fig. 24—Image by a Concave Mirror.

Let AB be the principal section of a mirror (Fig. 24 and Fig. 25) of which A is the *pole*, C the *centre of curvature*. AC

the principal axis and *F the principal focus*. Let *PQ* be the object perpendicular to the axis placed at *Q* beyond the centre of curvature. Draw a ray *PB* parallel to the axis which, after reflection, will pass through *F*, the principal focus. Then *BF* is the reflected ray in Fig. 24 (the reflected ray in Fig. 25 being produced backwards passes through *F*). Draw another ray *PC* through *C*, the centre of curvature, which will be reflected back along the same path. The point *P'* of intersection of the rays is the required image of *P*. So *P'Q'* is the image of *PQ*, where *P'Q'* is drawn perpendicular to *AC*. The size and position of *P'Q'* will depend on the position of *PQ* with respect to the mirror. Following the above method of construction, different cases may be examined.

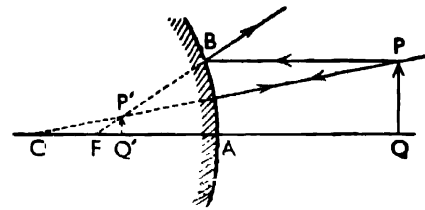


Fig. 25—Image by a Convex Mirror.

Note.—(*i*) In case the object *PQ* is very long, and the mirror is very small, the two rays *PB* and *PC* (or any one of them) may not meet the mirror. In such a case the image should be obtained graphically by extending the trace of the mirror. *It should be remembered that the image is actually formed by a large number of rays proceeding from each point of the object, and so it does not matter whether the two particular rays fall on the mirror or not.*

(*ii*) The focal length of a *concave mirror is positive*. The focal length of a *convex mirror is negative*. *The focus in the case of a convex mirror* is a *virtual focus*.

(*iii*) *Real images are always inverted, while virtual images are always erect.*

22. Linear or Transverse Magnification :—The linear magnification (*m*) of the image produced by a mirror is given by,

$$m = \frac{\text{size of image}}{\text{size of object}}.$$

Here *size* refers to linear dimension (*e.g.* length, or breadth, or height) and *not to area*, or *volume*. Also, *m* gives the value of the **linear magnification** only.

Magnification in the case of a Concave Mirror producing Real Image.—

In Fig. 24, join *PA* and *P'A*. Then *P'A* is the reflected ray corresponding to *PA*, since *P'* is the image of *P*. *AC* is the normal at *A*. The two triangles *PAQ* and *P'AQ'* are equiangular and, therefore, similar. So,

$\frac{P'Q'}{PQ} = \frac{Q'A}{QA} = \frac{v}{u}$. **This is deduced from the property of triangles. It is**

necessary to introduce at this stage a convention regarding the signs of erect and inverted bodies. Let us say that an erect body bears a positive sign and so, consistently, an inverted body must have a negetive sign attached to it. Since, $P'Q'$ is inverted with respect to PQ, it should then be mathematically represented by giving a negative sign before it.

$$\therefore \quad m = \frac{\text{size of image}}{\text{size of object}} = -\frac{P'Q'}{PQ} = -\frac{v}{u}.$$

Thus, for a concave mirror producing real image, $m = -\frac{v}{u}$. The magnification in this case is negative. It means that the image is inverted and real.

Similarly, it can be shown that the magnification in the case of a convex mirror is also given by $m = -\frac{v}{u}$. As v is negative here, m is really positive for the erect virtual image in this case.

Various Expressions for Linear Magnification.—We may also find other expressions for the magnification m, in the terms of u, v, f, and r as follows :—

(*i*) We have, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$.

Multiplying each term by v, we have $1 + \frac{v}{u} = \frac{v}{f}$.

$$\therefore \quad m = -\frac{v}{u} = -\left(\frac{v}{f} - 1\right) = -\left(\frac{v-f}{f}\right).$$

$$\therefore \quad m = -\frac{v}{u} = -\frac{v-f}{f} \qquad \ldots \quad \ldots \quad \ldots \quad (1)$$

(*ii*) Also, multiplying each term of the general equation by u, we get, $\frac{u}{v} + 1 = \frac{u}{f}$, or, $\frac{u}{v} = \frac{u-f}{f}$.

$$\therefore \quad m = -\frac{v}{u} = -\left(\frac{f}{u-f}\right).$$

$$\therefore \quad m = -\frac{v}{u} = \frac{-f}{u-f} \qquad \ldots \quad \ldots \quad \ldots \quad (2)$$

(*iii*) Again, taking the equation $\frac{1}{v} + \frac{1}{u} = \frac{2}{r}$.

we have, $\frac{1}{v} - \frac{1}{r} = \frac{1}{r} - \frac{1}{u}$, or, $\frac{r-v}{vr} = \frac{u-r}{ur}$.

$$\therefore \quad \frac{v}{u} = \frac{vr}{ur} = \frac{r-v}{u-r}, \text{ or, } m = -\frac{v}{u} = -\left(\frac{r-v}{u-r}\right)$$

$$\therefore \quad m = -\frac{v}{u} = -\frac{r-v}{u-r} \qquad \ldots \quad \ldots \quad (3)$$

Superficial or Surface Magnification.—If a small surface be placed perpendicular to the axis in front of a spherical

mirror, both its length and breadth will be magnified in the ratio, $-\frac{v}{u}$.

Hence the *surface magnification*,

$$m_s = -\frac{v}{u} \times -\frac{v}{u} = \frac{v^2}{u^2}.$$

Axial or Longitudinal Magnification :—If an object at a distance u from a spherical mirror occupies a length du along the axis and if its image in its turn extends over a length dv along the axis and is formed at a distance v from the mirror, then the *axial* or *longitudinal* magnification is defined by the relation, $m_l = \frac{dv}{du}$.

We know, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$.

Differentiating this relation with respect to u and remembering that f is a constant for a mirror, we have,

$$-\frac{1}{v^2}\frac{dv}{du} - \frac{1}{u^2} = 0 ;$$

$$\text{or,} \quad -\frac{1}{v^2}\frac{dv}{du} = \frac{1}{u^2} ;$$

$$\text{or} \quad \frac{dv}{du} = -\frac{v^2}{u^2} = m_l.$$

Thus the longitudinal magnification is equal to the square of the transverse or linear magnification.

The relation is valid only when the extensions of the object and its image are small compared to their distances from the mirror.

23. Nature, Position and Size of the Image in Spherical Mirrors.—The nature, position, and size of the image formed by a spherical mirror depend on the positions of the object with respect to the mirror.

(i) Geometrical method.

(a) Concave Mirror.—

(*i*) *Object at Infinity.*—Rays coming from any point of an object placed at an infinite distance are practically parallel. These parallel rays, after reflection, converge to a point in the focal plane of the mirror (Fig. 26). The image is *real*, *inverted*, and *very diminished*.

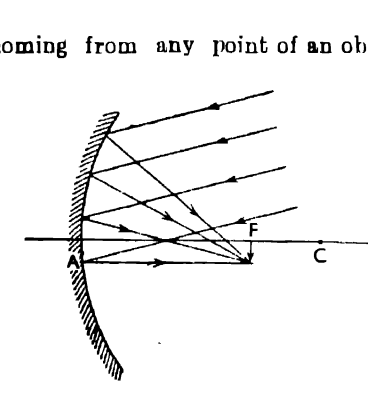


Fig. 26– Object at Infinity.

(*ii*) *Object between Infinity and the Centre of Curvature.* C (*vide* Fig. 24) —PQ is the object placed beyond C. $P'Q'$ is the image formed between C and the principal focus F, and is *real*, *inverted*, and *diminished*.

(*iii*) *Object at the Centre of Curvature* (*C*).—*PQ* is the object at *C* (Fig. 27). A ray from *P* parallel to the axis, after reflection, passes through *F*, and another ray *PB'* passing through *F*, after reflection, passes parallel to the axis. These two rays intersect at *P'* which is the image of *P*. As all rays from *Q* pass through *Q* after reflection, so the image of *Q* is formed at *Q*. *P'Q* is thus the image of *PQ* which is *real* and *inverted* ; and it can be easily proved that $PQ = P'Q$. So the image is of the *same size* as the object.

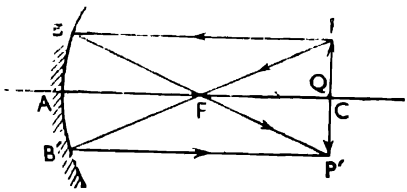


Fig. 27—Object at Centre of Curvature of a Mirror.

(*iv*) *Object between the Centre* (*C*) *and the Focus* (*F*). The ray *PB* is taken parallel to the axis and the other *CP* is taken through *C* (Fig. 28). These two rays after reflection intersect at *P'* which is the image of *P*, and *Q'*, is the image of *Q*. So the image is formed at *P'Q'* which is *real*, *inverted* and *enlarged*.

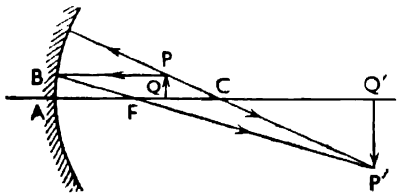


Fig. 28 - Object between *C* and *F*.

(*v*) *Object at Focus* (Fig. 29)—This case is the converse of case (*i*). Two rays from *P*, one parallel to the axis and the other in a direction through *C*, after reflection, become parallel and therefore meet at infinity. The image is *real*, *inverted*, and *infinitely enlarged*.

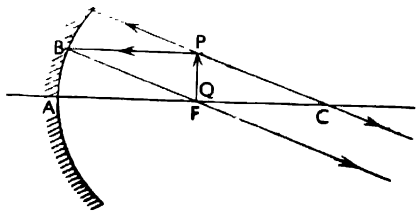


Fig. 29—Object at *F*.

(*vi*) *Object between Focus and Mirror* (*i.e. distance less than the focal length*)—The ray *PB* (Fig. 30), parallel to the axis, passes through *F* after reflection and the other ray incident on the mirror in a direction through *C* retraces its path after reflection and passes through *C*. The rays form a divergent pencil and meet at *P'* behind the mirror, when produced backwards ; *P'* is the virtual image of *P*. The image *P'Q'* in this case is *virtual*, *erect*, and *magnified*.

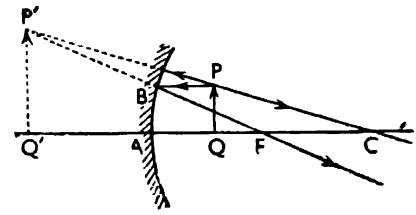


Fig. 30—Object between *F* and *A*.

(b) **Convex Mirror.**—This case is given in Fig. 25, where it has been found that the image $P'Q'$ is virtual, erect, and diminished, and it will be found also from Fig. 31 that for all positions of the object, the image is *virtual*, *erect* and *diminished*. Three different positions marked I, II and III of the object PQ and the corresponding positions marked 1, 2 and 3 of the images are shown in Fig. 31. As the object PQ is brought from infinity towards the pole A, the image $P'Q'$ increases from a point size to a size almost equal to PQ.

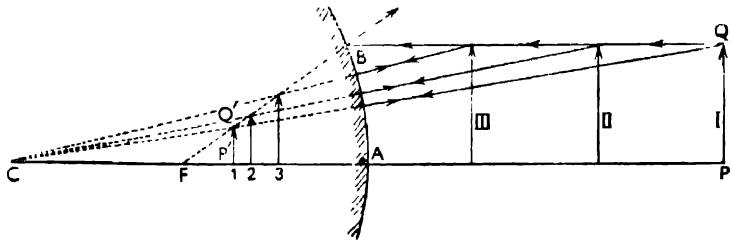


Fig. 31

(ii) Summary of Results.—

Concave Mirror

Position of Object	Position of Image	Nature of Image	Size
∞	At F	Real Inverted	Diminished
Between ∞ and C	Between F and C	" "	"
At C	At C	" "	Same size
Between C and F	Between C and ∞	" "	Enlarged
At F	∞	" "	"
Between F and pole	Behind the mirror	Virtual, Erect	"
At Pole	At Pole	" "	Same size

Convex Mirror

	F	Virtual, Erect	Diminished
Between pole and ∞	Between F and pole		

(iii) Verification from the Formula.—The position of the image formed by a **concave mirror** may be obtained from the *formula* as follows:

(*i*) *Object at infinite distance, here* $u=\infty$ $\quad \frac{1}{u}=0.$

$\therefore$ From $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, we get, $\frac{1}{v}=\frac{1}{f}$

$\therefore$ $v=f$, *i.e.* the image is at focus.

(*ii*) *Object between* ∞ *and centre* C *Here* $u>2f$ *and* $<$

$\therefore$ From the general formula (Art. 19), $\frac{1}{v}>\frac{1}{2f}$ and $<\frac{1}{f}$.

$\therefore$ $v<2f$ and $>f$, *i.e.* the image is between F and C.

(*iii*) *Object at* C. Here $u=r=2f$.

Hence, $\frac{1}{v}=\frac{1}{f}-\frac{1}{2f}=\frac{1}{2f}=\frac{1}{r}$, $\quad v=r$, *i.e.* the image is at C.

(*iv*) *Object between* C *and* F. Here $u>f$ and $<2f$.

$\therefore$ $\frac{1}{u}<\frac{1}{f}$ and $>\frac{1}{2f}$, $\therefore$ $\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$,

i.e. $\frac{1}{v}>\left(\frac{1}{f}-\frac{1}{f}\right)$ and $<\left(\frac{1}{f}-\frac{1}{2f}\right)$, *i.e.* $\frac{1}{v}>0$ and $<\frac{1}{2f}$.

$\therefore$ $v<\infty$ and $2f$, *i.e.* image between C and ∞.

(*v*) *Object at* F. Here $u=f$. $\quad \frac{1}{v}=\frac{1}{f}-\frac{1}{f}=0$,

i.e. $v=\infty$, the image is at ∞.

(*vi*) *Object between* F *and pole, i.e. mirror.* Here $u<f$.

$\therefore$ $\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$, $\frac{1}{v}$ is negative. $\therefore$ v is negative, *i.e.* the image is behind the mirror.

24. Conjugate Foci Relation with the Principal Focus as the the origin (Newton's Formula) :—

We have from the general conjugate foci relation,

$$\frac{f}{v}+\frac{f}{u}=1, \text{ or, } uv=uf+vf,$$

or, $uv-uf-vf+f^2=f^2$, or, $u(v-f)-f(v-f)=f^2$;

or, $(u-f)(v-f)=f^2$ ... ... (1)

So, if the distance of the object and its image due to a spherical mirror are measured from the principal focus instead of from the pole, and are represented by U and V respectively, then $U=(u-f)$ and $V=(u-f)$; so equation (1) becomes, $UV=f^2$ ... ... (2)

Here f^2 is always positive, whether f is positive or negative, so U and V must be of the same sign, *i.e.*, they are on the same side of the principal focus.

25. Experimental Methods of finding the Radius of Curvature and Focal length of Concave and Convex Mirrors :—

(1) CONCAVE MIRROR

(a) u-v Method.—Mount the mirror M on a stand S placed on a table (Fig. 32) and place a lighted candle A in front of it so that the tip of the flame is on the axis of the mirror. Inverted image C of the flame is received on the screen B on the same side as the object, when the object is placed at a distance greater than the focal length. Move the screen until the image is quite distinct. Measure the distances of the flame and its image from the mirror. Use the equation,

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{r}=\frac{1}{f},$$

to find r or f. The experiment may be better performed in a dark room on an *optical bench.*

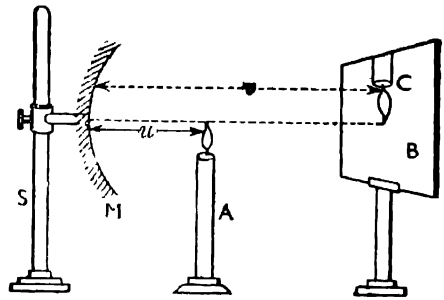


Fig. 32—u-v Method.

(A pin suitably mounted, can also be used as the object and another pin may be placed so that it coincides with the image and there is no parallax between the two, *i.e.* the image and the second pin move together on moving the eye side to side. In this case no dark room is necessary).

(b) Radius of Curvature and Focal Length from the Graph.—

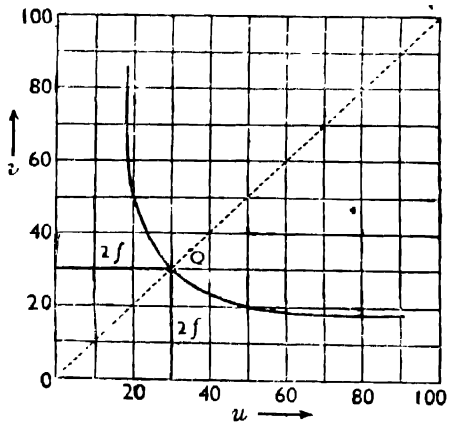


Fig. 33

(1) Determine at least 5 or 6 values of u and their corresponding values of v by the above method.

Draw a graph of u and v [Fig. 33] choosing the same scale for u and v.

From the graph of u and v, which is a rectangular hyperbola we can find r and f; for when $v=u$,

from relation, $\frac{1}{v}+\frac{1}{u}=\frac{2}{r}=\frac{1}{f}$

we have, $\frac{2}{v}=\frac{2}{u}=\frac{2}{r}=\frac{1}{f}$;

or, $u=v=r=2f$.

On the curve the point Q, for which $v=u$, represents the point of intersection of the curve and a straight line bisecting the angle between the axes at the origin. This bisecting line is inclined at an angle 45° with either of the co-ordinate axes. Thus drawing a line OQ making 45° with either of the axes, and finding out the co-ordinates of the point of intersection of OQ with the curve, the value of r and f can be found. The co-ordinates of this intersection point are,

$$\left.\begin{matrix} u \\ \text{and} \\ v \end{matrix}\right\} = r = 2f.$$

(*ii*) Determine a number of u's and v's by the method (*a*) and mark off v's along the Y-axis and u's along the X-axis on a graph-paper on the same scale. The corresponding points on the axes are then joined by straight lines (1), (2) etc., as shown in Fig. 34. All these straight lines intersect at a single point P both the co-ordinates of which are found to be equal to $f=\frac{r}{2}$.

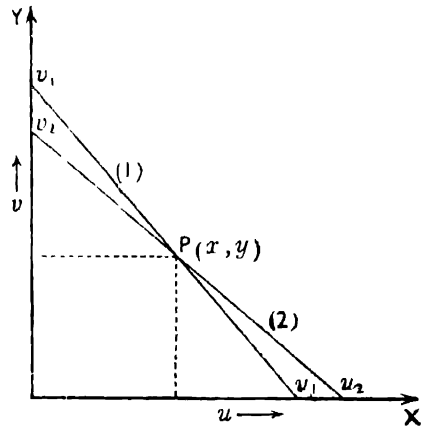


Fig. 34

Proof:—Let x and y be the co-ordinates of P. Then the equation of any straight line passing through P would be expressed as,

$y=mx+C$, where m is the gradient of the line and C is the intercept made by the line on the Y-axis.

For the line v_1u_1 [*i.e.*, (1)]. $y=-\frac{v_1}{u_1}x+v_1$ ... (1)

[the gradient of the line, $\left(\frac{v_1}{u_1}\right)$, being negative]

and for the line v_2u_2 [*i.e.*, (2)], $y=-\frac{v_2}{u_2}x+v_2$ ... (2)

Again, $\frac{1}{f}=\frac{1}{v_1}+\frac{1}{u_1}$, or, $\frac{v_1}{f}=1+\frac{v_1}{u_1}$; or, $\frac{v_1}{u_1}=\frac{v_1-f}{f}$.

and similarly, $\frac{v_2}{u_2} = \frac{v_2 - f}{f}$. Substituting in (1) and (2), we have,

$$y = \frac{f - v_1}{f} x + v_1 \quad \ldots \quad \ldots \quad (3)$$

and, $y = \frac{f - v_2}{f} x + v_2 \quad \ldots \quad \ldots \quad (4)$

$\therefore$ From (3) and (4), (by subtraction),

$$x\left(\frac{f - v_1}{f} - \frac{f - v_2}{f}\right) = (v_2 - v_1) \quad \text{Hence, } x = f.$$

Substituting this value in (3), we get, $y = f$ also. Thus the co-ordinates of P give the focal length of the mirror.

(c) **Parallax Method or the Method of Self-Conjugate point.**— In experiment (*a*) above, using two pins, one as object and the other for location of image, after getting the inverted image move the object pin in such a way that the image and the object become coincident in position, no matter from whatever position they are viewed, that is, find the position for *no parallax* (Fig. 35). This is possible only when the object is placed at the centre of curvature. Hence, the distance of the object from the mirror is the radius of curvature of the mirror, and one-half of this value will be the focal length.

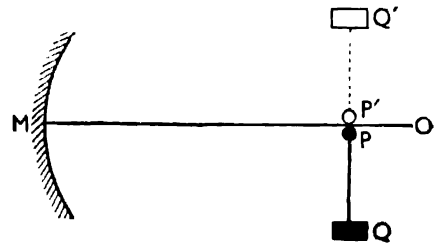


Fig. 35

(d) **Spherometer Method.**—A spherometer is taken and its least count is first determined. It is then placed on a planed glass plate and the central leg of the spherometer is adjusted so as to touch the plate. The reading of the instrument is noted. The spherometer is then placed on the concave mirror and its central leg is gradually lowered so as to touch the surface of the mirror. The reading of the spherometer is again noted. The difference between these two readings gives the depression, h, of the concave surface. Now an impression of the three outer legs of the spherometer is taken on a piece of paper and from that the mean distance between two legs is determined with the help of a divider and a diagonal scale. Let this distance be a. Then the radius of curvature of the mirror is given by,

$$r = \frac{a^2}{6h} + \frac{h}{2}, \text{ and } f = \frac{r}{2}.$$

(2) CONVEX MIRROR

(a) u-v method with the help of a plane mirror :—A long pin $P_1 P_2$ is placed in front of a convex mirror A (Fig. 36) such that it equally extends both above and below the axis of A. An erect virtual image diminished in size is seen by an observer with his eye placed along the axis. Now a plane mirror M is held vertically in front of the convex mirror so that its upper end reaches the axis of the spherical mirror. A virtual image of the lower half of the pin is formed as far behind M as the pin is in front of it. By adjusting the position M this image Q_2 is made to coincide exactly with the image Q_1 of the upper half of the pin as formed by the convex mirror. This position of coincidence is to be checked by the method of parallax. When exact coincidence of Q_1 and Q_2 is secured, we have,

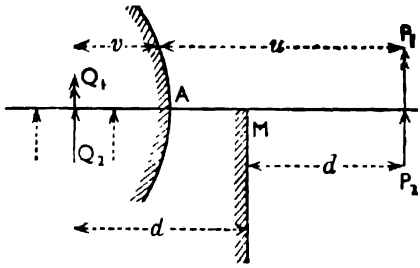


Fig. 36

$$AP_1 = u, \text{ and } AQ_1 = v.$$

But as $MQ_2 = MP_2 = d$, we have, $v = 2d - u$

Thus knowing u and v, f can be determined from the conjugate foci relation, $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$.

(b) Convex Lens Method.—Take a convex lens L of focal length shorter than the radius of curvature of the convex mirror M and placed it in front of the mirror (Fig 37). Then put a pin PQ in front of the lens at a distance greater than the focal length of the lens, when a real image of the pin will be formed on the same side of the lens after being reflected back by the mirror. Now adjust the position of the object pin till the image coincides with the object and there is *no parallax* between them. For this condition the rays of light from P, after refraction through L, must fall normally on the mirror, when they will be reflected back along the same path, and this would happen when the distance MO is the radius of curvature of the mirror.

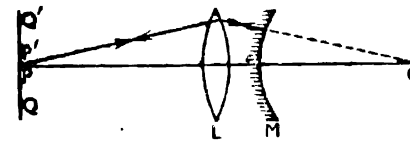


Fig. 37

Thus after proper adjustment is made, measure the distance LM, keeping everything undisturbed, and then remove the mirror.

Now take another pin and locate, by the method of parallax, the position O of the real and inverted image of PO formed by the lens alone without disturbing the position of the object PQ and the lens L. Therefore, the radius of curvature of M,

$$=(LO-LM)=MO=2\times \text{focal length of } M.$$

Take several readings as above and take the mean value as the correct focal length.

(c) **By Spherometer**—The method is exactly similar to method (*d*) used in the case of the concave mirror. Here the spherometer measures the elevation of the convex surface instead of the depression in the former case. If h be this elevation and a the mean distance between the outer legs of the spherometer,

$$r=\frac{a^2}{6h}+\frac{h}{2}, \text{ and } f=\frac{r}{2}.$$

26. Distinction between Mirrors:—A plane, a concave and convex mirror can be distinguished from each other by the *images* formed by them.

(1) **Plane Mirror.**—A plane mirror gives an *erect* image of the *same size* as the object situated as far behind the mirror as the object is in front. Our *looking glasses* are plane mirrors. A good looking glass should be of *uniform thickness* and have a *plane surface*. The *silvering* at the back should also be good. Costly mirrors are sometimes made by silvering the front face. Ordinary cheap looking glasses are neither plane nor of uniform thickness.

If the surface is not plane, concavity or convexity of different amounts, though small, will be present at different parts. As a result, different magnifications will be caused at different parts of the image which will thus appear distorted.

If the thickness is non-uniform, the angle between any two reflected rays will not be the same as between the relevant incident rays. As a result, the image and the object will not look alike.

If the silvering at the back is good, the image formed by rays, reflected at the back, will be much brighter than the faint image formed by reflection at the front face and there will be no confusion resulting from multiple images. Due to defects of silvering and use of low quality glass, ordinary cheap mirrors give unsatisfactory images.

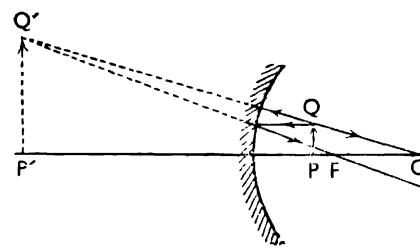


Fig. 38

(2) **Concave Mirror.**—A concave mirror gives a *magnified, erect* image (*virtual*), if it is brought very near to the eye (Fig. 38); other-

wise, an *inverted* image is formed, if the observer PQ stands farther from the mirror than the focus F.

(3) **Convex Mirror**—A convex mirror always gives a virtual, diminished, erect image situated within the focal length for all positions of the object.

27. **Mirror of Large Aperture and Spherical Aberration**:—So far we have confined our attention almost entirely to the paraxial rays. But the conditions of paraxial nature of the rays are rarely satisfied in many instruments *e.g.* in the cases of the 100-inch and the 200-inch reflectors at Mount Wilson and Palomar. In big aperture reflectors the paraxial conditions are definitely not satisfied by the rays which are incident at the edge or the peripheral region of the mirrors. For preliminary discussion of the effects of a large aperture, let us consider a point object at infinity on the axis of a large aperture mirror. The incident rays will all be parallel to the principal axis and after reflection from different regions of the mirror, they will intersect the axis at different points. Only the paraxial rays will intersect at the principal focus, F, of mirror. The marginal rays after reflection are brought to a focus at the point F_1 on this axis, the intersecting points of the intermediate rays with the axis, lying in between F and F_1. Thus the focus of the mirror in this case is not a fixed point on the axis. It rather shifts its position from F to F_1 as the rays gradually shift from the paraxial to the marginal region. This shift or wandering of the focus along the principal axis is called *longitudinal spherical aberration*. As stated above all the reflected rays do not come to a point focus at all, but there is a surface of maximum concentration of rays and of maximum illumination.

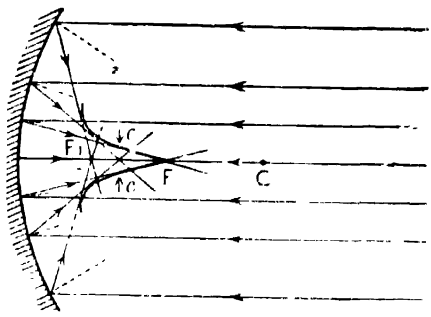


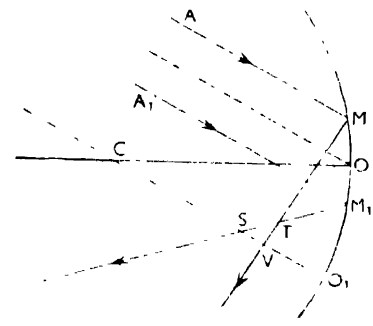
Fig. 39

If a curve is drawn such that the reflected rays are all tangent to it, the illumination will be maximum along this curve. This is known as the **Caustic** or the **Caustic Curve.**

Since the rays are incident all around the principal axis, the maximum illumination surface is obtained in the form of a conical surface by revolving the Caustic Curve about the axis. This surface is known as the **Caustic surface.** The

apex of this conical caustic surface is known as the *cusp*, which lies at the paraxial focus of the mirror. Now if a screen be placed at F, the image will be a circular patch of light with a bright centre the edge being of gradually faded intensity. Again if the screen be placed at F_1, the circular patch of light will have brighter rim but a fainter centre. But if the screen be gradually shifted from F to F_1, at about the position CC (shown by the arrow marks in Fig. 39) an almost uniformly illuminated circular patch will be obtained. This signifies the position of best image and the circular patch obtained here is called the *circle of least confusion*. The radius of this circle is a measure of the *lateral or transverse spherical aberration*. Thus we find that with a large aperture mirror even the image of a point object at infinity will spread over definite region and would no more be a point on the axis. This defect may however be removed by a few suitable methods as will be mentioned later in this chapter.

28. Image formed by an oblique-centric pencil on a spherical mirror : Astigmatism :—Whenever a pencil of rays falls on a mirror or lens such that the central ray of the pencil makes a sufficiently large angle with its principal axis, the pencil is called an oblique-centric pencil. In the cases of such oblique-centric pencils and of eccentric pencils, the paraxial conditions are violated even if a small aperture mirror is used. Let us consider a parallel pencil of rays, whose central ray makes a large angle with the axis, to be incident on a mirror MM_1 [Fig. 40(a)] whose axis, centre of curvature and pole are CO, C and O respectively. The extreme rays AM and A_1M_1 of the pencil fall on the mirror at large angles of incidence and as such our simple relation for the conjugate foci is no longer valid. The portion of the mirror MM_1 in this case may be considered to be a small marginal portion of a large aperture mirror of principal axis CO_1, parallel to AM. Here the marginal ray AM will, after reflection, cut the axis CO_1 at V, a point closer to the pole O_1, than the other parallel ray A_1M_1, which intersects the axis at S. The two reflected rays MV and M_1S intersect at the point T. Actually the pencil of rays extends in three dimensions and so also does the reflecting surface MM_1. So to have a true state of affairs, if the figure is slightly rotated about CO_1, the point T describes a line TR [Fig. 40 (b)] perpendicular to the plane of the paper. The



(a)

Fig. 40

reflected rays first concentrate along this line TR and then they again are focussed along a second line SV in the plane of the paper. Fig. 40(*b*) shows how the reflected rays are actually concentrated along two lines after reflection from the strip of mirror $M'M$. These two lines TR and SV are known as the *focal lines* and are respectively called the **tangential (or first)** and the **sagittal (or second) focal lines.** Thus we see that an incident oblique-centric pencil, after reflection, cannot be brought to a point focus but

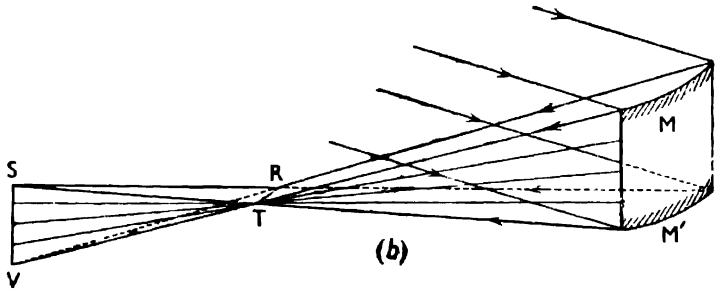


Fig. 40

are spread out along two focal lines separated by a distance. This phenomenon shown by an oblique pencil of rays wherein it cannot be concentrated to a single point focus is known as **astigmatism** (a—not ; stigma—point). When a screen is placed at T at right angle to the plane of the paper, a sharply illuminated line TR is visible. At SV again a sharp line in the plane of paper is seen. In between these two positions the reflected cone of rays forms a circle at right angles to the direction of propagation, wherein the intensity is almost uniform. This circle is the **circle of least confusion** and gives the best position for an uniformly illuminated image of the source.

From the above discussions it is clear that for a large aperture of the mirror spherical aberration creeps in while for an obliquely incident beam astigmatism arises. But actually for a wide angle beam both these defects simultaneously occur. So the image of an extended object near a mirror of large aperture suffers from both these defects, the result being a curved and blurred image.

29. Removal of the above defects :—The above defects in a spherical reflecting surface cannot be completely removed. They may, however, be reduced to a practical minimum by suitable arrangements. These arrangements generally consist in limiting only a small portion of the mirror near its pole to be effective in casting an image and also reducing the effective angles of incidence of the beam by putting a suitable stop of small diameter

at the centre of curvature of the mirror. This stop improves the definition of the image but itself introduces a curvature in the image. But if the dimension of the stop be very small this curvature may be neglected for all practical purposes.

The best way to remove the defects in a concave reflector is to utilise the property of an *aplanatic surface*. Normally a wide aperture reflecting or refracting surface cannot cast a point image of a point object for any relative positions on the axis. But surfaces can always be so constructed that for a particular position of a point object on the axis an exact point image is formed after reflection or refraction. Such surfaces are called **aplanatic surfaces** and the object and image point as the **aplanatic foci.** An ellipsoidal reflecting surface (obtained by the revolution of an ellipse about its major axis) behaves as a perfectly aplanatic surface for the object placed at one of its foci. If a point object is placed at S (Fig. 41), the rays starting from it after reflection from the mirror will be brought to a focus at S_1, which is the second focus of the ellipsoid. So S and S_1 are the aplanatic points here.

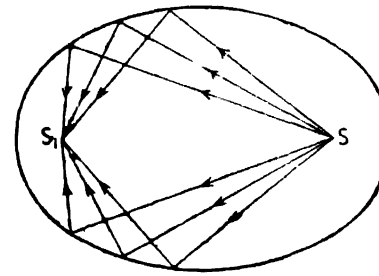


Fig. 41

This follows from the simple property of an ellipse wherein we know that the lines joining the two foci of an ellipse and any point on its surface are equally inclined to the tangent drawn to the ellipse at that point of the surface.

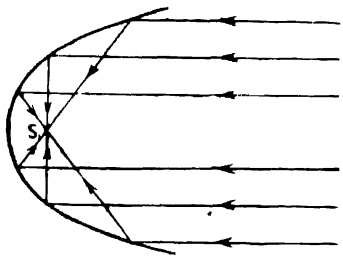


Fig. 42

Paraboloidal mirrors are also aplanatic surfaces when one of the aplanatic points is taken at infinity and the other at the focus of the parabola.

Thus when a beam of parallel ray falls on a paraboloidal mirror, it will evidently be concentrated at its focus (Fig. 42). Conversely if a source of light is placed at the focus of the paraboloidal mirror, the rays after reflection will be exactly parallel. This property is utilised in paraboloidal reflectors used in search-lights of steam ships and in head lights of motor cars.

The spherical concave reflectors may be considered to be aplanatic for a very small region near its pole, where this surface is practically coincident with an ellipsoidal surface. A plane reflector is also aplanatic

because here a point object always produces a point image—though virtual in nature.

30. Uses of concave and convex mirrors:—A *concave mirror* is sometimes used as a **shaving glass**. A virtual, magnified image of the face is produced when the mirror is brought near it, the distance being less than the focal length.

Sometimes surgeons, while examining the internal parts of the ear, nose, or throat, use a small *concave mirror* to throw a narrow, but sharp beam of light into the affected parts. A small concave mirror with a small hole in the centre is used in the doctor's **Opthalmoscope**.

Concave Mirrors consisting of bright surfaces are very often used behind a source of light as reflectors. *Concave paraboloidal mirrors* have uses in search-lights of steamers and also in the head lights of motor cars as mentioned in the previous article.

In the case of a *convex mirror* a larger field of view is obtained than in the case of other types of mirrors. So convex mirrors are used as *view finders* in automobiles. Such a mirror is mounted in front of the driver projecting slightly outside the car. All objects behind the car cast virtual, erect and diminished images within the focal length of the mirror. The driver can easily see what is happening at the rear by simply looking into the mirror. It covers a wide field of view. Street lights are also provided with convex reflectors so that light can be spread over an extended area by reflection.

Cylindrical Mirrors are used for window advertisements in shops because of the characteristic distortions they produce in the image of a body.

30(a). Instruction for Solution of Problems:—In solving numerical examples follow the rules given below :—

(*a*) *First put down the data of the example with their proper signs.* (*b*) *In the general formula of mirrors, substitute the numerical values of u, v, etc. with their proper signs. Do not change the sign of any of the distances whose numerical values are not given. Then solve the equation and draw the conclusion from the sign of the distance, or magnification, as the case may be.*

Conclusion.—(*i*) *The positive sign of the focal length (or radius of curvature) will mean that the mirror is concave, while the negative sign will mean that the mirror is convex.*

(*ii*) *If the value of* **m** *is negative, the image is real and inverted and is formed in front of the mirror, and if* **m** *is positive, the image is erect and virtual and is formed behind the mirror.*

Examples

1. The sun subtends an angle of half a degree at the pole of a concave mirror which has a radius of curvature of 15 metres. Find the size of the image of the sun formed by the concave mirror. [Patna U. 1943]

Ans. Here u = distance of sun, v = image distance $= 15/2 = 7.5$ metres $= (7.5 \times 100)$ cms. $= v$, the sun being at infinite distance. Now, $\left(\frac{1}{2}\right)^\circ = \frac{1}{2} \times \frac{\pi}{180}$ radians.

Again angle in radian $= \frac{\text{arc}}{\text{radius}} = \frac{\text{diameter of sun}}{\text{distance of sun}} = \frac{\pi}{360}$

But $\frac{\text{diameter of image}}{\text{diameter of sun}} = \frac{v}{u} = \frac{\text{distance of image}}{\text{distance of sun}} = \frac{7.5 \times 100}{\text{distance of sun}}$

$\therefore$ $\frac{\text{Diameter of image}}{7.5 \times 100} = \frac{\text{diameter of sun}}{\text{distance of sun}} = \frac{\pi}{360}$

$\therefore$ Diameter of image $= \frac{7.5 \times 100 \times 22}{360 \times 7} = 6.54$ cms.

2. An object is placed 18 cms away from a concave mirror whose focal length is 10 cms. , find the position and the size of the image, if the object be 4 cms. broad by 12 cms. long.

Ans. Here, $u = 18$ cms. , $f = 10$ cms. , $v = ?$

We have, $\frac{1}{v} + \frac{1}{18} = \frac{1}{10}$, whence $v = 22.5$ cms.

Magnification $= \frac{1}{u} = \frac{45}{2} \div 18 = \frac{5}{4}$. Length of the object $= 1.2$ cm. and breadth $= 0.4$ cm. $\therefore$ Length of the image $= 1.2 \times \frac{5}{4} = 1.5$ cm.

Breadth of the image $= 0.4 \times \frac{5}{4} = 0.5$ cm.

$\therefore$ Size of the image $= (1.5 \text{ cm.} \times 0.5 \text{ cm.}) = 0.75$ sq. cm.

3. The image of a gas flame standing at a distance of 6 ft. from the screen should be magnified three times. Where would you hold the mirror and what sort of mirror would you require?

Ans. Let x ft be the distance of the mirror from the gas-flame, then the distance of the mirror from the screen is $(6+x)$ ft. Here, $u = x$ ft. , $v = (6+x)$ ft.

Because the image is three times the size of the object, we have,

$$\frac{6+x}{x} = 3, \text{ whence } x = 3. \quad \text{So } u = 3 \text{ ft.} ; \ v = 3+6 = 9 \text{ ft.}$$

Hence, $\frac{1}{3} + \frac{1}{9} = \frac{1}{f}$ whence $f = +2.25$ ft.

The positive sign shows that the mirror is a concave one of 2·25 ft. focal length and it should be held 9 ft. from the screen or 3 ft. from the gas-flame. (No real image is possible in the case of a convex mirror).

4. An image produced by a convex mirror of focal length f is $\frac{1}{r}$ th times the size of the object. Show that the distance of the object from the mirror is $(r-1)f$.

Ans. Here focal length $= -f$.

$$m = \frac{1}{r} = -\frac{v}{u}, \text{ or, } v = -\frac{u}{r}, \quad \therefore \quad \text{From } \frac{1}{f} = \frac{1}{v} + \frac{1}{u} \text{ we get}$$

$$-\frac{1}{f} = \frac{-r}{u} + \frac{1}{u} = \frac{-(r-1)}{u}, \text{ or, } \quad u = (r-1)f.$$

5 A linear object 10 cm. long lies along the axis of a concave mirror whose radius of curvature is 30 cm., the near end of the object lying 18 cm. from the mirror. Find the magnification of the image.

Ans. For near end, $f = \frac{30}{2} = 15$ cm. and $u = 18$ cm.

So $\frac{1}{15} = \frac{1}{v} + \frac{1}{18}$. Hence $v = 90$ cm.

For the farther end, $u = 28$ cm.

$\therefore \quad \frac{1}{15} = \frac{1}{v} + \frac{1}{28}$ So, $v = 32{\cdot}3$ cm.

$\therefore$ Length of image $= 90 - 32{\cdot}3 = 57{\cdot}7$ cm. $\quad \therefore \quad m = \frac{57{\cdot}7}{10} = 5{\cdot}77.$

Questions (Chapter II)

1. Prove that if an object in front of a plane mirror moves through a distance '*d*' away from the mirror, the image moves through the same distance; whereas, if the mirror moves parallel to itself through a distance '*d*' (the object remaining fixed), the image will move through a distance $2d$.

2. Calculate the minimum size of a plane mirror fixed on the wall of a room in which an observer at the centre of the room can see the full image of the wall behind him.

3. What is the relation between the turning of a mirror and that of a ray of light reflected from it? Apply this property to the construction of the sextant.

4. Two plane mirrors are inclined at an angle to each other. A ray of light parallel to one of the mirrors travels after two reflections, parallel to the other. Find the angle between the mirrors.

[*Ans.*: 60°]

5. Explain how a number of images is visible when a bright object is held in front of a thick mirror silvered at the back. Illustrate your answer by neat diagram for a given position of the eyes.

6. A horizontal ray of light is successively reflected once at each of two plane vertical mirrors inclined at a fixed angle α. If the mirrors are rotated about the vertical axis coinciding with their line of intersection show that the final direction of the ray is unchanged.

7. A ray is reflected in succession from two plane surfaces inclined to each other at a constant angle. Show that the deviation of the ray is independent of the angle of incidence on the first surface.

8. If two mirros be inclined to each other at an angle of 72°, find the number of images formed of an object placed midway between them.

[*Ans.*: 4]

9. Explain "Conjugate Foci" as applied to a concave mirror. Describe a method of determining the focal length of a concave mirror by finding the distances of the conjugate foci. If these distances are 5" and 10", calculate the focal length.

[*Ans.* . 10/3 inches]

10. Explain how with the help of the laws of reflection the position and size of an image formed by reflection at a convex surface can be determined geometrically.

11. An object is at a distance of 10 cms. from a mirror and the image of the object is at a distance of 30 cms. from the mirror on the same side as the object. Is the mirror concave or convex ? What is the focal length ?

[*Ans.* . $f=7{\cdot}5$ cm.]

12. x and y are the distances of an object and its image form the focus of a spherical mirror. Show that $xy=f^2$, where f is the focal length of the mirror.

13. The driver of a motor-car is supplied with a convex mirror in order that he may see the roadway behind him. Explain how he is able to do this. Will a plane mirror serve the purpose equally well ?

14. Show that an object at a distance x from a concave spherical mirror of radius r produces an image at a distance y given by $y=\frac{rx}{2x-r}$.

15. Explain clearly how spherical aberration and astigmatism arise due of reflection from a spherical mirror of large aperture.

16. What do you mean by aplanatic points and an aplanatic surface ? What are the advantages of a paraboloidal mirror over a sperical one ?

17. A convex and a concave mirror of radii 10 cm. each are placed facing each other and 15 cm. apart. An object is placed midway between them. Find the position of the final image if the reflection first takes place in the concave and then in the convex mirror. (Pat. U. 1930)

[*Ans.* : At the pole of the convex mirror.]

31. Refraction of light :—When a ray of light travels in the same homogeneous medium, it travels straight, but when it passes *from one optical medium to another*, it suffers a change of direction at the surface of separation between the two media and is said to be *refracted*, except when the ray is normally incident. If the second medium be optically **denser** than the first, the ray is bent in the second medium **towards** the normal at the point of incidence, and if **rarer** than the first, the ray is bent **away** from the normal. *This phenomenon is known as* **refraction** of light.

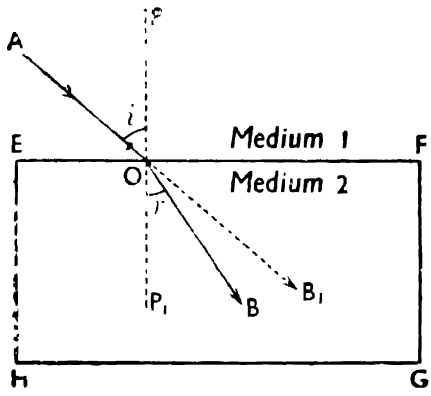


Fig 43

The path of the ray in the first medium is called the **incident ray**, and that in the second medium is called the **refracted ray**. In Fig 43, *AO* is the incident ray, *OB* is the corresponding refracted ray, and, because medium 2 (glass) is denser than medium 1 (air), the refracted ray *OB* is bent towards the normal OP_1.

32. The Laws of Refraction :—

1. The incident ray, the refracted ray, and the normal to the refracting surface at the point of incidence all lie in one plane.

2. The sine of the angle of incidence bears to the sine of the angle of refraction a constant ratio for the same two media for the same colour of light.

The second law was formulated by Snell, and is known as **Snell's Law**, or simply **the Law of Sines.**

33. Refractive Index :—When a ray of light passes from a medium '*a*' into another medium '*b*', the ratio of the sine of the angle of incidence '*i*' to the sine of the angle of refraction '*r*' is a constant whose value depends only on the colour of the light and nature of the two media. This constant is called the refractive index of medium '*b*' relative to medium '*a*' and is commonly denoted by $_a\mu_b$.

That is, $_a\mu_b = \frac{\sin i}{\sin r}$. It is also often called the relative index of refraction.

When the first medium 'a' is vacuum, *i.e.*, the ray travels from vacuum to medium 'b', the value of the constant is maximum, and is called the **absolute index of refraction** of medium 'b'.

It is to be noted, as already pointed out, that the refractive index ($R.I$) depends on the nature of the media and the colour of the light only. It is independent of the angle of incidence of the light and its value increases from the red end of the spectrum to the violet. When no mention of the colour is made in giving the value of the $R.I.$, it is to be taken for the mean colour, *i.e.* yellow. The $R.I.$ of a material, unless otherwise mentioned, is to be taken as relative to air.

34 Verification of Laws of Refraction by Hartle's Optical Disc :—A semi-circular glass plate CED is held at the centre of the disc with its plane face CD lying along the $90°-90°$ diameter of the optical disc in such a way that the $0°-0°$ diameter passes normally through the centre O of the plate (Fig. 44). A narrow pencil of light AO is adjusted through the slit in the screen S to trace its path along the disc and made incident at the centre O of the plate obliquely. The refracted pencil being radial meets the curved face, CED, normally and passes out undeviated as OB. The angle of incidence AON and the angle of refraction BON_1 are read off directly from the graduations of the circular scale on the disc. This angle of incidence is then changed by rotating the disc keeping the screen S fixed, and the corresponding angle of refraction is again found out. The operation is repeated for different angles of incidence. The ratio of the sine of the angle of incidence to the sine of the angle of refraction is found to be the same in each case. This proves the Second Law.

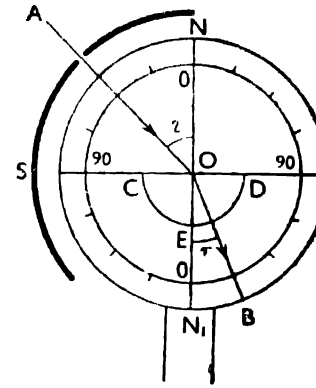


Fig 44

The incident ray, the normal at the point of incidence ($0°-0°$ diameter), and the refracted ray all lie on the plane of the disc. *i.e.* in the same plane. This proves the First Law.

35. Cause of Refraction :—According to the wave theory of light the $R.I.$ (μ) of one medium 'b' relative another 'a' is given by,

$$_a\mu_b = \frac{\text{velocity of light of medium '}a\text{'}}{\text{velocity of light in medium '}b\text{'}} = \frac{v_a}{v_b}$$

If these velocities are equal, $R.I.$ will be equal to unity, *i.e.* there will be no refraction or bending of the light at the interface between the two media. If in medium 'b' the velocity is less, $R.I.$ will

be greater than unity, as in the case of glass relative to air, *i.e.* the light will bend towards the normal drawn at the point of incidence in passing from air to glass. All these have been verified experimentally and, therefore, the cause of refraction may be thought of as due to the velocity of light being different in different media.

36. Refractive Index and Consequence of Reversibility of Light :—In Fig. 43, a ray AO travelling in a medium (1) is refracted along OB in a second medium (2). Since the path of light is reversible, a ray travelling along BO in the second medium will refract along OA in the first medium.

So it follows that if ${}_1\mu_2$ denotes the index of refraction of medium '2' with respect to medium '1', and ${}_2\mu_1$ that of '1' with respect to '2', $\frac{\sin i}{\sin r} = {}_1\mu_2$ and $\frac{\sin r}{\sin i} = {}_2\mu_1$. $\therefore \quad {}_2\mu_1 = \frac{1}{{}_1\mu_2}$

Hence, if medium '1' be air and '2' be glass, and if ${}_1\mu_2$ represents the refractive index of glass with respect to air, the refractive index of air with respect to glass is given by $\frac{1}{{}_1\mu_2}$. Thus the refractive index of air relative to glass will be $\frac{2}{3}$, if that of glass to air is $\frac{3}{2}$.

Therefore, *the R.I. of medium 'a' relative to medium 'b' is the reciprocal of the R.I. of medium 'b' relative to medium 'a'.*

37. Refraction through a Plate with parallel Faces :—

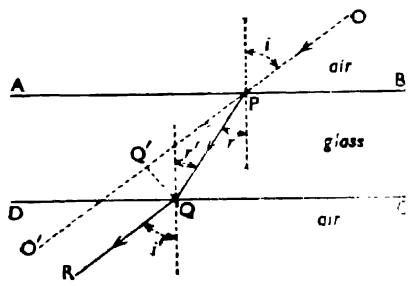


Fig. 45

Let AB, DC be parallel faces of a glass plate, and let OP be a ray incident at the point P making an angle i with the normal at P, and PQ the corresponding refracted ray in glass (Fig. 45). It is bent towards the normal having r as the angle of refraction. The ray PQ emerges out into air in the direction QR bending away from the normal at Q.

We have, from Fig. 45, $\frac{\sin i}{\sin r} = {}_a\mu_g$ ... ... (1)

Again, for refraction at the second face, $\frac{\sin r'}{\sin i'} = {}_g\mu_a$ ... (2)

But $\frac{\sin r'}{\sin i'} = 1 \Big/ \frac{\sin i'}{\sin r'} = 1 \Big/ {}_a\mu_g$ ... ... (3)

i.e. $\frac{\sin i'}{\sin r'} = {}_a\mu_g$. Hence, from (1) and (3), $\frac{\sin i}{\sin r} = \frac{\sin i'}{\sin r'}$.

But the two normals are parallel to each other, as the sides AB and DC are parallel ; so the angle r = the angle r'

Hence $\sin i = \sin i'$; or, $i = i'$, or, QR is parallel to OP.

Thus, *a ray of light in passing through a parallel-faced plate emerges* **parallel** *to the original direction having suffered a lateral displacement only.*

38. Lateral Displacement of an oblique ray due to refraction through a parallel-faced transparent plate :—In Fig. 45 the emergent ray QR is parallel to the incident ray OP and it is not deviated by the parallel-sided block but is only *displaced* sideways. This sideways shifting is known as the *lateral displacement* whose magnitude is measured by the perpendicular distance QQ' between the path of the incident and the emergent rays. If i and r be the angles of incidence and refraction, t the thickness of the plate and μ the refractive index of the material of the plate then,

$QQ' = PQ \sin QPQ' = PQ \sin (i - r)$. But $PQ \cos r = t$.

Hence $QQ' = t \sin (i-r)/\cos r = t (\sin i \cos r - \cos i \sin r)/\cos r = t [\sin i - \cos i \sin r/\cos r]$ ... ... (1)

We know, $\sin r = (\sin i)/\mu$.

$$\therefore \quad \sin r/\cos r = \frac{\sin i}{\mu} \Big/ \left(1 - \frac{\sin^2 i}{\mu^2}\right)^{\frac{1}{2}} = \frac{\sin i}{(\mu^2 - \sin^2 i)^{\frac{1}{2}}}.$$

Substituting in (1), $QQ' = t \sin i \left[1 - \dfrac{\cos i}{(\mu^2 - \sin^2 i)^{\frac{1}{2}}}\right]$. (2)

Thus from eqn. (2), we see that the amount of lateral displacement depends on thickness t, angle of incidence i, and the refractive index μ.

39. Passage of a Ray through several Media :—If instead of two media, we have three media (1), (2), (3) and the last medium be again (1) (Fig. 46). then an incident ray OP after passing through these media will emerge out parallel in medium (1) as RS, the refracting faces of these media being supposed to be parallel to one another. If ϕ_1, ϕ_2, ϕ_3, be the angles of incidence at the successive interfaces and ϕ_2, ϕ_3, ϕ_1, the corresponding angles of refraction as shown in the figure, then,

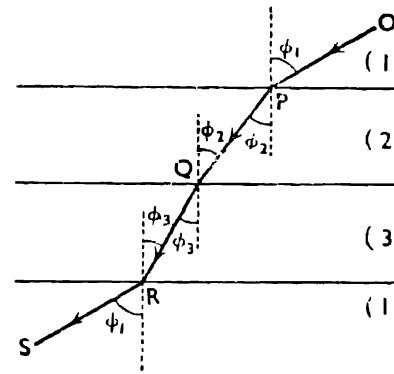


Fig. 46

$$ {}_1\mu_2 = \frac{\sin\phi_1}{\sin\phi_2}, \quad {}_2\mu_3 = \frac{\sin\phi_2}{\sin\phi_3}, \text{ and } {}_3\mu_1 = \frac{\sin\phi_3}{\sin\phi_1}. $$

$$ \therefore \quad {}_1\mu_2 \times {}_2\mu_3 \times {}_3\mu_1 = \frac{\sin\phi_1}{\sin\phi_2} \times \frac{\sin\phi_2}{\sin\phi_3} \times \frac{\sin\phi_3}{\sin\phi_1} = 1 \qquad (A) $$

Generalising for n media, we may write,

$$ {}_1\mu_2 \times {}_2\mu_3 \times \ldots\ldots \times {}_n\mu_1 = 1 \qquad \ldots \qquad (B) $$

Generalised form of Snell's law :—

If we consider medium (1) in Fig. 46 to be vacuum, then from relation (A), ${}_v\mu_2 \times {}_2\mu_3 \times {}_3\mu_v = 1$, where subscript v refers to vacuum.

Hence $${}_2\mu_3 = \frac{1}{{}_v\mu_2} \times \frac{1}{{}_3\mu_v} = \frac{1}{{}_v\mu_2} \times {}_v\mu_3 = \frac{\mu_3}{\mu_2},$$

where ${}_v\mu_3 = \mu_3$ and ${}_v\mu_2 = \mu_2$ are the absolute refractive indices of media (3) and (2).

Again, $${}_2\mu_3 = \frac{\sin\phi_2}{\sin\phi_3}.$$

$$ \therefore \quad {}_2\mu_3 = \frac{\mu_3}{\mu_2} = \frac{\sin\phi_2}{\sin\phi_3}; $$

or, $$\mu_2 \sin\phi_2 = \mu_3 \sin\phi_3.$$

Generalising for n media,

$$\mu_1 \sin\phi_1 = \mu_2 \sin\phi_2 = \ldots = \mu_n \sin\phi_n.$$

This is known as the generalised Snell's law, which states that the product of the absolute index of refraction and the sine of the angle made by the ray with the normal in a particular medium is constant when the ray is refracted from one medium to another.

40. Deviation of a Refracted Ray :—

The angle of deviation of a refracted ray is the angle through which the refracted ray has been deviated from the original

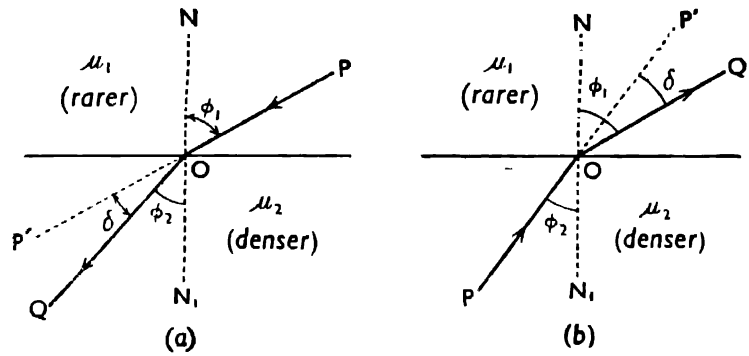


Fig 47

direction of incidence. In Fig. 47 (*a*), the ray *PO* passes from a

rarer to a denser medium. So here the refracted ray OQ is *deviated towards the normal.* The angle of deviation, $\delta = P'OQ = (\phi_1 - \phi_2)$.,

In Fig. 47(*b*), the ray PO passes from a denser to a rarer medium. Hence after refraction, the ray OQ is *deviated away from the normal.* If ϕ_1 and ϕ_2 be the angles the ray makes with the normal in the denser and the rarer media, then, here also the deviation is given by the angle, $\delta = (\phi_1 - \phi_2)$.

Variation of Deviation with angle of incidence :

Whenever a ray suffers refraction at the interface of two different media and makes angles, ϕ_1 and ϕ_2 with the normal in the two media of refractive indices μ_1 and μ_2 (Fig. 47), the deviation of the ray is given by, $\delta = \phi_1 - \phi_2$.

$$\text{But } \frac{\sin \phi_1}{\sin \phi_2} = \frac{\mu_2}{\mu_1}; \text{ or, } \frac{\mu_2 + \mu_1}{\mu_2 - \mu_1} = \frac{\sin \phi_1 + \sin \phi_2}{\sin \phi_1 - \sin \phi_2}$$

$$= \frac{2 \sin \frac{1}{2}(\phi_1 + \phi_2) \cos \frac{1}{2}(\phi_1 - \phi_2)}{2 \cos \frac{1}{2}(\phi_1 + \phi_2) \sin \frac{1}{2}(\phi_1 - \phi_2)} = \frac{\tan \frac{1}{2}(\phi_1 + \phi_2)}{\tan \delta/2}$$

$$(\because \quad \delta = \phi_1 - \phi_2).$$

$$\therefore \quad \tan \delta/2 = \frac{\mu_2 - \mu_1}{\mu_2 + \mu_1} \tan \tfrac{1}{2}(\phi_1 + \phi_2).$$

When ϕ_1 increases, ϕ_2 also increases and so does $\tan \frac{1}{2}(\phi_1 + \phi_2)$. Thus with increase of ϕ_1, $\tan \delta/2$ increases.

So δ also increases with the angle of incidence ϕ_1.

[Alternative method (by calculus) :—Since $\delta = \phi_1 - \phi_2$ its rate of variation with the angle of incidence ϕ_1 is given by,

$$\frac{d\delta}{d\phi_1} = 1 - \frac{d\phi_2}{d\phi_1} \qquad \ldots \qquad \ldots \qquad (1)$$

But, $\mu_1 \sin \phi_1 = \mu_2 \sin \phi_2$.

Differentiating this with respect to ϕ_1, we get,

$$\mu_1 \cos \phi_1 = \mu_2 \cos \phi_2 \frac{d\phi_2}{d\phi_1};$$

$$\text{or, } \frac{d\phi_2}{d\phi_1} = \frac{\mu_1 \cos \phi_1}{\mu_1 \cos \phi_2}$$

Substituting this value of eqn. (1),

$$\frac{d\delta}{d\phi_1} = 1 - \frac{\mu_1 \cos \phi_1}{\mu_2 \cos \phi_2} \qquad \ldots \qquad \ldots \qquad (2)$$

Now, since $\phi_1 > \phi_2$, $\cos \phi_2 > \cos \phi_1$.

Again μ_2 being greater than μ_1, $\mu_1 \cos \phi_1$ is less than $\mu_2 \cos \phi_2$.

$$\therefore \quad \frac{\mu_1 \cos \phi_1}{\mu_2 \cos \phi_2} \text{ is less than unity.}$$

Thus the right hand side of eqn. (2) is positive.

$\therefore$ $\dfrac{d\delta}{d\phi_1}$ is positive, that is, if ϕ_1 increases δ will also increase.]

Since δ increases with ϕ_1 and $\delta=\phi_1-\phi_2$, it is evident that the rate of increase of ϕ_1 is greater than the corresponding rate of ϕ_2, for if the rate of increase of ϕ_1 and ϕ_2 were identical, δ would not have increased with increase of ϕ_1. Similarly if ϕ_2 increased at a greater rate than ϕ_1, δ could neither increase with ϕ_1. Thus ϕ_1, the angle which the ray makes with the normal in the rarer medium increases at a greater rate than that made by the ray in the denser medium.

41. The Effect of Refraction on the position of an object placed in a denser medium :—Let P be an object placed inside a denser medium of refractive index μ_2 at a depth of $PO_1=u$. When P is viewed obliquely from a rarer medium of refractive index μ_1 in a direction OA, a ray of light from P is incident at O in a direction PO and refracted in the rarer medium along OA. As shown in Fig. 48 the angle of incidence in the denser medium is ϕ_2 and the angle of refraction in the rarer medium is ϕ_1. Since the object is viewed along the direction OA, its position will be apparently raised to Q due to refraction, where Q lies in the line AO produced in the denser medium vertically above P. So the apparent depth of the object or the image distance is $QO_1=v$.

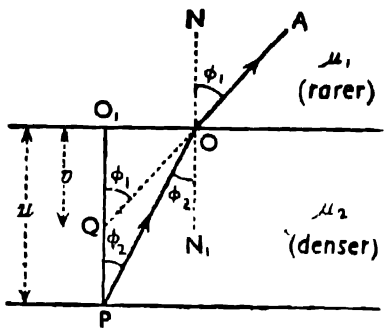


Fig. 48

To establish a relation between the real depth u and the apparent depth v, let us start with Snell's law in its general form, *i.e.*

$$\mu_1 \sin\phi_1=\mu_2 \sin\phi_2 ;$$

$$\text{or,}\quad \frac{\sin\phi_1}{\sin\phi_2}=\frac{\mu_2}{\mu_1} \qquad \ldots \qquad \ldots \qquad (1)$$

With reference to Fig. 48, we get,

$$\frac{OO_1}{O_1P}=\frac{OO_1}{u}=\tan\phi_2 \qquad \ldots \qquad \ldots \qquad (2)$$

$$\text{and}\quad \frac{OO_1}{O_1Q}=\frac{OO_1}{v}=\tan\phi_1 \qquad \ldots \qquad \ldots \qquad (3)$$

$\therefore$ Dividing (3) by (2)

$$\frac{u}{v}=\frac{\tan\phi_1}{\tan\phi_2}=\frac{\sin\phi_1}{\sin\phi_2}\cdot\frac{\cos\phi_2}{\cos\phi_1}=\frac{\mu_2}{\mu_1}\cdot\frac{\cos\phi_2}{\cos\phi_1}$$

As ϕ_1 refers to the angle made by the ray in the rarer medium, ϕ_1 increases at a greater rate than ϕ_2.

$\therefore$ $\cos\phi_1$ decreases more rapidly than $\cos\phi_2$.

$\therefore$ v also decreases as ϕ_1 increases.

So with the increase of the obliquity of vision, the apparent depth of the object in the denser medium will be decreasing, *i.e.* the object will seem to be more and more raised. When the object is viewed normally from the rarer medium, $\phi_1=\phi_2=0$.

$$\therefore \quad \frac{u}{v}=\frac{\mu_2}{\mu_1}.$$

If the rarer medium be air, $\mu_1=1$ and $\mu_2=\mu$, the relative refractive index of the denser medium with respect to air, then

$$\mu=\frac{u}{v}=\frac{\text{real depth}}{\text{apparent depth}}.$$

This relation is often utilised in determining the refractive indices of solids and liquids.

The above treatment is approximate and is valid only when the object is viewed almost normally. For a normal view only PO_1 and QO_1 become identical with PO and QO. When the observer looks obliquely from the rarer medium, the image Q would no more be seen at the same position. Here the refracted rays instead of appearing to come from a point P would appear to come from an envelope QC, called the **Caustic curve** of P (Fig. 49). As the refracted rays are more and more oblique, the position of the image of P on this caustic curve would gradually approach the separating surface of the two media. When the eye of the observer is very close to the separating

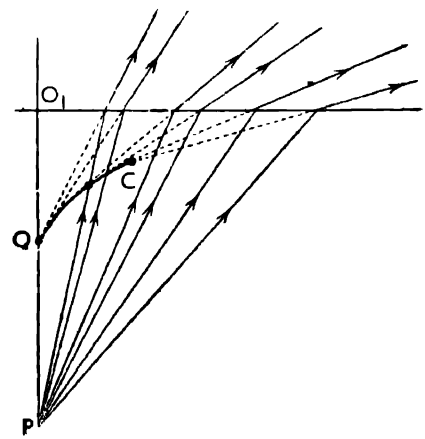


Fig. 49

surface and the view is sufficiently oblique a distorted image of a large object at P will be seen.

42. The case of an object placed in a rarer medium and viewed from a denser medium.— Let P be an objet placed in a rarer medium of refractive index μ_1 at a distance $O_1P = u$ from the surface of separation of two media. If an eye be placed at E in the denser medium, a ray from P in order that it may reach E will follow the path PO in the rarer medium, then get refracted at O along OE and will ultimately reach E. So to the eye at E the object will seem to be situated along EO produced, *i.e.* at Q. Hence $O_1Q = v$ = apparent distance of the object. As evident from Fig. 50, $OO_1 = u \tan \phi_1$ and $OO_1 = v \tan \phi_2$

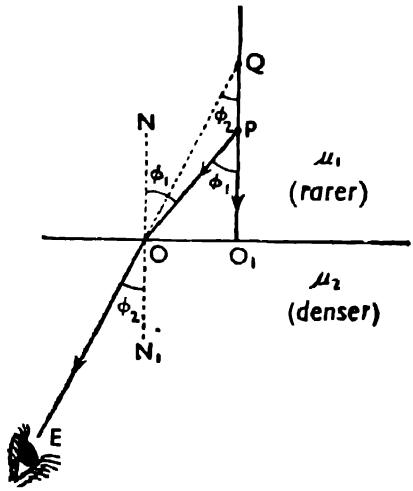


Fig. 50

Hence proceeding exactly in the same way as in the last article, we get,

$$\frac{v}{u} = \frac{\mu_2 \cos \phi_2}{\mu_1 \cos \phi_1}.$$

When viewed normally $\cos \phi_2 = \cos \phi_1 = 1$, and $\frac{v}{u} = \frac{\mu_2}{\mu_1}$.

If $\mu_1 = 1$ (for air) and $\mu_2 = \mu$ = refractive index of the denser medium, we have,

$$\mu = \frac{v}{u} = \frac{\text{apparent distance}}{\text{real distance}}.$$

43. Displacement of the image when viewed normally through a transparent parallel-sided block :—Let $ABCD$ be a transparent parallel-sided block of thickness t and of refractive index μ_2 and let it be placed in a medium of refractive index

μ_1 (Fig. 51). If a point object P be placed at a distance from the separating surface CD of the two media, a ray of light from P proceeds along PP_1 and gets refracted at P_1 into the medium of the block. P_1O_2 is the direction of this refracted ray which again emerges out into the original medium along O_2R such that O_2R is parallel to PP_1. If ϕ_1 and ϕ_2 be the angles subtended by the ray with the normal, in the media of refractive indices μ_1 and μ_2 respectively,

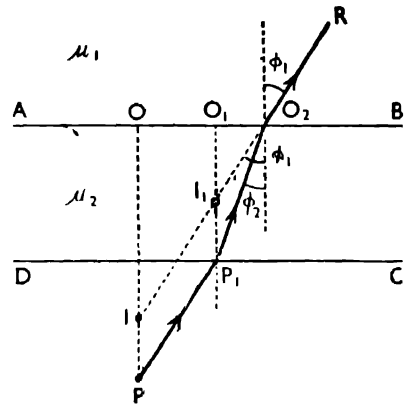


Fig. 51

$$\frac{\mu_1}{\mu_2}=\frac{\sin\phi_2}{\sin\phi_1} \text{ (from Snell's law).}$$

As evident from the figure, the position of the image of P as viewed along O_2R will be I. Hence PI is the displacement of the image. Since the line II_1O_2R is parallel to PP_1 and as OP and O_1P_1 are also parallel being normals to the same surface, the displacement PI is equal to the distance P_1I_1.

Hence the displacement $P_1I_1=O_1P_1-O_1I_1=t-O_1I_1$ (1)

But $\frac{\mu_1}{\mu_2}=\frac{\sin\phi_2}{\sin\phi_1}=\frac{\sin O_2P_1O_1}{\sin O_2I_1O_1}=\frac{O_1O_2/O_2P_1}{O_1O_2/O_2I_1}=\frac{O_2I_1}{O_2P_1}$.

When veiwed normally O_1 and O_2 coincide,

$\therefore\quad O_2P_1=O_1P_1=t$ and $O_2I_1=O_1I$.

$\therefore\quad \frac{\mu_2}{\mu_1}=\frac{t}{O_1I_1}$; or $O_1I_1=\frac{\mu_1 t}{\mu_2}$ ... (2)

Substituting (2) in relation (1), the displacement,

$$P_1I_1=t-\frac{\mu_1 t}{\mu_2}=t\left(1-\frac{\mu_1}{\mu_2}\right)$$

If the outer medium be air, $\mu_1=1$ and the refractive index of the block-material be $\mu_2=\mu$, then the displacement,

$$PI=P_1I_1=d=t\left(1-\frac{1}{\mu}\right)$$

$$=\frac{\mu-1}{\mu}t \qquad (3)$$

Hence, $\mu d=\mu t-t$; or, $\mu=\frac{t}{t-d}$ (4)

This relation (4) may be utilised for determining μ of a solid block or of a liquid, with the help of a travelling microscope.

44. Apparent position of an object when normally viewed through a number of media of different depths :— Let μ_1, μ_2 and μ' be the refractive indices of the two denser media (1), (2) and air respectively (Fig. 52). Let t_1 and t_2 be the thicknesses of the two media (1) and (2). If an object P be placed at the bottom of medium (1) and viewed normally from E, due to refraction at the surface of separation of (1) and (2), P will apparently be raised to Q,

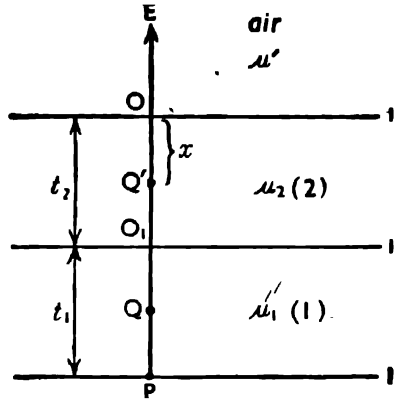


Fig. 52

where, $O_1P/O_1Q=\frac{\mu_1}{\mu_2}$.

Hence $O_1Q=O_1P\times\frac{\mu_2}{\mu_1}$

$=t_1\frac{\mu_2}{\mu_1}$, taking $\mu_1>\mu_2$.

This Q now serves as the object for the second refraction at the interface of medium (2) and air. When viewed normally, Q will be apparently raised to Q', so that OQ' $(=x)$ will ultimately be the *apparent distance* of the object from the upper surface of separation.

Here again, $\frac{OQ}{OQ'}=\frac{OO_1+O_1Q}{x}=\frac{\mu_2}{\mu'}$;

or, $\frac{t_2+t_1\frac{\mu_2}{\mu_1}}{x}=\mu_2$, (since $\mu'=1$);

or, $x=\frac{t_2+t_1\frac{\mu_2}{\mu_1}}{\mu_2}=\frac{t_2}{\mu_2}+\frac{t_1}{\mu_1}$.

If we consider n different media of thickness, t_1, t_2.....t_n and refractive indices, μ_1, μ_2....., μ_n, the apparent depth of an object situated at the bottom of the lowest medium is given by,

$$X=\frac{t_1}{\mu_1}+\frac{t_2}{\mu_2}+\frac{t_3}{\mu_3}+.....+\frac{t_n}{\mu_n}$$

$$=\sum_1^n\frac{t}{\mu}.$$

This result is true when the medium from which the observation is made in air.

45. Interesting Illustrations of Refraction :—

(i) **Atmospheric Refraction.**—The atmosphere, we all know, is less and less dense as its height increases above the sea-level and it is also known from Gladstone and Dale's law that for a gas $\frac{\mu-1}{\rho}$ is a constant, so that the refractive index of a gas decreases with the decrease of density. So the refractive index of air decreases upwards owing to the diminution in density. For this reason the rays of light proceeding from a heavenly body, such as a star, cannot travel in straight lines, but are refracted more and more towards the normal as they penetrate into the successive lower layers of the atmosphere. Since an observer on the earth's surface sees any heavenly body in the direction of the rays reaching him, the altitudes of heavenly bodies always appear to be too great, *i.e.* they always appear higher up in the sky than they are. Thus, an observer A on the earth surface sees the star S in the direction AS' (Fig. 53). Due to such atmospheric refraction, *the sun becomes visible some time before it rises and remains visible for some time after it sets*, *i.e.* when it goes below the horizon. Due to the same reason the sun, when near the horizon, *appears to be slightly oval-shaped*, because rays from the lower edge, which have to pass through a greater thickness of air than the rays from the upper edge, are refracted more, and so the vertical diameter of the sun appears to be diminished in size, whereas the horizontal diameter remains unaltered.

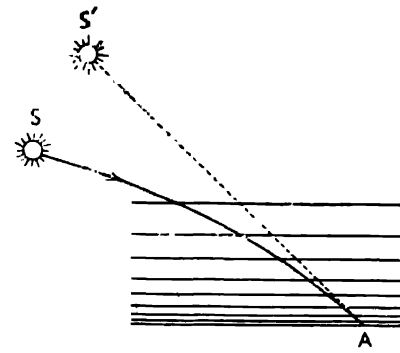


Fig. 53

(ii) **Twinkling of Stars.**—It is caused by the changes in atmospheric refraction. The rays of light from a star before they reach an observer on the earth travel through the atmospheric air the layers of which have different densities at different levels and these densities are also never steady due to convection currents resulting from temperature differences and other causes. Consequent on such changes in mass density of the air, the refractive index of the layers of the air also changes, though such changes are rather small.

Now a good concentration of rays in a particular direction must reach an observer to enable him to see a star brightly. This can not happen continuously because of the changes of the refractive index of the air, on account of which the courses of the rays alter from instant to instant. The amount of light reaching the eye of an observer in a given direction being thus not constant every moment a star appears to the observer to twinkle.

The planets, however, do not appear to twinkle. This is because the amount of light received from them by an observer on the earth is very much greater, the planets being much nearer comparatively, and the fractional change in brightness due to changes in atmospheric refraction is inappreciable.

(iii) **Powdered or Ground Glass** —Glass is ordinarily transparent, but when it is powdered it appears white and opaque, because in that case light is reflected from the surfaces of innumerable tiny particles of glass and causes the powder to appear white. If now water is poured into the powder, the rays are bent or refracted, and very little reflection takes place at the surfaces and so the glass appears transparent again.

Powdered coloured glass also appear white because the colour of the glass is due to the light which passes through it, but in this case the rays of light are reflected at the surfaces before they penetrate far into the glass, and so the powder appears white. The colour is restored by moistening the powder, as in this case less reflection takes place.

(iv) **Oiled Paper.**—Ordinary paper becomes translucent when it is oiled because, in this case, reflection at the rough surface is reduced and more light passes through the paper.

(v) It is known that the glare of an ordinary electric bulb, where the light enters the eye only from the direction of the filament is reduced by frosting, *i.e* by artificially roughening the inside surface of the bulb by which the light from the filament is reflected in all directi ns. If, however, the outside of the bulb is roughened, much of the l ght meets the surfaces of glass at angles greater than the critical angle (*vide* Art. 46), and thus the rays get reflected back into the bulb.

(vi) If a colourless transparent object like glass is placed in a liquid having the same refractive index as that of the solid, neither reflection nor refraction takes place at the surface of the object, and so the object remains completely invisible. An optical illusion may be produced by keeping a ball of iron on a glass rod dipped in a bottle of glycerol (which has got the same refractive index as that of glass) ; then the ball will appear to be floating on the liquid.

46. Total Internal Reflection : Critical Angle :—When a ray of light BA passes from a denser medium like glass or water, into a rarer medium like air, it is refracted away from the normal, *i.e.* as AF in Fig. 54, and *so the angle of refraction is always greater than the angle of incidence.* If the angle of incidence increases from $\angle BAN_1$ to $\angle CAN_1$, the angle of refraction also increases from $\angle NAF$ to $\angle NAG$; and for a certain value of the angle of incidence, say $\angle DAN_1$, the angle of refraction becomes 90°, that is, the refracted ray AY grazes the surface of separation XY. This particular angle of incidence DAN_1 for which the angle of refraction is 90° is called the **critical angle**, the value of which *depends upon the two media concerned and the colour of light used.*

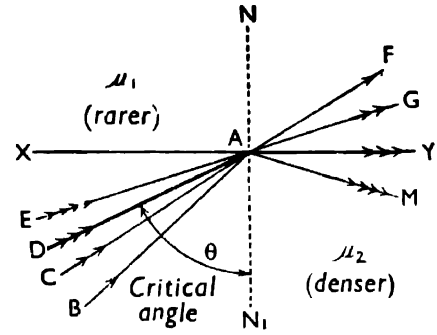


Fig 54

If the angle of incidence be increased still further, say, to $\angle EAN_1$, there will not be any corresponding refracted ray. The ray AM, instead of emerging out into the rarer medium, will be reflected back into the same medium obeying the ordinary laws of reflection. The ray AM is said to be totally reflected and such reflection is called **total internal reflection** to distinguish it from all other cases of reflection. The reflection is *total* and *internal*, because here no part of the incident light is refracted, but the incident light is wholly reflected at the internal surface.

If μ_1 and μ_2 be the refractive indices of the rarer and the denser media respectively and θ be the critical angle for the two media, then since the light passes from the denser medium to the rarer one, we have by Snell's law,

$$\frac{\mu_1}{\mu_2} = \frac{\sin\theta}{\sin 90^\circ}. \qquad \therefore \quad \sin\theta = \frac{\mu_1}{\mu_2}.$$

If the rarer medium be air, $\mu_1 = 1$ and if the refractive index of the denser medium be $\mu_2 = \mu$, then

$$\sin\theta = \frac{1}{\mu}; \quad \text{or,} \quad \mu = \frac{1}{\sin\theta}.$$

So the sine of the critical angle is equal to the reciprocal of the refractive index of the denser medium with respect to the rarer one. Thus if θ is known, μ can be calculated and vice versa.

The mean value of μ for water $= \frac{4}{3}$. So the critical angle for water and air is $\theta = \sin^{-1}\frac{1}{\mu} = \sin^{-1}\frac{3}{4} = \sin^{-1} 0.75 = 48°36'$. Similar calculations show that the critical angle for crown glass is about 41°·5, that for flint glass is 36°·5 and that for diamond is 24°·5 approximately.

Critical Angle.—*When a ray of light of any given colour travels from a denser to a rarer medium in such a way that the angle of refraction is 90°, the corresponding angle of incidence is called the critical angle for those two media. It is so called, because, if the angle of incidence is lesser, the ray refracts into the rarer medium obeying the ordinary laws of refraction, while if the angle of incidence is greater, the ray gets wholly reflected back into the denser medium obeying the ordinary laws of reflection (without any part of the ray being refracted).*

Total Internal Reflection.—*When a ray of light of any given colour travelling in a denser medium meets the surface of a less denser medium in such a way that the angle of incidence is greater than critical angle for those two media, the ray is wholly reflected back into the denser medium, and is said to undergo total internal reflection.*

Conditions of Total Internal Reflection.—(*i*) Total reflection is possible only for a ray of light passing from a denser medium on a rarer one. (*ii*) The second condition is that the angle of incidence must exceed the critical angle for the two media for the given colour of light.

47. **Mirage**:—The mirage seen in deserts or over any flat extensive heated surface is an example of total reflection. The layer of air nearest the surface of the desert is heated most; it is thus the least dense; and, in the absence of wind, the atmosphere may be thought of as consisting of a series of layers of air of density increasing upwards as the distance above the surface of the earth increases.

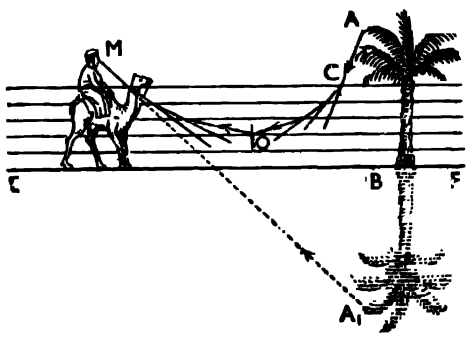


Fig. 55—Mirage of a Desert.

In Fig. 55, a ray of light coming from a tree *BA* is shown to be refracted away from the normal in passing down from one layer to the next below,

which is less dense. The refraction is thus increased as the ray passed down to successively lower layers and so it travels along a curved path as shown in the figure. The angle of incidence, increasing in this way may attain ultimately a value at which it just exceeds the critical angle for the two successive layers. Let this layer be at O, when the ray will be totally internally reflected. The ray then passes upwards being continually refracted into denser and denser layers and this up-going ray finally enters the traveller's eye (at M), who sees the tree in the direction of the light last reaching his eye, *i.e.* he sees an inverted image BA_1 of the tree below the ground. The inversion of the image produces, by the law of association, in the mind of the traveller, an impression that the image is formed by a near pool of water. This optical illusion is called **mirage**. Sometimes a thirsty traveller, in a desert sees a palm tree and its inverted image by reflection in a pool, but on approaching it, the pool disappears and he becomes a cruel victim to the illusion.

Mirage in Cold Regions—In very cold regions, the lower layers of air are gradually more and more cold and so the density of the air gradually increases downwards. In this case, light from a distant object (the ship in Fig. 56), while proceeding upwards, is refracted away from the normal at each upper layer, and the refraction increases more and more in the successive layers until it is totally reflected and begins to travel downwards. The object, therefore, appears to an observer to be hanging inverted in the air as shown in Fig. 56.

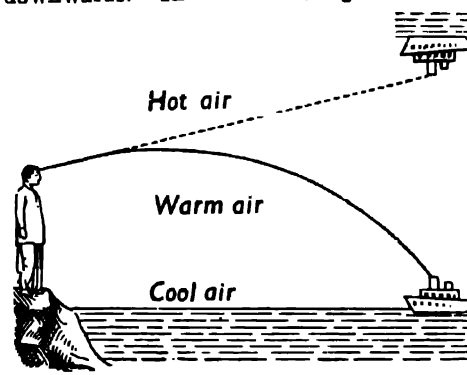


Fig. 56—Mirage in Cold Regions.

Examples

(*1*) *The refractive indices of glass and water are 1·52 and 1·33 respectively. Calculate the critical angle when light passes from glass to water.*

Ans. Refractive index of water with respect to glass, ${}_g\mu_w = 1{\cdot}33 \div 1{\cdot}52 = 0{\cdot}8750$. Now from the condition of total reflection (light travelling from glass to water),

$${}_w\mu_g = \frac{1}{\sin C}$$ where C is the critical angle ;

or, $\sin C = 1/{}_w\mu_g = {}_g\mu_w = 0{\cdot}8750$.

$\therefore \quad \angle C = \sin^{-1} 0{\cdot}8750 = 61°3'$.

(2) *An eye is placed at a certain depth below the calm surface of water. Show that, to the eye, the surface appears like a reflecting plane mirror with a circular hole through which objects situated outside the water can be seen.*

Also prove that the radius of the hole is $h/\sqrt{\mu^2-1}$ *cm. where* μ *is the refractive index of water and h cm. the depth at which the eye is placed.*

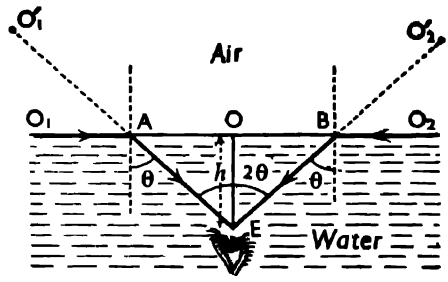


Fig. 57

Ans. Let the critical angle for water-air interface be θ. This means that objects on the water surface, say O_1 on one side and O_2 on the other, as shown in Fig. 57, will apparently crowd into a cone whose slant side AE and BE meet at an angle of 2θ to an eye (E) placed in water. If now a normal EO be drawn from E to the water surface, it will be equivalent to saying that the surface will behave as having a hole of radius AO or BO through which objects outside the water can be seen by the eye.

Now $AO=BO=EO\times\tan AEO=h\tan\theta$. Now, $\sin\theta=\frac{}{}$; or, $\mu^2=\operatorname{cosec}^2\theta=1+\cot^2\theta$; or, $\cot\theta=\sqrt{\mu^2-1}$. $\quad AO=BO=h\tan\theta=\dfrac{}{\sqrt{\mu^2-1}}$

48. Optical Path or Reduced Path :—The path described by light in vacuum in the same interval of time as it takes to traverse a certain length of path in a medium, is called the *optical path* or the *reduced path* of the latter. If l be the length of the path in a medium of refractive index μ and if C_m be the velocity of light in this medium, the time taken by light to traverse this path is, $t=\dfrac{l}{C_m}$.

If C_v be the velocity of light in vacuum, the path traversed by light in the vacuum during the above interval of time, is given by

$$l_o=C_v\times t=\frac{C_v}{C_m}\,l=\mu l.$$

(From the wave theory of light the absolute refractive index of the medium is, $\mu=\dfrac{C_v}{C_m}$.)

This vacuum path l_o is the optical or the reduced path corresponding to l. This path may as well be termed as the *equivalent vacuum path of l.*

49 Fermat's Principle :—The principle states that the actual path followed by light has a stationary value When a ray of light traverses several media undergoing a number of reflections and refractions, the path from one point to another is such that the total optical path $\Sigma\mu l$ is stationary for small variations in the path, where l is the length of the path in the medium of refractive index μ. So it is evident that the principle demands

that the actual path differs from a neighbouring path only by the second order of small quantities when the starting and the ending point are the same for both the paths.

Hence the principle may be written as,

$$\Sigma\mu l = \text{stationary, } i.e.$$
$$\delta\Sigma\mu l = 0, \quad \ldots \quad \ldots \quad (1)$$

where $\delta\Sigma\mu l$ indicates an infinitesimal variation in the total optical path which is same as the difference between the actual path and a neighbouring path. The above optical path which is stationary may as well be a minimum or a maximum. Since in ordinary cases of reflection and refraction at plane surfaces the optical path is a minimum, the principle is sometimes known as the **principle of least path.** As optical path is the product of the velocity of light in vacuum and the time, the former being a constant, the principle is often referred to as the **principle of least time.** In many cases of reflection and refraction *at surfaces other than plane the optical path is*, however, *not always the least but is merely stationary.* The validity of the law may be established easily from the laws of reflection and refraction. Assuming the validity of the law, the laws of reflection and refraction may as well be established.

[For "Fermat's Principle and Wave Theory", see Art. 15, Physical Optics]

(i) Fermat's Principle from the laws of reflection and refraction :—

(a) From laws of reflection :—Let MM_1 be a plane mirror and PO be an incident ray coming from the point source P (Fig. 58). The ray suffers regular reflection at O and finally reaches the point Q such that the path followed by the regularly reflected ray in coming from P to Q is $PO+OQ$. Let PS be any other ray from P incident at S. This ray also reaches Q after suffering an irregular reflection and the path followed by this irregularly reflected ray is $PS+SQ$. We are to show that the path, $(PO+OQ)$, followed by the ray obeying the laws of reflection is minimum.

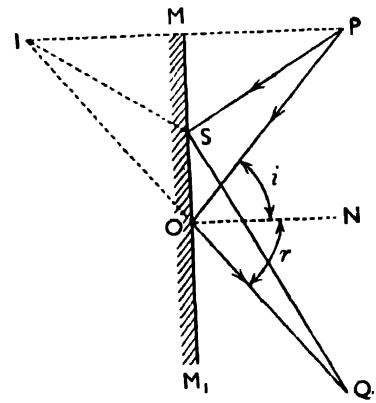


Fig. 58

If a perpendicular PM be drawn on MM_1 from P, the image I of P will lie on PM produced such that $IM = MP$. Again since OQ is the regularly reflected ray, I will be on QO produced. IS is joined. Now since IP is parallel to ON.

$$\angle MPO = \angle PON = \angle i = \angle r = \angle MIO.$$

$\therefore$ $IO = PO$ and $\angle IOS = \angle POS$.

$\therefore$ In $\triangle s$ PSO and ISO, $PO = IO$, SO is common to both and $\angle IOS = \angle POS$. $\therefore$ $IS = SP$.

$\therefore$ The path $(PO + OQ) = (IO + OQ) = IQ$.
and the path $(PS + SQ) = (IS + SQ)$.

So in the $\triangle$ ISQ, $IS + SO > IQ$, *i.e.* $> (IO + OQ)$.

Hence, $PS + SQ > PO + OQ$.

So the path $POQ (= PO + OQ)$ followed by the ray obeying laws of reflection is minimum. This establishes Fermat's principle of least path in the case of reflection. It is evident from the figure that if S is a point very close to O, the difference between the two paths becomes infinitesimally small.

(b) From laws of refraction :—Let RS be the separating surface between two media of absolute refractive indices μ_1 and μ_2 (Fig. 59). P is a point source, from which a ray PO is incident on RS at O. It suffers regular refraction and finally reaches the point Q along OQ. Here $\angle PON = \angle i$ and $\angle QON' = \angle r$, such that $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$.

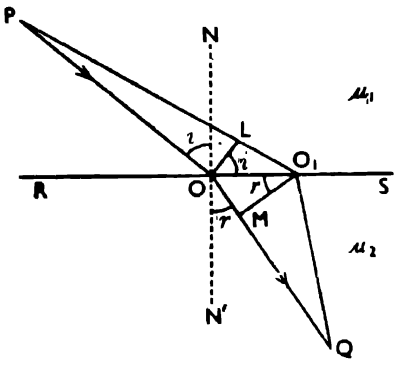


Fig. 59

The equivalent optical path traversed by this ray, POQ in between P and Q is evidently $\mu_1 . PO + \mu_2 . QO$. According to Fermat's principle this path must be minimum. To establish this principle let us consider another ray PO_1 from P which finally reaches Q after irregular refraction. The optical path of this ray, PO_1Q, is $\mu_1 . PO_1 + \mu_2 . QO_1$.

At O draw a line $OL \perp$ to PO so as to intersect PO_1 at L. Again from O_1 draw $O_1M \perp$ to OQ.

Then, $\angle LOO_1 = \angle i$ and $\angle MO_1O = \angle r$.

$$\therefore \quad \frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r} = \frac{\sin LOO_1}{\sin MO_1O} = \frac{LO_1/OO_1}{OM/OO_1} = \frac{LO_1}{OM}.$$

$$\therefore \quad \mu_2 \times OM = \mu_1 \times LO_1.$$

The optical path followed by the regularly refracted ray is $\mu_1 . PO + \mu_2 . QO = \mu_1 . PO + \mu_2 . OM + \mu_2 MQ$, and that followed by the secondary is

$$\mu_1 PO_1 + \mu_2 . QO_1 = \mu_1 . PL + \mu_1 . LO_1 + \mu_2 . QO_1.$$

In these two expressions, $PL > PO$, (since PL is the hypotenuse of the rt. $\angle d$. ΔPOL)

$QO_1 > MQ$ ($\because$ QO_1 is hypotenuse of ΔC_1MQ

and $\mu_2.OM = \mu_1.LO_1$.

$\therefore$ The path, $(\mu_1.PO + \mu_2.OM + \mu_2.MQ)$ < the path, $(\mu_1 PL + \mu_1 LO_1 + \mu_2 QO_1)$

or, $(\mu_1.PO + \mu_2.QO) < (\mu_1.PO_1 + \mu_2.QO_1)$.

Thus the path followed by the regularly refracted ray is less than that followed by the other ray. This is true for any other ray which does not suffer regular refraction. So, Fermat's principle of least path is also established in case of refraction. It is evident from the figure that if O_1 is very close to O, PO and PL become practically equal and so are QO_1 and QM. The paths POQ and PO_1Q are therefore practically identical or stationary. If there be any difference between the two paths, that will merely be an infinitesimally small quantity.

(ii) Deductions of the Laws of Reflection and Refraction from the Fermat's Principle of Least Path :—

(a) Laws of Reflection from Fermat's Principle :

Let MM_1 be a reflecting surface, PO an incident ray and OQ the corresponding reflected ray (Fig. 60). The path POQ between P and Q is minimum according to Fermat's principle. Let PL and QR be the perpendiculars drawn on MM_1 from the points P and Q respectively. Let $PL = h_1$, $QR = h_2$, $LO = x$ and $LR = e$. Then $OR = (e - x)$. Here since the starting and terminal points P and Q are fixed, $LR = e$ is a fixed distance and as the irregularly reflected rays reaching Q from P may be incident at any point O on the surface, the position of point only changes and hence its distance x from L is the only variable.

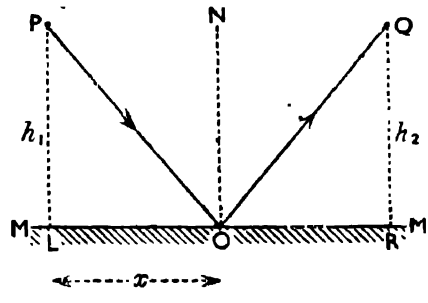


Fig. 60

As evident from Fig. 60, the path, $POQ = s = PO + OQ$
$\sqrt{h_1^2 + x^2} + \sqrt{h_2^2 + (e-x)^2}$ ... ... (1)

But since s is stationary according to Fermat's principle, $\frac{ds}{dx}=0$.

$$\therefore \quad \frac{ds}{dx}=0=\tfrac{1}{2}(h_1^2+x^2)^{-\frac{1}{2}}.\,2x-\tfrac{1}{2}\{h_2^2+(e-x)^2\}^{-\frac{1}{2}}\,2(e-x)$$

$$=\frac{x}{\sqrt{h_1^2+x^2}}-\frac{e-x}{\sqrt{h_2^2+(e-x)^2}}=\frac{LO}{PO}-\frac{OR}{QO}.$$

$$\therefore \quad \frac{LO}{PO}=\frac{OR}{QO}\text{; or, } \sin OPL=\sin OQR. \qquad \therefore OPL=\angle OQR.$$

Hence the angle of incidence, $\angle PON$ = the angle of reflection $\angle QON$. Again PO and OQ must lie in a plane perpendicular to the reflecting surface containing the normal ON as shown in Fig. 61; for if they were in different planes like PO_1 and QO_1, their projections on the normal plane through O would evidently be less than PO_1 and O_1Q themselves. The sum of these projections will be either equal to $(PO+OQ)$ when OO_1 is perpendicular to the POQ-plane or greater than $(PO+OQ)$ when the perpendicular from O_1 falls at any other point of the line MM_1. So the sum of PO_1 and QO_1 would always be greater than the sum of PO and OQ lying in the normal plane. So the principle of least path which demands that the sum of the paths must be minimum automatically implies that PO and OQ must lie in one plane. Thus both the laws of reflection follow from the principle of least path of Fermat.

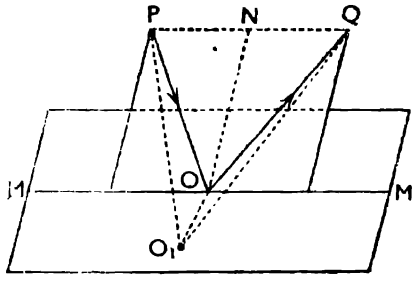


Fig. 61

(b) Laws of Refraction from Fermat's Principle :—Let the incident ray PO in a medium of refractive index μ_1 meet the separating surface TS of two media and after refraction let it proceed along OQ in the second medium of refractive index μ_2. Then the equivalent optical path between P and Q is given by,

$$s=\mu_1.PO+\mu_2.OQ.$$

This path s, according to Fermat's principle, is a stationary one (or often minimum). Let PL and QR be the perpendiculars drawn from P and Q on TS (Fig. 62). Let $PL=$

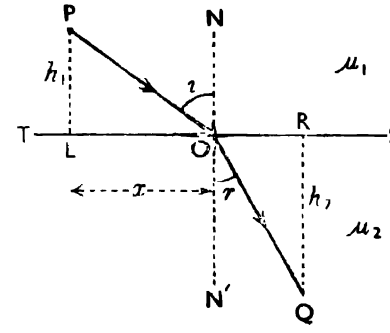


Fig. 62.

μ_1, $QR=h_2$, $LR=e$, $LO=x$ and $OR=(e-x)$. Here since the point of incidence for different rays varies, x is the only variable.

Now, $s=\mu_1 . PO+\mu_2 . OQ$

$$=\mu_1\sqrt{h_1^2+x^2}+\mu_2\sqrt{h_2^2+(e-x)^2}.$$

As s is stationary, $\frac{ds}{dx}=0$.

$$\therefore \quad \frac{ds}{dx}=0=\mu_1 . \frac{1}{2}\frac{2x}{\sqrt{h_1^2+x^2}}$$

$$-\mu_2 \frac{1}{2}\frac{2(e-x)}{\sqrt{h_2^2+(e-x)^2}}$$

$$=\mu_1\frac{x}{\sqrt{h_1^2+x^2}}$$

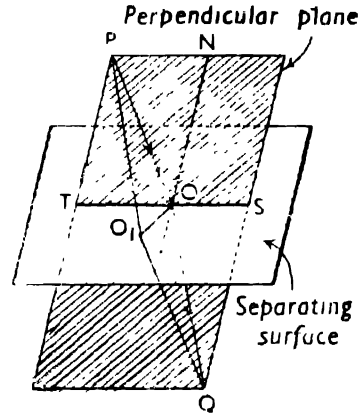


Fig 63

$$-\mu_2\frac{e-x}{\sqrt{h_2^2+(e-x)^2}}=\mu_1\frac{LO}{PO}-\mu_2\frac{OR}{OQ}=\mu_1 \sin OPL-\mu_2 \sin OQR$$

$$\therefore \quad \mu_1 \sin OPL=\mu_2 \sin OQR$$

or, $$\frac{\mu_1}{\mu_2}=\frac{\sin OQR}{\sin OPL}=\frac{\sin r}{\sin i};$$

or, $\mu_1 \sin i=\mu_2 \sin r$.

Thus Snell's law is deduced from Fermat's principle. Again, PO and OQ must lie in one plane. If not, suppose they lie in different planes and the ray follows the path PO_1Q (Fig. 63). The projections of PO_1 and QO_1 on the plane containing PO, OQ and ON must be less than PO_1 and QO_1 themselves. But the sum of those projections are equal to or greater than the regularly refracted path POQ by Fermat's principle. Hence PO_1+O_1Q is also greater than $PO+OQ$. So for the path to be minimum PO and QO must be coplanar. Thus both the laws of refraction are deduced from Fermat's principle of least path.

Example :

Deduce the relation $\frac{1}{v}+\frac{1}{u}=\frac{2}{r}$ for concave mirror from Fermat's principle.

Ans. Let LQ represent the concave mirror whose pole O is the origin of the co-ordinate axes OX and OY. C is the centre of curvature of the mirror and the X-axis coincides with its principal axis.

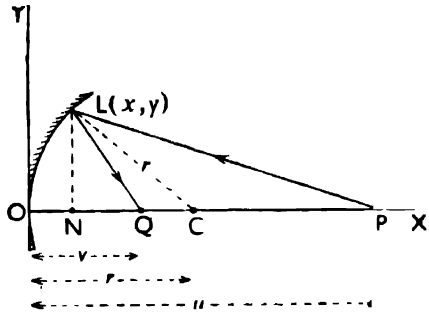


Fig 63(*a*)

P is a point object, a ray PL from which is incident on the mirror at L and is reflected along LQ forming a point image at Q. Let the co-ordinates of L be x and y and $OP = u$, $OQ = v$ and $OC = r = LC$.

Draw $LN = y$ perpendicular to OX, when $ON = x$.

Then $y^2 = LC^2 - NC^2$

$= r^2 - (r - x)^2 = 2rx - x^2$.

Optical path from P to Q $= PL + LQ$

Now $PL = \sqrt{y^2 + (u-x)^2} = \left\{2ux - x^2 + u^2 - 2ux + x^2\right\}^{\frac{1}{2}} = \left\{u^2 - 2(u-r)x\right\}^{\frac{1}{2}}$

$$= u\left\{1 - \frac{(u-r)}{u^2}x\right\} = \left\{u - \frac{(u-r)}{u}x\right\} \text{ (approx.)}$$

Similarly $QL = v + \frac{(r-v)}{v}x$.

$\therefore$ Optical path $\Delta = (u+v) + \left(\frac{r-v}{v}\right)x - \left(\frac{u-r}{u}\right)x$

By Fermat's principle, $\frac{d\Delta}{dx} = 0$

$$\therefore \quad \frac{d}{dx}\left[(u+v) + \frac{r-v}{v}x - \frac{u-r}{u}x\right] = 0$$

$$\text{or,} \quad \frac{r-v}{v} - \frac{u-r}{u} = 0$$

$$\text{or,} \quad \frac{r}{v} - 1 - 1 + \frac{r}{u} = 0, \quad \text{or} \quad \frac{r}{u} + \frac{r}{v} = 2$$

$$\therefore \quad \frac{1}{u} + \frac{1}{v} = \frac{2}{r}.$$

PRISMS

prism and other terms related to it :—A prism is a some transparent medium lying between two plane faces an angle.

faces are called the **refracting faces** of the prism and

the line along which the two refracting faces meet is called the **refracting edge** or simply the edge of the prism. The angle of inclination between the two refracting faces is called the refracting angle or simply the **angle** of the prism. Any section of the prism made by a plane perpendicular to the edge of the prism is called a **principal section** of the prism. The face of the prism opposite to the refracting edge is called the **base** of the prism.

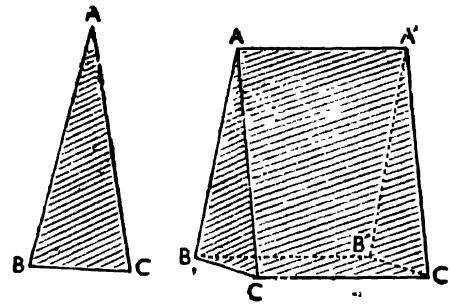


Fig. 64.

In Fig. 64, a prism has been shown where if we consider the faces $ABB'A'$ and $ACC'A'$ as the refracting faces, the line AA' along which they meet will be the edge of the prism and the angle BAC included between the refracting faces will represent the angle of the prism. The face $BB'C'C$ which is opposite to the edge will be the base of the prism. The section ABC, made by a plane perpendicular to the edge AA', represents a principal section of the prism. In a prism only the refracting faces through which light passes need be transparent.

51. Refraction through a prism in its principal section :—Let ABC be the principal section of a prism (Fig. 65), whose material is optically denser than air. $PQRS$ represents a ray passing through the principal section of the prism. PQ is a ray coming through air and incident on the prism. NQO is the normal at the point of incidence Q and i is the angle of incidence. QR is the refracted ray in the principal section of the prism, r is the angle of refraction. QR is incident on the second refracting face AC at an angle r' with the normal ORN' at R. RS is the emergent ray and i' the angle of emergence. It is evident from the figure that the incident ray PQ, which should have proceeded along PQT in

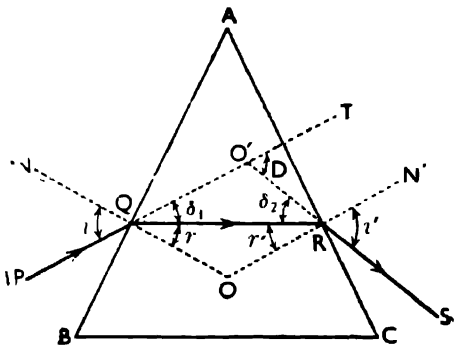


Fig. 65—Refraction through a prism.

absence of the prism, is emergent in the direction RS. So as a result of refraction through the prism, the emergent ray deviates towards the base BC. Now producing SR to cut QT in O', we get the angle $TO'S=D$, through which the incident ray has been deviated due to refraction through the prism.

The angle of deviation, $D=\angle O'QR+\angle O'RQ=\delta_1+\delta_2$
$=(i-r)+(i'-r')=(i+i')-(r+r')$ ... ... (1)

But since in the quadrilateral $AQOR$, the angles at Q and R are rt. $\angle s$, $\angle A+\angle O=2$ rt. $\angle s$.

Again in the triangle QOR, $(r+r')+\angle O=2$ rt. $\angle s$.
$\therefore \quad \angle A=r+r'$.

Hence from (1), we get, $D=(i+i')-(r+r')=(i+i')-A$ ... (2)

Note: In the above consideration of the deviation of a ray through a prism, the prism material has been supposed to be optically denser than the surrounding medium. If the prism material be optically rarer than the outside medium, the ray will after refraction through the prism be deviated towards the edge of the prism.

Angle of Minimum Deviation.—The angle of incidence of a ray and the corresponding angle of deviation due to refraction in a prism may be conveniently determined by a spectrometer and also by other suitable methods. By finding out a number of such values of i and the values of the corresponding D, a graph (Fig 66) is drawn plotting the angles of incidence (i) as abscissæ against the corresponding angles of deviation (D) as ordinates. From this i-D curve, it will be seen that the deviation at first diminishes with the increase of the angle of incidence until it attains a *minimum value*, represented by the lowest position A' of the curve, after which the deviation increases again with further increase of the angle of incidence. The value of the *minimum deviation* is represented by the ordinate AA' corresponding to the lowest point of the curve.

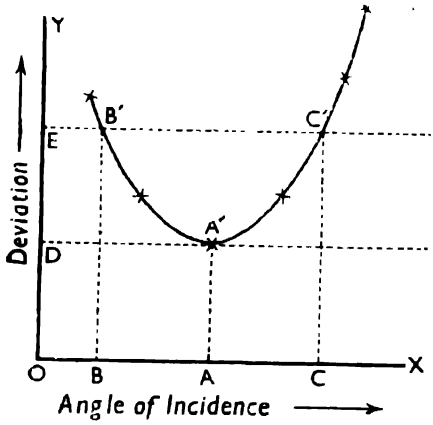


Fig. 66—i-D Curve.

Thus the deviation is minimum corresponding to the angle of incidence OA. For every prism *there is a definite angle of incidence*.

such as the angle OA in Fig. 66, for which the deviation suffered by light of a definite colour in passing through the prism is minimum, *i.e.* the deviation would be greater both when the angle of incidence is greater or less than the above angle of incidence. It can be shown, both mathematically and experimentally, that when the deviation is minimum, the angle of emergence i' is equal to the angle of incidence i, *i.e.*, the refracted ray QR (Fig. 65) within the prism passes symmetrically through the prism.

Plotting of Incidence *vs.* Deviation (i-D) Curve of a prism with the help of a Spectrometer.—(The description of a spectrometer and the methods of levelling and focussing it for parallel rays are given in Chapter XI).

The spectrometer is first levelled and focussed for parallel rays. Now bringing the telescope in position T_1 (Fig. 67) the direct image of the illuminated slit of the collimator is adjusted exactly on the cross-wire of its eye-piece and the position T_1 is noted by means of the vernier and the circular scale.

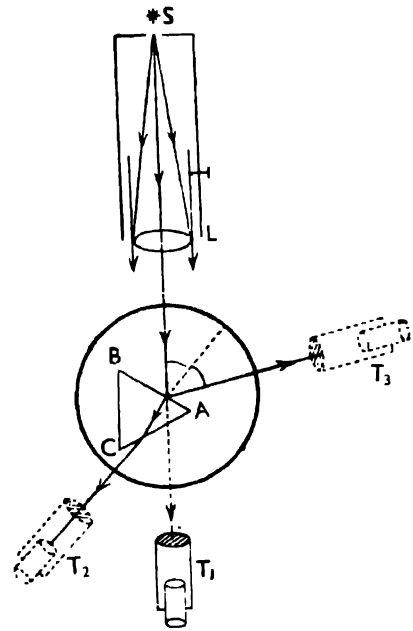


Fig. 67

By means of the fixing screw attached to the prism table the verniers (which are generally attached to the prism table in the ordinary laboratory spectrometers) are kept fixed in their positions. The platform of the prism table is now detached from the fixing attachments so that when the prism is placed on this platform, it can be rotated independently of the movement of the verniers. The prism ABC is now placed such that the centre of the prism table is in the plane of the face AB as shown in Fig. 67. The parallel rays falling on the prism are then refracted. The refracted image of the slit is then received on the cross-wire of the telescope, say in the position T_2 which is again recorded from the reading of the vernier against the scale. The difference of this vernier reading (for T_2) and the former vernier reading (for T_1) gives the angle of deviation D. Now the telescope is rotated to the position T_3 to receive the reflected ray from the face AB and the position T_3 is again noted from the vernier. Now the

difference between the readings corresponding to T_1 and T_2 is supplementary to the angle contained between the incident and the reflected rays. Hence 180° minus this difference gives $2i$. Thus i is determined corresponding to the above D. Now slightly rotating the platform of the prism another value of D is found for a different value of i. In this way a number of values of i and D being obtained, the i-D curve as shown in Fig. 66 can be drawn, which shows a definite minimum value for D, as has been referred to above.

Condition of Minimum Deviation in a Prism.—We know that the deviation suffered by a ray due to refraction in a prism is given by (Art. 51)

$$D = i + i' - A, \quad \ldots \quad \ldots \quad (1)$$

where i = angle of incidence, i' = angle of emergence and A = angle of the prism.

Again as has been already shown,

$$A = r + r' \quad \ldots \quad \ldots \quad (2)$$

$\therefore$ Differentiating (2) and remembering that A is constant, we get, $0 = dr + dr'$,

$$\text{or,} \quad dr = -dr' \quad \ldots \quad \ldots \quad (3)$$

Now when D becomes minimum, $\frac{dD}{di} = 0$.

$\therefore$ Differentiating equation (1), $\frac{dD}{di} = 1 + \frac{di'}{di} = 0$,

$$\text{or,} \quad \frac{di'}{di} = -1 \quad \ldots \quad \ldots \quad (4)$$

Again considering the refractions at the points of incidence and emergence of the prism (Fig. 65), we get,

$$\sin i = \mu \sin r \quad \ldots \quad \ldots \quad (5)$$

$$\text{and } \sin i' = \mu \sin r' \quad \ldots \quad \ldots \quad (6)$$

where μ is the refractive index of the material of the prism. Differentiating (5) & (6) we get,

$$\cos i \, . \, di = \mu \cos r \, . \, dr$$

$$\text{and } \cos i' \, di' = \mu \cos r' \, . \, dr'.$$

$$\therefore \quad \frac{\cos i}{\cos i'} \cdot \frac{di}{di'} = \frac{\cos r}{\cos r'} \cdot \frac{dr}{dr'}$$

$$\therefore \quad \frac{di}{di'} = \frac{\cos i'}{\cos i} \cdot \frac{\cos r}{\cos r'} \cdot \frac{dr}{dr'} \quad \ldots \quad \ldots \quad (7)$$

Taking $dr = -dr'$ from relation (3) and using relation (4), we have from equation (7),

$$\frac{\cos i'}{\cos i} \cdot \frac{\cos r}{\cos r'} = 1 \; ; \text{ or } \frac{\cos i'}{\cos i} = \frac{\cos r'}{\cos r}.$$

Squaring, we get,

$$\frac{1 - \sin^2 i'}{1 - \sin^2 i} = \frac{1 - \sin^2 r'}{1 - \sin^2 r} \quad (a)$$

Multiplying both the numerator and the denominator of the right hand side of relation (8) by μ^2,

$$\frac{1-\sin^2 i'}{1-\sin^2 i} = \frac{\mu^2-\mu^2\sin^2 r'}{\mu^2-\mu^2\sin^2 r} = \frac{\mu^2-\sin^2 i'}{\mu^2-\sin^2 i} \text{ (by Snell's law)}$$

$$= \frac{(\mu^2-\sin^2 i')-(1-\sin^2 i')}{(\mu^2-\sin^2 i)-(1-\sin^2 i)} = \frac{\mu^2-1}{\mu^2-1} = 1.$$

$\therefore \quad \sin^2 i = \sin^2 i'$; or, $i = i'$

Thus the minimum deviation occurs when the angles of incidence and emergence are equal.

From relations (5) and (6) it follows that when $i = i'$,

$$r = r' \qquad \ldots \qquad \ldots \qquad (9)$$

Now referring to Fig. 65, since the angles AQO and ARO are rt. $\angle$s and $\angle r = \angle r'$, we have,

$$\angle AQR = \angle ARQ. \qquad \ldots \qquad \ldots \qquad (10)$$

$\therefore$ Q and R are equidistant points from A. Thus the ray *passes symmetrically through the prism* when the deviation is minimum.

Since the above condition (*i.e.*, $i = i'$) applies equally to the case of a minimum or a maximum value for D, the exact nature of D is found by doubly differentiating D with respect to i, which yields,

$$\frac{d^2D}{di^2} = \frac{2(\mu^2-1)\tan i}{\mu^2\cos^2 r}.$$

This being a $+ve$ quantity, the condition ($i = i'$) deduced above definitely refers to a minimum value of D.

Expression for μ of the material of a prism.

Since, $r = r'$ and $A = r + r'$,

we have, $r + r' = 2r = A$; or $r = A/2 \qquad \ldots \qquad \ldots \qquad (11)$

Again as, $D = i + i' - A$ and

$i = i'$ when D is the minimum deviation D_m, we may write,

$$D_m + A = 2i \text{ ;}$$

$$\text{or,} \quad i = \frac{A+D_m}{2} \qquad \ldots \qquad \ldots \qquad (12)$$

Substituting i and r from (12) and (11) in relation (5), we have,

$$\sin\frac{A+D_m}{2} = \mu \sin\frac{A}{2} \text{ ;}$$

$$\text{or,} \quad \mu = \sin\frac{A+D_m}{2} \Big/ \sin\frac{A}{2} \qquad (13)$$

This relation is often used for finding the refractive index of the material of prism.

[**Alternative Trigonometrical method for finding the Condition of minimum deviation** :—Applying Snell's law at the points of incidence and emergence (Fig. 67), we have,

$$\mu=\frac{\sin i}{\sin r}=\frac{\sin i'}{\sin r'}=\frac{\sin i+\sin i'}{\sin r+\sin r'}.$$

$$=2\sin\frac{i+i'}{2}\cos\frac{i-i'}{2}\Big/\,2\sin\frac{r+r'}{2}\cos\frac{r-r'}{2}$$

$$=\frac{\sin\frac{A+D}{2}}{\sin\frac{A}{2}}\times\frac{\cos\frac{i-i'}{2}}{\cos\frac{r-r'}{2}}=x\times y\ (\text{say}),$$

where x and y are both variable factors. With the changes of deviation D, the value of x changes. The value of y also changes with changes of i and i' and consequently of r and r'. But the product of x and y is equal to μ and is constant. So when x is minimum, y must be maximum. But x is minimum when the deviation D is minimum. Thus, the condition of minimum deviation is identical with the condition of the maximum value of y.

Now, $\quad y=\dfrac{\cos\frac{i-i'}{2}}{\cos\frac{r-r'}{2}}=\dfrac{\cos\theta}{\cos\phi}$, where $\theta=\dfrac{i-i'}{2}$ and $\phi=\dfrac{r-r'}{2}$.

(*a*) Suppose $i>i'$, and consequently $r>r'$. Again $(i-i')>(r-r')$, because the rotation of the incident ray in air is greater than the rotation of the refracted ray in the denser medium. Thus $\theta>\phi$. If $i>i'$, $\theta>\phi$ and consequently $y<1$.

(*b*) Suppose $i<i'$, and consequently $r<r'$. Again $(i'-i)>(r'-r)$, since the rotation of the incident ray in air is greater than the rotation of the refracted ray in the denser medium. Hence $\theta>\phi$.

That is, $\quad y=\dfrac{\cos(-\theta)}{\cos(-\phi)}=\dfrac{\cos\theta}{\cos\phi}$; but $\theta>\phi$. So, $y<1$.

(*c*) Suppose $i=i'$, *i.e.*, $r=r'$. Then $\quad y=\dfrac{\cos 0}{\cos 0}=1$.

From the above considerations it is clear that y is maximum when $i=i'$, *i.e.*, $r=r'$; it has smaller values both when i is greater and smaller than i'. So, the deviation D, is also minimum when $i=i'$ (or $r=r'$).]

52. Cases of Normal incidence on a Prism :—

(a) When the Prism is of any angle and the incidence is exactly normal :—When the incident ray is exactly normal to the refracting face AB (Fig. 68),

$\angle i = 0$ and $\angle r = 0$.

Again since $A = r + r'$, $r' = A$ ... (1)

We know that in a prism, the deviation, $D = i + i' - A$.

$\therefore$ In case of normal incidence, $D = i' - A$.

or, $i' = A + D$ ... (2)

Considering the refraction at point of emergence, R, we have,

$$\sin i' = \mu \sin r' \quad \ldots (3)$$

Now substituting the values of r' and i' from relations (1) & (2) in relation (3), we get,

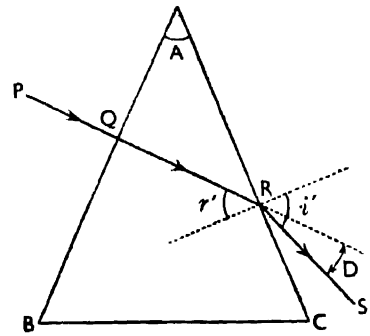


Fig. 68

$$\mu = \frac{\sin i'}{\sin r'} = \frac{\sin A + D}{\sin A} \quad (4)$$

This relation is valid for all types of prisms—thin or thick—when the incident ray is exactly normal. In general the expression is used for determining the refractive index of the material of a prism whose angle is small but not very small (*i.e.* not below 5°).

(b) When the Prism is thin and the incidence is exactly normal :—A prism is considered to be thin when its refracting angle is 5° or less. So in this case the angle of the prism, A, is small. Again since the incidence is normal $\angle i = \angle r = 0$.

$\therefore$ $r' (= A)$ is a very small angle and so is i'. In such a case relation (3) can be written as, $i' = \mu r'$ ... ... (5)

Hence substituting the values of i' and r' from (2) and (1), we have,

$$A + D = \mu A \text{ , or, } D = A(\mu - 1) \quad \ldots \quad (6)$$

The relation is generally used for determining μ in the case of a thin prism.

(c) When the angle of the prism is very small but the incidence is nearly normal.—Since the incidence is not exactly normal, both i and r are not zero. They are, however, very small, so that $\sin i = i$ and $\sin r = r$. Again as A is also very small, angles i' and r' are also very small and $\sin i = i'$ and $\sin r' = r'$.

$\therefore$ Considering refractions at the two faces of the prism, we get,

$$\sin i = \mu \sin r, \text{ or, } i = \mu r \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

$$\text{and } \sin i' = \mu \sin r' \text{ or, } i' = \mu r' \quad \ldots \quad \ldots \quad (8)$$

We know that, $D = i + i' - A$

Substituting the values of i and i' from (7) & (8) in the expression for D, we get,

$$D=\mu(r+r')-A\,,$$

$$\text{or,}\quad D=\mu\,A-A,\ (\because\ A=r+r')$$

$$=A(\mu-1).$$

Thus the deviation is independent of the angle of incidence provided the incidence is very nearly normal.

Note.—*When the angle of incidence (i) is large and the prism is thin.*

We have deviation $D=i+i'-(r+r')=i+i'-A$ ($\because\ A=r+r'$).

Since $\sin i'=\mu \sin r'$, $i'=(D+A-i)$ and $r'=A-r$, we can write $\sin(D+A-i)=\mu \sin(A-r)$,

$$\text{or,}\quad \sin(A+D)\cos i-\cos(A+D)\sin i$$

$$=\mu(\sin A\cos r-\cos A\sin r)\,;$$

$$\text{or,}\quad (A+D)\cos i-\sin i=\mu\,(A\cos r-\sin r)=\mu\,A\cos r-\sin i,$$

(since A and D are small and $\mu \sin r=\sin i$).

$$\text{Rearranging,}\quad D=\left(\mu\frac{\cos r}{\cos i}-1\right)A=\left\{\left(\mu^2\frac{\cos^2 r}{\cos^2 i}\right)^{\frac{1}{2}}-1\right\}A$$

$$=\left\{\left(\mu^2\frac{(1-\sin^2 r)}{\cos^2 i}\right)^{\frac{1}{2}}-1\right\}A=\left\{\left(\frac{\mu^2-\sin^2 i}{\cos^2 i}\right)^{\frac{1}{2}}-1\right\}A$$

$$=\left\{\left(\frac{\mu^2-1+\cos^2 i}{\cos^2 i}\right)^{\frac{1}{2}}-1\right\}A=[\{(\mu^2-1)\sec^2 i+1\}^{\frac{1}{2}}-1]A\ldots(1)$$

This is the general expression for deviation in a thin prism. If i is small, $\sec^2 i=1$ and hence $D=(\mu-1)\,A$.

53. Condition of no emergence (in the case of a Prism) :—

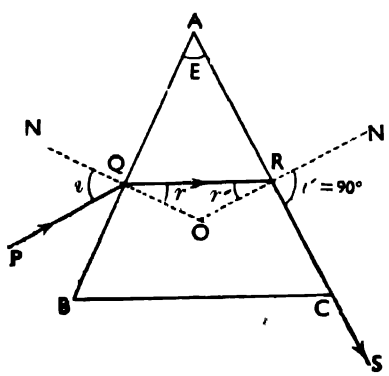


Fig. 69

Let ABC be the principal section of a prism (Fig. 69). If a ray PQ be incident at an angle i on the face AB of the prism, after refraction it will be deverted along QR, where R is the point of emergence. If the ray on emergence grazes along the face AC of the prism, the corresponding angle of incidence r' inside the prism gives the limiting value of the condition of no emergence.

Here, since $i'=90°$,

$$\mu=\frac{\sin i'}{\sin r'}=\frac{1}{\sin r'}$$

or, $r' = \sin^{-1}\frac{1}{\mu} = \theta_c$, where θ_c is the critical angle for the prism material and external medium (air).

We know, $A = r + r'$ ... ... (1)

or, $r = A - r' = \left(A - \sin^{-1}\frac{1}{\mu}\right)$.

But at Q, $\sin i = \mu \sin r$ ... ... (2)

$\therefore$ $\sin i = \mu \sin\left(A - \sin^{-1}\frac{1}{\mu}\right)$ ... ... (3)

This angle of incidence i gives the limiting value of the angle of incidence corresponding to no emergence.

Again from relation (2), $\sin r = \frac{\sin i}{\mu}$; or, $r = \sin^{-1}\frac{\sin i}{\mu}$.

$\therefore$ From relation (1), it follows that,

$$A = \sin^{-1}\frac{\sin i}{\mu} + \sin^{-1}\frac{1}{\mu} \quad \ldots \quad \ldots \quad (4)$$

Relation (4) gives the expression of A for no emergence. If the angle of incidence (i) is greater than the limiting angle given by eqn. (3), the corresponding value of the angle, r, will also increase. But as $A = r + r'$, r' will diminish and become less than $\theta_c = \sin^{-1}\frac{1}{\mu}$ and the ray after refraction will once again emerge out of the prism. But if i is less than the limiting value, r' will be greater than θ_c and there will be no emergence. So if we want no emergence for any angle of incidence, let us start with the maximum possible angle of incidence (*i.e.* 90°) so that for any angle of incidence the condition remains unaltered.

Then, $\sin i = \mu \sin r$; or, $r = \sin^{-1}\frac{\sin i}{\mu} = \sin^{-1}\frac{\sin 90^\circ}{\mu}$

$$= \sin^{-1}\frac{1}{\mu} = \theta_c$$

Substituting this value in relation (4), we get,

$$A = \sin^{-1}\frac{1}{\mu} + \sin^{-1}\frac{1}{\mu} = 2\theta_c \quad \ldots \quad \ldots \quad (5)$$

So if the angle of the prism be twice the critical angle for the prism material and air or greater, there will be no emergence for any angle of incidence.

Again if the incident ray falls normally on the prism $i = r = 0$.

$\therefore$ $A = r'$.

So for no emergence, $A = r' = \sin^{-1}\frac{1}{\mu} = \theta_c$.

54. Formation of Image by a Prism :—Let PQ and PQ_1 be two rays (Fig. 70) coming from a point source P and let them be incident on the face AB of a prism at angles i and $i+di$ respectively. These two rays, after refraction through the prism, emerge out as RS and R_1S_1, the corresponding angles of emergence being i' and $i'+di'$. These emergent rays, when produced meet at P_1. So P_1 is the virtual image of P. Now since the angular inclination of the two incident rays at P is di and that of the emergent rays at the position of the image P_1 is $-di'$, the negative sign evidently indicating the fact that while the incident rays move towards the edge of the prism the emergent rays move away from it. So the angular magnification of the prism is given by,

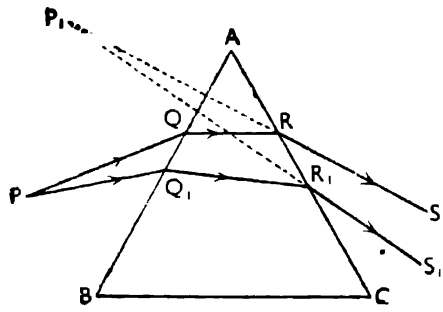


Fig. 70

$$m_a = \frac{\text{angle of inclination in the image space}}{\text{angle of inclination in the object space}}$$

$$= -\frac{di'}{di} \qquad \ldots \qquad \ldots \qquad (1)$$

Since $\sin i = \mu \sin r$, we have by differentiation,

$$\cos i \,.\, di = \mu \cos r \,.\, dr \qquad (2)$$

Similarly since $\sin i' = \mu \sin r'$, $\qquad \cos i' \, di' = u \cos r' \, dr' \qquad (3)$

$\therefore$ Dividing (3) by (2), $\dfrac{\cos i' . di'}{\cos i . di} = \dfrac{\cos r' . dr'}{\cos r . dr} \qquad (4)$

But since $A = r + r'$, $dr + dr' = 0$, or, $dr = -dr'$.

$$\therefore \quad \frac{\cos i'}{\cos i}\frac{di'}{di} = -\frac{\cos r'}{\cos r} \,.$$

$$m_a = -\frac{di'}{di} = \frac{\cos i}{\cos i'} \cdot \frac{\cos r'}{\cos r} \qquad (5)$$

When the ray is incident on the prism, at such an angle that the deviation is minimum, then $i = i'$ and $r = r'$.

$$\therefore \quad m_a = 1.$$

Thus the angular magnification is unity which means that the emergent rays are inclined at the same angle as the incident rays. So in the minimum deviation position of the prism, a point object would always produces a point image, which is one of the essential conditions for the formation of a pure spectrum.

When the angle of incidence i is greater than the angle of emergence i' m_a is less than unity, *i.e.*, the angle of inclination

of the emergent rays becomes smaller. So the image is formed at a distance greater than the object. The position of $i > i'$ is known as the *inclined* (or *slanting*) *position* while focussing a prism spectrometer.

Again when the angle of incidence i is less than the angle i', m_a is greater than unity, *i.e.* the angle of inclination of the emergent rays becomes greater. So the image is formed at a nearer point than the object. This position is known as the *normal position*.

55. Different types of prisms and their uses :—

(i) Totally Reflecting Prisms :—

(a) Deviation of Rays through 90°.—In Fig. 71, a right-angled isosceles glass prism is represented by the principal section *ABC* having equal sides *AC* and *BC* and right-angle at *C*. If a parallel beam of light falls normally on the face *AC*, the rays will pass undeviated through this face but on meeting the hypotenuse face *AB*, they will be totally reflected. The reason for total reflection is that they will be incident at *AB* at an angle of 45° which is greater than the critical angle (42°) for glass to air refraction.

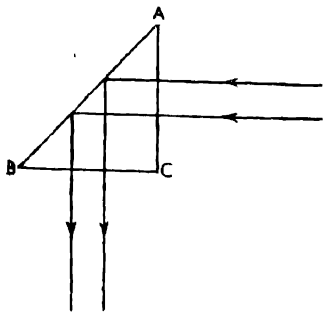


Fig. 71

So the rays, after total reflection, will be turned vertically downwards, because the angle of reflection will also be 45°, *i.e.* the reflected rays will be deviated through 90° with respect to the original incident beam. These reflected rays will strike the face *BC* normally and will, therefore, pass out straight.

(b) Inversion of Image with the rays deviated through 180°.—In Fig. 72, light enters normally through the hypotenuse face *AB* of a right-angled isosceles prism whose principal section is given by *ABC*. The light will be totally reflected successively at the faces *AC* and *CB* and will finally emerge out of the face *AB* parallel to the original direction of incidence. The image produced by the emergent rays will be inverted as shown in the figure, and the deviation, of the rays is evidently 180°. It may be noted that, if the *edge* of the prism

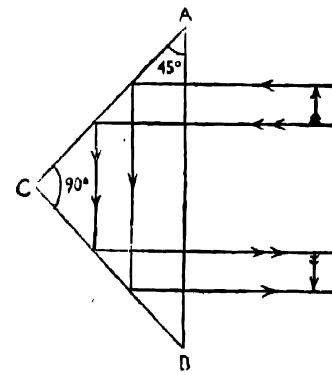


Fig. 72

is vertical, the image will be inverted laterally while, if the *edge* is horizontal, the image will be inverted vertically.

(c) Inversion of Image without Deviation of rays (Erecting Prism).—In Fig. 73, an inverted image is shown to be made erect by the use of a right-angled isosceles glass prism, where total internal reflection takes place at the hypotenuse. From the tracing of the rays it is evident how inversion of the image can take place without deviation of the rays.

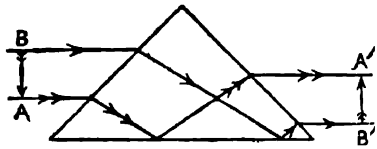


Fig. 73

A prism used in this way is called an **erecting prism.** Such an erecting prism is often used in *projection lanterns*, and other optical instruments, where inverted images are formed by lenses, which are then made to appear in the right way upon the screen by placing an erecting prism like this in the path of the emergent rays.

Advantages of a Totally Reflecting Prism as a Reflector.—As compared with a plane mirror, a totally reflecting prism used as a reflector has some advantages. An ordinary plane mirror has two surfaces of which the back surface is usually silvered for reflection to take place. Reflection, however, takes place also at the front face on account of which the image formed by silvered surface cannot be as bright as is possible by single reflection. Besides the loss of brightness of the image, there is another defect. If the mirror is thick, and the incidence of light oblique, a number of images may be formed by successive reflections at the two faces. This is undesirable for a good reflector. If the silvering is done at the first face, or a metallic reflector is used, these two above defects may, however, be reduced. But in such cases, the surface becomes liable to tarnish on account of direct exposure to the atmosphere, which ultimately results in diminishing the brightness of the image formed. When a 45° totally reflecting prism is used as a reflector, there is no doubt that some loss of light takes place due to reflection at the place of entrance and emergence, and by absorption within the glass. But they are much smaller than in a plane mirror, for the entrance and emergence take place *normally*. The reflecting surface being an internal face is protected from tarnishing and the reflection that takes place there is *total*. The result is that such a prism provides a permanent reflecting surface producing a bright and clear image.

(ii) Constant Deviation Prisms :—A constant deviation prism is such a prism by means of which a ray incident on it can be deviated through a desired constant angle.

Fig. 74 shows one type of constant deviation prism whose principal section is a pentagon. It is, therefore, commonly known as the *penta prism*.

It works on the principle that two reflecting surfaces inclined at an angle θ deviate the incident rays through 2θ. If two faces AB and CD of the prism be inclined at 45° and if the angle AED be 90°, the rays falling on the prism in the way shown in Fig. 74, will be deviated through 90°. Here since the rays do not suffer total internal reflection at faces AB and CD, these faces must be silvered. When any angle of deviation δ is desired, the prism is so constructed that the angle between AB and CD is $\delta/2$ and the angle AED is $(180° - \delta)$.

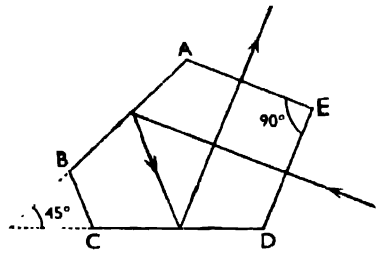


Fig. 74

Such prisms are used in range-finders and also for setting an exact right angle over long distances in engineering works.

The second type of the constant deviation prism consists of a single block of glass whose principal section $ABCD$ is shown in Fig. 75. It is called a *constant deviation dispersive prism*. It may be considered to be built up of two 30° right-angled prisms (ABO and ACD) and a 45° right-angled prism (BOC). Here the incident ray PQ is so directed that it suffers minimum deviation after refraction through the prism. If at the point of incidence, the ray is refracted as QR making an angle $r = 30°$ with the normal, it will be parallel to AC and will be incident at R at an angle of 45° which is greater than the critical angle. So after internal reflection at R the ray is turned through 90° and proceeds along RS parallel to CD. At S it makes an angle of 30° with the normal and comes out along ST making an angle of emergence $i' = i$, the angle of incidence at Q. So the ray under the above conditions suffers minimum deviation. Again since the normals at Q and S are mutually perpendicular and $i = i'$ the incident ray PQ must also be

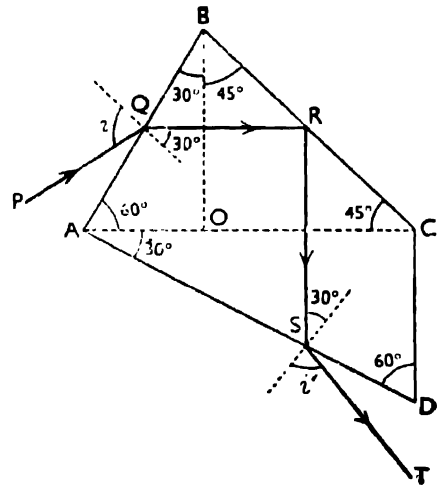


Fig. 75

perpendicular to the emergent ray ST. So when a ray passes through this prism set in the minimum deviation position, the emergent ray will always be deviated through 90°.

56. Determination of Refractive Index :—

(A) μ of solids

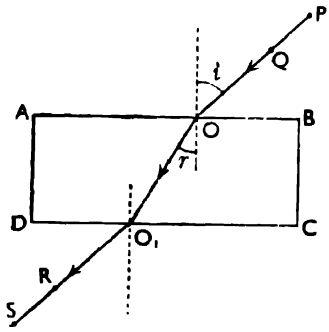


Fig. 76

(i) Transparent block or plate.—

(a) By ray-tracing or pin method.—Here the block is placed on a piece of drawing paper fixed to a drawing board. By means of four pins P,Q,R,S, the incident and emergent rays are traced and hence also the refracted ray OO_1. Then from the simple relation $\mu = \sin i/\sin r$, μ is determined by actually measuring i and r.

(b) By the Travelling Microscope.—A travelling microscope EO (E is the eye-piece and O the objective) is first adjusted so that its tube is exactly vertical (Fig. 77). It is moved up and down a vertical scale S and is focussed on a fine pencil mark on a piece of paper placed on the table. The reading d_1 of the vernier V_1 on the vertical scale S is noted. A plate of glass being then placed on the mark, the microscope is focussed again on the mark as seen through the glass and the position d_2 of the vernier on the scale S is noted. A few grains of lycopodium powder are then sprinkled on the top of the glass plate and the reading d_3 on the scale is again noted when the powder is best visible. Then

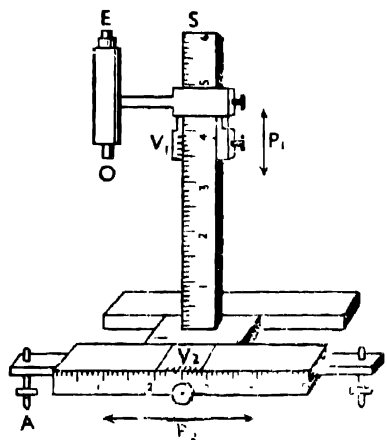


Fig. 77—Travelling Microscope.

$$_a\mu_g = \frac{\text{real thickness}}{\text{apparent thickness}} = \frac{d_3 - d_1}{d_3 - d_2}.$$

(ii) Transparent material available in the form of a prism.—

(a) By ray-tracing —The refractive index is found by this method with the help of the relation,

$$\mu = \frac{\sin \frac{A+D_m}{2}}{\sin \frac{A}{2}} \qquad (1)$$

So the method has two different parts in the experiment, namely (i) the determination of D_m and (ii) the determination of A.

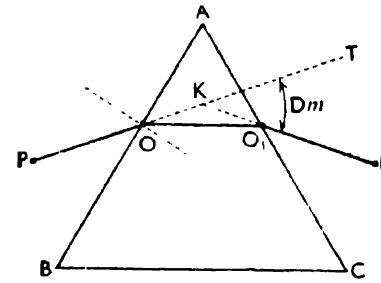


Fig. 78

To determine D_m the prism ABC (Fig. 78) is placed on a piece of drawing paper fixed to a drawing board. Drawing the outline of the prism on the paper two points O and O_1 are chosen on two refracting faces, equidistant from A. Two pins are vertically fixed at O and O_1, then with the help of two other pins P and R, the incident ray PO and the emergent ray O_1R are traced. Producing PO to T and RO_1 to cut PT at K, the angle of minimum deviation D_m (*i.e.* $\angle TKR$) is actually measured. The mean value of a number of such measurements is taken.

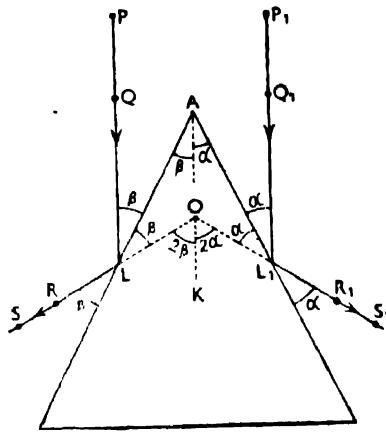


Fig 79

To measure A, the prism is as usual placed on a drawing board and a piece of drawing paper and its outline is drawn. Two parallel lines PQ and P_1Q_1 are drawn on the paper as shown in Fig. 79. The prism is placed on the outline and four pins P, Q, and P_1,Q_1 are placed on these lines as shown. With the help of four other pins the reflected rays RS and R_1S_1 are traced. RS and R_1S_1 are produced to intersect at O after removing the prism from the drawing paper. The angle at O (*i. e.* $\angle LOL_1$) is measured directly. This is equal to $2A$. Hence A is found

Then putting the values of D_m and A in relation (1), μ can be determined.

[To prove that the angle at O (*i.e.* $\angle LOL_1$) is $2A$, let us draw lines through A and O parallel to PQ and P_1Q_1. If $\angle PLA=\beta$ and $\angle P_1L_1A=\alpha$, it is evident from the geometry of Fig. 79, that $A=\alpha+\beta$. Again since $\angle AL_1O=\alpha$ and $\angle ALO=\beta$, $\angle LOK=2\beta$ and $\angle L_1OK=2\alpha$.

$$\therefore \quad \angle LOL_1=2(\alpha+\beta)=2A. \ ]$$

(b) **By Spectrometer.**—[The description of a spectrometer and the methods of levelling and focussing it are given in Chapter XI].

In the method also the refractive index of the material of the prism is determined with the help of relation (1),

$$i.e. \quad \mu=\frac{\sin\frac{A+D_m}{2}}{\sin\frac{A}{2}}.$$

So here also D_m and A are to be measured. By a spectrometer these two angles can be very accurately measured and as such this method is merely an improved and accurate modification of the former *ray-tracing* method.

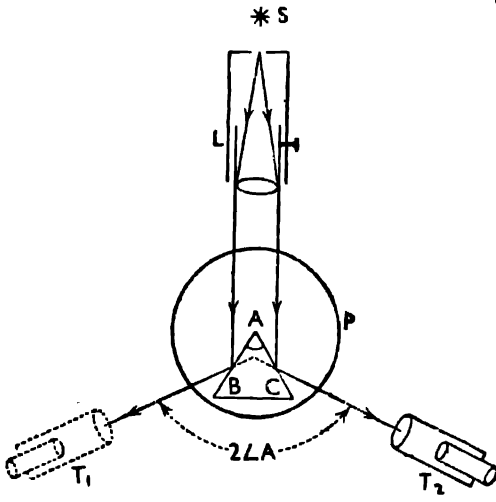


Fig. 80

First of all the spectrometer is levelled and focussed for parallel rays. To determine the angle of the prism, the prism is placed with its edge at the centre of the prism table P (Fig. 80) and its base BC opposite to the collimator L. The parallel beam of light from the collimator falling on the edge of the prism will be evidently divided into two parts One portion of this split-up beam will be reflected from the face AB of the prism and this reflected component is observed through the telescope and is made to coincide with the cross-wire of the eye-piece. This position of the telescope T_1 is noted with the help of the angular

vernier attached to the telescope. The telescope is then rotated and brought to the position T_2 where the other part of the beam reflected from the face AC is just brought in coincidence with the cross-wire. This position T_2 of the telescope is again noted from the vernier. The difference of the two vernier readings gives an angle equal to twice the angle of the prism. This determination is repeated at least thrice and the mean value of $2A$ is found. Thus A is determined accurately.

To find out the angle of minimum deviation the prism is placed with its centre coinciding with the centre of the prism table P (Fig. 81). The refracting face AB is then turned towards the collimator and the emergent ray from the face AC is received by the telescope. The prism table is now slowly rotated so that the emergent ray gradually approaches the direction of the incident ray (ST_2). During this rotation of P, the telescope is also to be rotated so as to keep the image of the slit S (as seen by the emergent rays) always in the field of view of the telescope. As P is rotated more and more, at a certain position of the telescope it is found that the image of S is temporarily steady and then turns back in a direction away from ST_2. This evidently is the position of minimum deviation. The cross-wire of the telescope is to be fixed on this turning point of the image by trial and the corresponding reading is to be taken from the vernier. [This adjustment for minimum deviation position may also be detected roughly by naked eyes at first and then bringing the telescope to this roughly adjusted position, the final adjustment may be accurately done]. The prism is now removed from the prism table and the direct image of the slit is made to coincide with the cross-wire of the telescope at T_2, the position of which is also recorded by the vernier. The difference of the two vernier readings gives D_m.

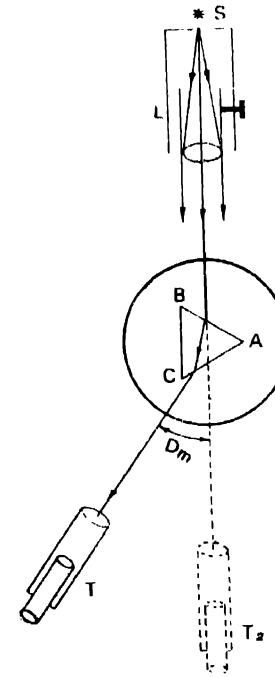


Fig. 81

Thus knowing A and D_m, μ can be calculated. In ordinary spectrometers μ can be measured accurately upto 3 places of decimal and with a precision type of spectrometer, the accuracy reached is upto the 4th place of decimal.

(c) **Thin Prism by the method of normal incidence.**—In this method generally a thin prism of the material is chosen, the refracting angle of the prism being less than 5°. Since the method of normal incidence is adopted, the refractive index of the material is determined from the relation $\delta = A(\mu - 1)$. So the experiment involves the measurement of the angle of the deviation through the prism while the incident ray is normal and also the determination of the angle of the prism. For this purpose, a spectrometer is to be used. First of all the spectrometer is to be levelled and focussed for parallel rays. Then the direct image of the slit is received on the cross-wire of the telescope eye-piece and the position of the telescope is noted from the circular vernier moving against the main scale of the spectrometer. The telescope is then rotated through exactly 90° and fixed there. The thin prism is then placed with its centre approximately coinciding with the centre of the prism table. Now by rotating the prism table P (Fig. 82) the face AB of the prism is so placed that the incident ray falling on it will be deviated through 90° by reflection and will be received by the telescope T. In this position AB is inclined at 45° to both the incident and the reflected rays. This position of the prism table is noted from the vernier. The prism table is now rotated so that the ray reflected from the face AC is received by T. Here again AC is inclined at 45° to the incident ray (and also to the reflected ray). This position of the prism table is again noted. The difference between the readings of the two positions of P gives an angle θ, which is short of two right angles by the angle A of the prism. Hence,

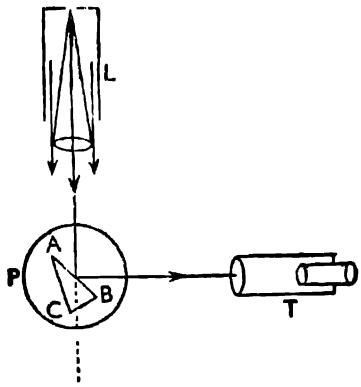


Fig. 82

$$A = 180° - \theta.$$

Now as the face AC in the new position is inclined at 45° to the incident ray, if the prism table is now further rotated through 45° to the greater inclination direction of the prism and fixed there, the ray will now fall exactly normally on the face AC. Now by rotating the telescope the refracted image of the slit is received on its cross-wire and the position is noted. The thin prism is now removed from the prism table and the direct image is again received on the cross-wire of T. The difference of these two positions of T gives δ. Hence knowing A and δ, μ can be calculated from the above relation.

If, however, the prism is not very thin, μ is to be determined from the following general relation,

$$\mu = \frac{\sin (A + D_m)}{\sin A},$$

which has already been established in Art. 52(*a*).

***(d) By Total reflection.**—In order to find out the refractive index of the material of a prism by the method of total reflection, one of the faces *BC* of the prism *ABC* (Fig. 83) should be ground. In case a prism with a ground face is not available a piece of ground glass is to be attached to a face (say, *BC*) by an India rubber band. Now this ground face *BC* is illuminated by condensing the monochromatic light from a sodium flame on to this face with a converging lens. Every point of *BC* becomes a source of light sending rays in all directions. If a ray from such a point source falls on the face *AB* at an angle equal to the critical angle (θ_c) for prism material and air, it will be just totally reflected along *PQ* and will emerge out along *QRT* (Fig. 83). Other rays from the same point incident on *AB* at angles greater than θ_c will be totally reflected and these reflected rays will make the region *AQ* of the prism bright. Again the rays incident at angles less than θ_c will be mostly refracted from the face *AB* on to air and as a result the portion *QC* of the prism will receive only a small amount of light due to reflection from face *AB*. So *QC* will be much darker compared to *AQ*. Hence a sharp line of demarcation will appear on the face *AC* which passes through *Q* and is parallel to the edge of the prism. This line of demarcation may be ascertained by the ordinary pin method or by means of a spectrometer. Hence the angle of emergence i' can be determined. Knowing i' and finding out the angle of the prism ($\angle A$) by any of the two methods (pin method or spectrometer) previously discussed, the refractive index of the material of the prism can be determined from relation (2) given hereafter.

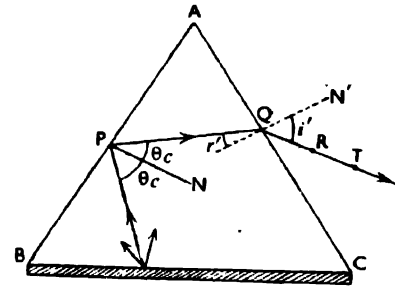


Fig. 83

Since $A = r + r'$, here r being equal to θ_c, $A = r' + \theta_c$;

or, $r' = A - \theta_c$.

But $\mu = \frac{\sin i'}{\sin r'}$. $\therefore$ $\sin i' = \mu \sin r' = \mu \sin (A - \theta_c)$;

or $\sin i' = \mu \sin A \cos \theta_c - \mu \cos A \sin \theta_c$ . . (1)

But as $\sin\theta_c = \frac{1}{\mu}$ and $\cos\theta_c = \sqrt{1-\frac{1}{\mu^2}} = \frac{1}{\mu}\sqrt{\mu^2-1}$, we can rewrite equation (1) as,

$$\sin i' = \mu \sin A.\frac{1}{\mu}\sqrt{\mu^2-1} - \cos A,$$

or, $\sin i' = \sin A.\sqrt{\mu^2-1} - \cos A$;

or, $(\sin i' + \cos A)^2 = (\mu^2-1)\sin^2 A.$

$$\therefore \quad \mu = \sqrt{1+\left(\frac{\sin i' + \cos A}{\sin A}\right)^2} \qquad \ldots \qquad (2)$$

The method can be modified by making the incident monochromatic ray to fall on AB at a grazing incidence (Fig. 84) when the refracted ray will pass along PQ making the angle of refraction equal to θ_c. Now finding out the angle of emergence (i') and the angle of the prism (A). μ of the material of the prism can be found from relation (2).

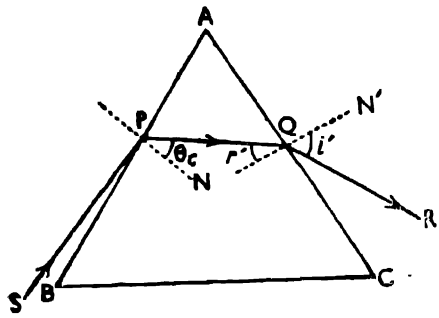


Fig. 84

(iii) Transparent material available in the form of a lens.— (For theory see Chapter on Lens).

In this method the focal length of the lens is first determined by any convenient method as described in Chapter IV. Then from the relation, $\frac{1}{f} = (\mu-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$, the value of μ can be calculated if r_1 and r_2 are determined by auxiliary experiments. r_1 and r_2 of a lens are generally determined in the laboratory by a spherometer with the help of the relation,

$$r = \frac{d^2}{6h} + \frac{h}{2},$$

where d is the mean distance between two of the three fixed legs of the spherometer and h is the elevation of the central movable leg of the spherometer from the plane on which the fixed legs stand. In the actual computation of μ, proper signs of r_1 and r_2 must be put in the above relation.

(iv) Transparent material available in the form of a thin sheet or slab.— The determination of the refractive index of such a sheet or slab is conveniently carried out with the help of

Jamin's and Rayleigh's refractometers. The details of such determinations will be dealt with in the Chapter on Interference (Physical Optics).

(v) **Irregular transparent solid.**—To determine μ of such a solid, it is placed in a container containing a colourless solution whose density can be varied. By adjasting the density when the solid body becomes invisible, the refractive index of both the solution and the solid becomes identical. Now determining μ of the solution by a convenient method, μ of the solid body is obtained.

(B) μ of Liquids

(i) **By a Travelling Microscope** —The method adopted here is almost similar to that used in the case of a solid. The travelling microscope is first adjusted so that its tube is exactly vertical. On the horizontal platform of the microscope is placed a shallow glass beaker which is provided with a scratch mark on the inner surface of its base. The microscope is then focussed on the scratch mark. Let R_1 be the reading of this position of the microscope vernier. The liquid, whose refractive index is to be determined, is then poured in the beaker The microscope is then again focussed on the apparently raised position of the scratch mark. The reading of the microscope vernier now is again noted. Let R_2 be this reading. Now a little lycopodium or cork powder is sprinkled on the surface of the liquid. These powder granules, which evidently float on the surface of the liquid, are then focussed by the microscope. Let R_3 be the reading of the microscope vernier when the microscope is focussed on the lycopodium powder. The real depth of the liquid poured in the beaker is $R_3 - R_1$ and the apparent depth is given by $R_3 - R_2$. So the refractive index of the liquid can be computed from the relation,

$$\mu = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{R_3 - R_1}{R_3 - R_2}.$$

Note :—When the scratch mark is on the outer surface of the base of the beaker, the above relation for μ is valid only when the thickness of the base is very small. When the base has a considerable thickness the method should be modified as follows :—

(1) Place the empty beaker on the platform and pour some quantity of the liquid under test in the beaker. (2) Focus the microscope on the apparent position of the scratch mark as seen through the liquid. Let R_1 represent the vernier reading at this position. (3) Now sprinkle a few grains of lycopodium powder over the liquid and focus the microscope on them. Let R_2 be this reading. (4) Pour some more liquid in the beaker and adjust the microscope so as to have the scratch mark once again in focus. Let R_3 be this reading. (5) Again scatter some lycopodium powder on the liquid surface and focus the microscope on the powder granules. Let R_4 be this reading.

The depth of the newly added liquid column is evidently $=t=R_4-R_2$.

Due to this depth the elevation of the apparent position of the scratch mark $=d=R_3-R_1$.

Hence from Art. 41, $\mu=\frac{t}{t-d}$.

(ii) **By a Concave mirror.**--In this method first of all a concave mirror is placed on the table with its concavity upwards and its centre of curvature C is determined with the help of a horizontal pin mounted on a vertical upright by the method of coincidence. A small quantity of the liquid is then poured into the mirror and the new position C_1 of coincidence of the pin with its real image is found out (by avoiding parallax). Then μ is determined from the relation,

$$\mu=\frac{PC}{PC_1}.$$

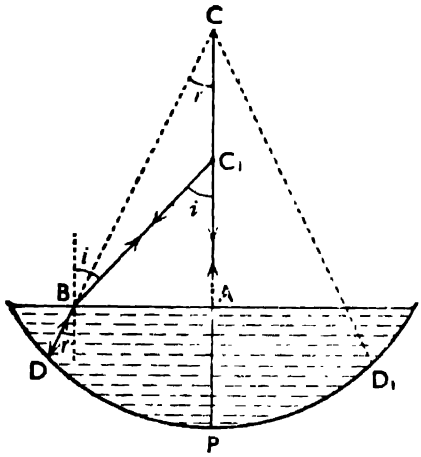


Fig. 85

The above relation can be deduced from the simple principle of refraction at the surfaces of liquid In the absence of liquid, the rays like CD and CD_1 were incident normally on the mirror and were reflected back in the same direction. But when liquid is poured, a ray from the pointer at C_1 proceeds along C_1B and gets refracted at B and if after refraction it follows the path BD it will be normally reflected and retrace its path and finally come back to C_1 (Fig. 85). Thus the image of the pointer and the pointer itself will coincide only under the above condition.

If μ be the refractive index of the liquid, then,

$$\mu=\frac{\sin i}{\sin r}=\frac{\sin BC_1A}{\sin BCA}=\frac{BA/C_1B}{BA/CB}=\frac{CB}{C_1B}.$$

When the quantity of liquid taken is so small as to cover up a small region near the pole P only, then,

$$CB=AC\doteq PC \text{ and } C_1B=C_1A\doteq PC_1.$$

$$\mu=\frac{AC}{AC_1}=\frac{PC}{PC_1}$$

This method is not accurate because of the fact that the surface of a small quantity of liquid is never plane as has been supposed in the calculation. Moreover, as the small thickness of the liquid layer is neglected, an automatic inaccuracy creeps in the result. So Bedford suggested that the mirror should be submerged in the liquid at a depth of one or two centimetres below the surface and μ should be found out from the relation,

$$\mu = \frac{AC}{AC_1}.$$

(iii) **By a plane mirror and a convex lens.**—The experimental arrangements of the method is shown in Fig. 86. A movable horizontal pin is attached to a vertical stand fixed to a horizontal base. On the base is placed a plane mirror M on which a double convex lens L_1 is placed. Now by adjusting the position of the pointer, the real and inverted image of the pointer, as formed by the rays refracted through L_1 and reflected from M, is made to coincide with the pointer itself avoiding parallax During this adjustment the tip of the pin should preferably move along the axis of the lens. Let P be this position of the pointer. Now the focal length f_1 of the lens L_1 is given by the distance between P and the upper surface of L_1 plus *one-third the thickness of the lens* as measured by a slide callipers.

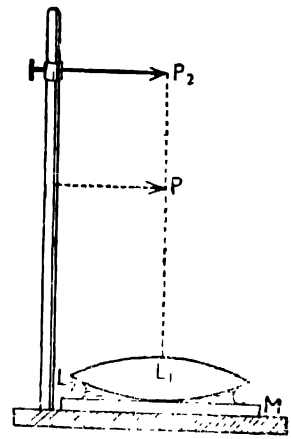


Fig. 86

Now the lens L_1 is removed and a few drops of the liquid under test is put on the surface of the plane mirror. On this liquid the lens L_1 is then placed, the liquid between L_1 and M thereby forming a plano-concave liquid-lens L_2. The focal length of this combination of L_1 and L_2 is again determined by adjusting the pointer at the position of exact coincidence which, suppose, is at P_2. The distance between P_2 and the upper surface of L_1 plus one-third the thickness of L_1 gives the focal length F of the combination.

But from the theory of equivalent lens we know,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \quad \ldots \quad (1)$$

where f_2 = focal length of the liquid lens.

$$\therefore \quad \frac{1}{f_2} = \frac{1}{F} - \frac{1}{f_1}.$$

Thus f_2 is determined.

But the focal length of the plano-convex lens is

$$\frac{1}{f_2}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{\infty}\right)=\frac{\mu-1}{r_1};$$

$$\text{or,}\quad \mu=\frac{r_1+f_2}{f_2} \qquad (2)$$

Thus from relation (2) μ can be determined if r_1, the radius of curvature of the upper surface of the liquid lens, *i.e.* the lower surface of the lens L_1 be determined with the help of a spherometer.

This method is very useful when only a small quantity of liquid is available because the experiment requires only a few drops of liquid as has been referred to above.

(iv) By Total Reflection Method.—

(a) Air-cell method.—When a large quantity of a liquid is available, the refractive index μ of the liquid can be determined by the air-cell method. An air-cell AB (Fig. 87) is constructed by enclosing a thin film of air between two thin parallel plates of glass, A and B, which are separated along the edges by paper and sealed all around. The cell is so mounted that it can be turned about a vertical axis (R) and the angle between any two positions can be measured by a pointer (P), which moves over a graduated circular scale M fixed to R horizontally. The cell, which is attached below the scale, is immersed in the experimental liquid, say, water, contained in a cubical trough (V) of glass. A beam of light from a sodium burner (F), transformed into a parallel beam by the lens L_1 is passed through a slit in the screen S_1 and falls on the air-cell (Fig. 88). The image of the slit is finally received by a telescope (T) focussed for parallel rays and placed on the other side of the trough.

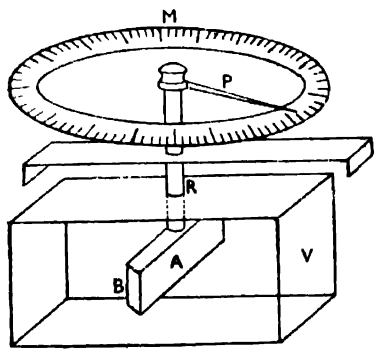


Fig. 87

The observer can see the bright image of the slit when the plates are normal to the beam of light. But if the plates are turned, the brightness of the image gradually diminishes and a position comes when the angle of incidence from glass to air is equal to the critical angle, and any further turning of the plates gives rise to total reflection when the image just disappears. The position of the pointer here is noted. The plates are now

rotated in the opposite direction until the image of the slit just disappears again. This case is shown in the figure by the dotted position of the air-cell.

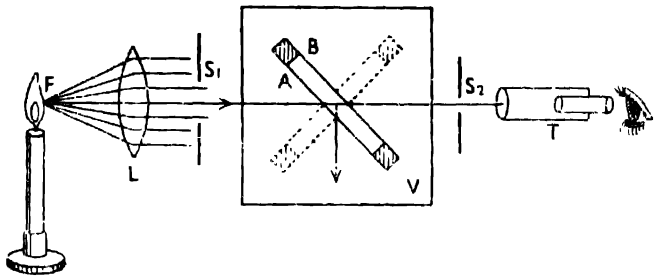


Fig. 88

The angle between the two positions of the air-cell, as given by the pointer, is twice the angle of incidence θ corresponding to the total internal reflection at the interface of glass and the liquid. The refractive index μ of the liquid under test is given by, $\mu = \frac{1}{\sin \theta}$ ... (1)

To establish this relation let us suppose that a ray PQ (Fig. 89) from the sodium flame is incident on the external face of the plate A *of the air-cell* AB at an angle θ. If this ray after refraction along QR is incident on the internal face of the same plate A at the critical angle θ_c for glass and air, there will occur total internal reflection at the interface of glass and air and the ray will just graze along the surface of separation as RS.

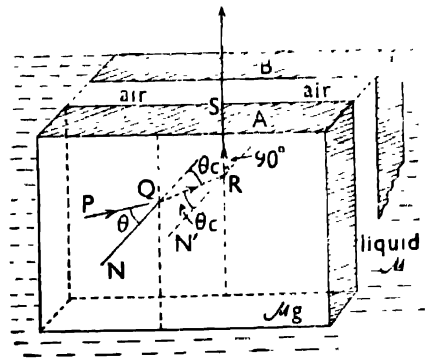


Fig. 89

If μ and μ_g be the refractive indices of the liquid and glass with respect to air respectively, then at Q, we have, $\mu \sin \theta = \mu_g \sin \theta_c$ (2)

($\because$ as the glass plates of the cell are parallel the angle of refraction is also equal to θ_c.)

Again at R, $\mu_g \sin \theta_c = \sin 90^\circ$; or (3)

So substituting in (2) we have,

$$\mu \sin \theta = \frac{1}{\sin \theta_c} \times \sin \theta_c = 1 ;$$

$$\text{or,} \quad \mu = \frac{1}{\sin \theta} .$$

*(b) **Prism method** :—In this method a prism ABC is taken the face BC of which is ground (Fig. 90). If there is no such ground face, a sheet of ground glass should be attached to BC. To the face AC is attached a plate of brass the central portion of which is removed to a depth of about 1 mm. The experimental liquid is to be poured into this scooped out region before attaching the plate to AC. Now light from a monochromatic source (say, a sodium flame) is made to fall on BC, every point of which now becomes a source of light sending rays in all directions. If from one of such points (say, S) a ray falls on the face AC making the angle of incidence equal to the critical angle θ_c for glass and the liquid, it will be totally reflected. The ray will finally come out through the face AB making an angle of emergence i'. When viewed through a telescope pointed towards the direction of the emergent beam the field of view will be divided into two halves by a boundary parallel to the edge of the prism. The cross-wire of the telescope is focussed on this line. Then ascertaining the direction of the normal the angle of emergence i' is found out. Determining the angle A of the prism by a subsidiary experiment and knowing the refractive index μ_1 of the material of the prism the refractive μ_2 of the liquid can easily be found from the relation (3) deduced below.

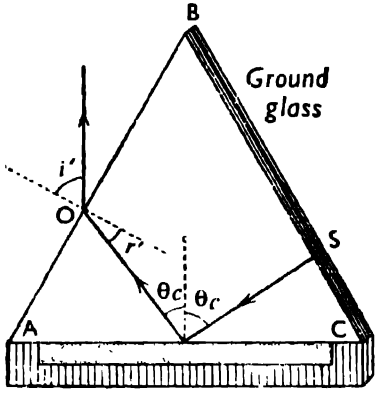


Fig. 90

From the condition of total reflection at the glass-liquid interface at AC, we have,

$$\mu_1 \sin \theta_c = \mu_2 \sin 90^\circ$$

$$\text{or,} \quad \mu_1 \sin \theta_c = \mu_2 \qquad (1)$$

To express θ_c in (1) in term of A and i', we use the relation $A = r + r'$.

Here $r = \theta_c$, $\qquad A = r' + \theta_c ;$

$$\text{or,} \quad \theta_c = A - r' \qquad (2)$$

Again since at O, $\mu_1 = \sin i'/\sin r'$, we have from (1),

$$\mu_2 = \mu_1 \sin(A - r') = \mu_1 \sin A \cos r' - \mu_1 \cos A \sin r'$$

$$= \mu_1 \sin A \sqrt{1 - \frac{\sin^2 i'}{\mu_1^2}} - \cos A \sin i'$$

$$= \sin A \sqrt{\mu_1^2 - \sin^2 i'} - \cos A \sin i' \quad \ldots (3)$$

This method is very convenient when only a small quantity of liquid is available for experimental purpose.

(c) Wollaston's Method.—When only a small quantity of the liquid is available, its refractive index can be conveniently determined by Wollaston's method. The arrangement of the method is shown

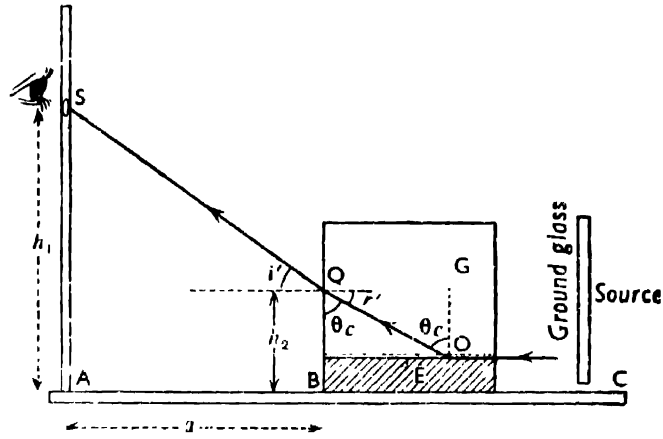


Fig. 91

in Fig. 91, where AC represents a horizontal platform on which is fixed a vertical stand AS. On AS is mounted a slit S whose position on the stand can be adjusted. On the platform a rectangular block of ebonite E (or any other suitable black substance—not acted on by the experimental liquid) is placed at a distance $AB = x$ from the stand. Over this block are put a few drops of the experimental liquid which are made to form a thin film under a rectangular block of glass G placed over the liquid drops. The block G is provided with a horizontal scratch mark at Q. Stray light is made to fall on G through a ground glass sheet illuminated by white light. An eye looking into the left hand face of G will see a horizontal boundary due to total internal reflection. But with white light the line of demarcation will not be sharp. This horizontal boundary will be coloured violet. For violet μ is largest and hence the corresponding critical angle least. So the angle of

emergence i' will be largest. Now this angle of emergence i' corresponding to the critical angle θ_c is noted by adjusting the position of the slit at a height $AS = h_1$ so that the horizontal line of demarcation (*i.e.*) boundary coincides with the scratch mark when viewed through S. If $BQ = h_2$, we have,

$$\tan i' = \frac{h_1 - h_2}{x} \quad \ldots \quad (1)$$

Hence $\sin i'$ can be determined from (1).

Again as total internal reflection takes place at O, we have,

$$\mu = \mu_g \sin \theta_c \quad \ldots \quad (2)$$

where μ = refractive index of the liquid with respect to air and μ_g = refractive index of glass with respect to air.

As evident from the simple geometry of the figure,

$$r' = (90^\circ - \theta_c).$$

$\therefore$ Considering refraction at Q,

$$\mu_g = \frac{\sin i'}{\sin r'};$$

or, $\mu_g \sin (90^\circ - \theta_c) = \sin i'$;

or, $\sin i' = \mu_g \cos \theta_c \quad \ldots \quad \ldots \quad (3)$

Squaring and adding (2) and (3), $\mu^2 + \sin^2 i' = \mu_g^2$;

or, $\mu = \sqrt{\mu_g^2 - \sin^2 i'} \quad \ldots \quad \ldots \quad (4)$

Thus knowing μ_g, μ of the liquid can be determined because i' is known from (1).

***(d) Pulfrich Refractometer.**—Pulfrich refractometer is a commercial type of instrument meant for quick measurement of μ of a solid or a liquid. It consists of a rectangular or triangular block of glass of high refractive index. The top surface and a vertical face (BC) of the block must be mutually at right angles and at the same time highly polished [Fig. 92(a)]. One plane face of the solid substance X whose refractive index is to be determined is placed on the horizontal face AB of the glass block. In the case of a liquid, a transparent cylinder is cemented to the surface AB and the liquid is to be poured in it. In the

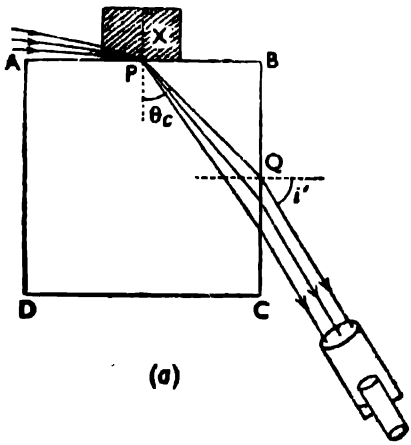


Fig. 92

case of a comparatively thicker liquid simply a drop or two of the same placed on AB may serve the purpose. Monochromatic

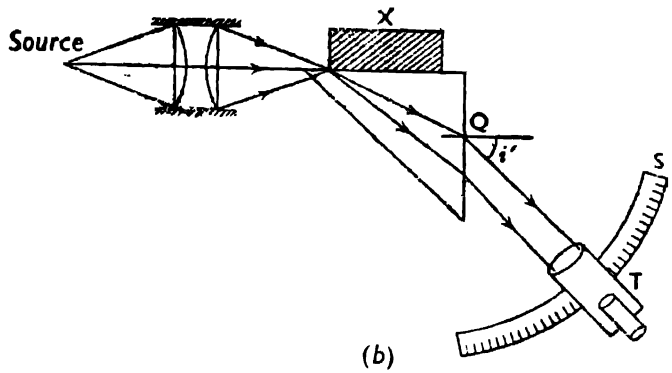


Fig. 92 (*b*)

light from a source is condensed by a converging system of lens L on to the interface of X and AB almost at the grazing incidence. When the corresponding emergent beam at Q is seen through an autocollimating telescope a distinct line of demarcation is seen and this line is made to coincide with the cross-wire of the telescope T [Fig. 92 (*b*)]. The telescope can slide along a vertical circular scale S and its position is noted from the attached vernier. The telescope is then slided up till with the help of the autocollimating device, the position of the normal is ascertained. The reading of the vernier attached to the telescope is again noted. The difference between these two vernier readings gives the angle of emergence i. Hence the refractive index μ_x of the substance X or of the liquid is determined from the relation,

$$\mu_x^2 = \mu_g^2 - \sin^2 i,$$

where μ_g = the refractive index of the glass block. So μ_x can be found since μ_g is known from the instrumental data supplied.

(v) By a Hollow Prism and a Spectrometer.—

In this method a hollow prism with thin walls is to be taken. It is filled up with the experimental liquid and placed on the prism table of a spectrometer, which has been previously levelled and focussed for parallel rays. Now proceeding exactly in the same way as in the case of a solid prism, the angle of minimum deviation D_m and the angle of the prism A are to be determined. Then μ of the liquid is to be calculated from the relation,

$$\mu = \sin\left(\frac{A+D_m}{2}\right) \Big/ \sin\frac{A}{2}.$$

(C) μ of Gas

The refractive index of a gas is generally found with the help of refractometers constructed on the principle of interference. Two such refractometers—Jamin's and Rayleigh's—are commonly used for the purpose. The construction and the working principles of these refractometers have been dealt with in the Chapter on Interference (Physical Optics).

Examples

1. Calculate the limiting angle of incidence for a 60° prism of glass of refractive index 1·5 so that no ray emerges out of the prism.

Ans. The critical angles $\theta_c = \sin^{-1}\frac{1}{\mu} = \sin^{-1}\frac{1}{1{\cdot}5} = 42°$ approx.

Since $A = r + r'$ and $r' = \theta_c$ for no emergence.

$$r = A - \theta_c = 60° - 42° = 18°.$$

At the point of incidence, $\sin i = \mu \sin r = 1{\cdot}5 \times \sin 18°$, $\therefore \quad i = 28°$.

2. The refracting angle of a prism is 60° and the index of refraction is $\sqrt{\frac{7}{3}}$. *What is the limiting angle of incidence of a ray that will be transmitted through the prism?* (C. U. 1945)

Ans. The critical angle $\theta_c = \sin^{-1}\frac{1}{\mu} = \sin^{-1}\sqrt{\frac{3}{7}}$.

Since $A = r + r'$ and $r' = \theta_c$ just for emergence.

$$r = 60° - \sin^{-1}\sqrt{\frac{3}{7}}.$$

At the point of incidence,

$$\sin i = \mu \sin r = \sqrt{\frac{7}{3}} \sin\left(60° - \sin^{-1}\sqrt{\frac{3}{7}}\right)$$

$$= \sqrt{\frac{7}{3}}\left[\sin 60° \cos\left(\sin^{-1}\sqrt{\frac{3}{7}}\right) - \cos 60° \sin\left(\sin^{-1}\sqrt{\frac{3}{7}}\right)\right]$$

$$= \sqrt{\frac{7}{3}}\left[\frac{\sqrt{3}}{2}\cdot\sqrt{\frac{4}{7}} - \frac{1}{2}\sqrt{\frac{3}{7}}\right]$$

$$= \sqrt{\frac{7}{3}}\left[\sqrt{\frac{3}{7}} - \frac{1}{2}\sqrt{\frac{3}{7}}\right] = \sqrt{\frac{7}{3}} \times \sqrt{\frac{3}{7}}\,\frac{1}{2} = \frac{1}{2}.$$

$\therefore \quad i = 30°$.

3. A thin prism of glass of refractive index μ is dipped in a liquid of refractive index μ' contained in a rectangular glass vessel. If a ray of light be almost normally incident on the prism in a direction perpendicular to the emergent face

of the glass vessel show that deviation suffered by the ray on emergence is given by $\delta=(\mu-\mu')A$.

Ans. We know that the deviation suffered by a ray in a prism is,

$$D=(i+i')-(r+r') \quad \ldots \quad (1)$$

Considering refraction at P and Q (Fig. 93),

$\mu' \sin i=\mu \sin r$ and $\mu' \sin i'=\mu \sin r'$

Since the angles i, r, r', i' are small,

$$i=\frac{\mu}{\mu'}r \text{ and } i'=\frac{\mu}{\mu'}r'$$

$\therefore$ From (1),

$$D=\frac{\mu}{\mu'}(r+r')-(r+r')$$

$$=A\left(\frac{\mu-\mu'}{\mu'}\right). \quad [\because A=r+r'].$$

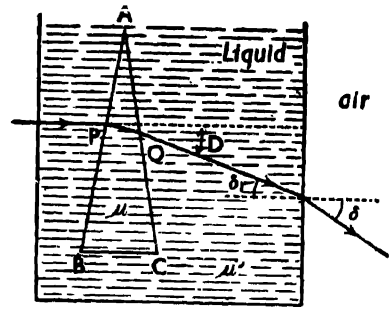


Fig. 93

Again it is evident from the geometry of the figure that $D=\delta_1$.

Considering refraction at R,

$$\frac{\sin \delta}{\sin \delta_1}=\mu', \text{ or, } \delta=\mu'\delta_1=\mu'.D. \quad [\text{ as } \delta \text{ and } \delta_1 \text{ are small }].$$

$$\therefore \quad \delta=\mu' \times A\left(\frac{\mu-\mu'}{\mu'}\right)=A(\mu-\mu').$$

4. *When a convex lens is placed above an empty tank, the image of a mark at the bottom of the tank, which is 45 cms. from the lens, is formed 36 cms. above the lens. When a liquid is poured in the tank to a depth of 40 cms., the distance of the image of the mark above the lens is 48 cms. Find the refractive index of the liquid.*
(C. U. 1944)

Ans. Considering the image formation when the tank is empty,

we get, $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$,

where f = the unknown focal length of the lens, $u=45$ cms. and $v=-36$ cms.

$$\therefore \quad \frac{1}{f}=-\left(\frac{45+36}{45\times 36}\right)=-\frac{81}{45\times 36}=-\frac{1}{20}.$$

$$\therefore \quad f=-20 \text{ cms.}$$

When the liquid is poured to a depth of 40 cms., the liquid surface is 5 cm. below the lens and the apparently raised position of the object is now given by

$$\frac{v_1}{u_1}=\mu \; ; \text{ or, } u_1=\frac{v_1}{\mu}=\frac{40}{\mu}$$

where u_1 = apparent depth, v_1 = real depth and μ is the refractive index of the liquid.

$\therefore$ The distance of this apparently raised object from the lense $\frac{40}{\mu}+5=u_2$

(say), the corresponding image distance v_2 being -48 cm.

$$\therefore \quad -\frac{1}{20}=-\frac{1}{48}-\frac{1}{5+\frac{40}{\mu}}; \text{ or, } \frac{14}{480}=\frac{1}{5+\frac{40}{\mu}}, \text{ or, } \frac{240}{7}=5+\frac{40}{\mu}$$

or, $205\mu=280$; or, $\mu=1{\cdot}366$.

5. A tank contains a parallel-faced block of glass of refractive index 1·6 and of thickness 8 cm. On this is poured a transparent heavy liquid of refractive index 1·5 to a depth of 4·5 cm. above the glass surface. On this liquid floats a column of water of thickness 6 cm. To an observer looking vertically from above, what is the apparent position of a mark on the bottom of the tank ? $\left(\mu \text{ for water} = \frac{4}{3}\right)$.

Ans. Here since the boundaries of the media are parallel the total displacement of the mark at the bottom of the tank is the sum of the displacement due to each medium.

For glass, displacement, $d_1 = t_1\left(1 - \frac{1}{{}_1\mu_g}\right) = 8\left(1 - \frac{1}{1{\cdot}6}\right) = 3$ cm

For liquid, $d_2 = t_2\left(1 - \frac{1}{\mu_l}\right) = 4.5\left(1 - \frac{1}{1{\cdot}5}\right) = 1{\cdot}5$ cm.

For water, $d_3 = t_3\left(1 - \frac{1}{\mu_w}\right) = 6\left(1 - \frac{3}{4}\right) = 1{\cdot}5$ cm.

$\therefore$ Total displacement $= 3 + 1{\cdot}5 + 1{\cdot}5 = 6$ cm.

Hence the apparent position of the mark is 6 cm. from the bottom.

5. Calculate the limiting angle of incidence in a 60° prism so that no light may emerge out of it. (Given μ of the prism is 1·5).

Ans. Since for the limiting angle of incidence the corresponding angle of emergence $i' = 90°$, we have $r' = \theta_c$, the critical angle. But $A = r + r'$.

$$\therefore \quad r = A - r' = A - \theta_c.$$

Since $\sin \theta_c = \frac{1}{\mu}$, we have $\cos \theta_c = \sqrt{\frac{\mu^2 - 1}{\mu}}$.

Hence, $\sin i = \mu \sin r = \mu \sin (A - \theta_c) = \mu \sin A \cos \theta_c - \mu \cos A \sin \theta_c$

$$= \sin A \sqrt{\mu^2 - 1} - \cos A.$$

This gives the limiting value of i in terms of A and μ.

Hence $\sin i = \sqrt{1{\cdot}5^2 - 1} \cdot \sin 60° - \cos 60°$

$$= \sqrt{1{\cdot}25} \times \frac{\sqrt{3}}{2} - \frac{1}{2} = 0{\cdot}4683,$$

$$\therefore \quad i = 27°55'.$$

Questions (Chapter III)

1. The refractive index of carbon bisulphide for red light is 1·634 and the difference between the critical angles for red and blue light at a carbon bisulphide-air interface is 0°56'. What is the refractive index of carbon bisulphide for blue light ?

[*Ans.* 1·669]

2. Show that when a ray is incident on a prism at a given angle i, the deviation of the ray increases with the angle of the prism.

[**Hints :** $D = i + i' - A$; or, $\frac{dD}{dA} = \frac{di'}{dA} - 1$, (since i is constant).

Again, $\sin i' = \mu \sin r'$ $\quad \therefore \quad \cos i' \cdot \frac{di'}{dA} = \mu \cos r' \cdot \frac{dr'}{dA}$.

Since $A = r + r'$ and r is a constant as i is constant, we have, $1 = \frac{dr'}{dA}$

$$\therefore \quad \frac{di'}{dA} = \mu \frac{\cos r'}{\cos i'},$$

$$\text{Thus,} \quad \frac{dD}{dA} = \mu \frac{\cos r'}{\cos i'} - 1.$$

Here $\mu > 1$ and $i' > r'$, *i.e.*, $\cos r' > \cos i'$

Hence, $\frac{dD}{dA}$ is $+ve$]

3. A small object is placed on the principal axis of a concave spherical mirror of radius 20 cm. at a distance of 30 cm. By how much will the position and size of the image alter if a rectangular slab of glass 6 cm. thick of refractive index 1·5 is introduced between the centre of curvature and the object with its faces perpendicular to the principal axis ?

[**Hints :** Due to the introduction of the slab the apparent position of the object shifts by $6\left(1-\frac{1}{1{\cdot}5}\right)=2$ cm. So the new object distance becomes $30-2=28$ cm. Now apply relation $\frac{1}{v}+\frac{1}{u}=\frac{2}{r}$.]

[*Ans.* Image changes its position by $\frac{8}{9}$ cm. and is 1·11 times increased in size.]

4. What is meant by the principal section of a prism ? Show that the deviation of a ray which passes symmetrically through a prism in a principal section is less than that of any other ray. (*cf.* C. U. '65) (C. U. 1945)

5. A parallel beam of light falls normally on the first face of a thin prism and undergoes a deviation of 1°15′ in passing through the prism. The part of the beam which is reflected internally, at the second face and emerges again at the first surface, makes an angle of 6° 30′ with the incident beam. Calculate the angle of the prism and the refractive index of the glass.

[*Ans.* 2° ; 1·625]

6. If the refractive indices of water and glass are $\frac{4}{3}$ and $\frac{8}{5}$ respectively, find the critical angle for a glass and water interface.

Fully describe a method for determining the refractive index of a liquid based on the measurement of the critical angle.

[*Ans.* 62° 45′]

7. Deduce the relation between the true and apparent positions of an object when viewed through a glass plate. Indicate how the formula can be used to measure the refractive index of a liquid. (C.U. 1954)

8. The refracting angle of a glass prism ($\mu = 1{\cdot}5$) is 60° and the angle of incidence of a ray is 45°. Find the deviation.

[*Ans.* 37°22′]

9. Explain the principle of finding the refractive index of a thin prism with a spectrometer by the method of normal incidence giving the adjustments in detail. (C. U. 1939)

10. A glass prism of angle 72° and refractive index 1·66 is immersed in a liquid of refractive index 1·33. What is the angle of maximum deviation for a parallel beam of light passing through the prism.

[*Ans.* 22°22′]

11. State Fermat's principle and deduce the laws of refraction from it. (*cf.* Del. U. '67) (C. U. Hons. 1956 ; Del. U. 1963)

12. The minimum deviation produced by a hollow prism filled with a certain liquid is 30°. If the angle of the prism is 60°, find the refractive index of the liquid.

[*Ans.* $\mu = 1{\cdot}41$]

13. The refractive index of a prism is 1·6434 for a red line in the lithium spectrum and 1·6852 for a violet line in the mercury spectrum. If the angle of the prism be 60°, calculate the angle of minimum deviation for each line.

[*Ans.* 50°30′ (for red), 54°48′ (for violet)]

14. What is a total reflecting prism? Explain with a sketch how such a prism can be used to deflect a beam of light by 90° from its original course.

Deduce an expression for the limiting angle of a prism for which there will be no emergent ray. (C.U. 1959)

15. Calculate the angle of minimum deviation of light through a prism of refracting angle 60′ and of material whose refractive index is 1·5. Also calculate the deviation when light is incident on one face of the prism at glancing incidence.

[*Ans.* 37°10′, 57°55]

16. Describe an experiment for finding the refractive index of a liquid by measuring its apparent depth.

A vessel of depth d cms. is half filled with alliquid of refractive index μ_1 and the other half is occupied by another liquid or refractive index μ_2. Show that the apparent depth of the vessel, viewed perpendicularly, is $\frac{d}{2}\left(\frac{1}{\mu_1}+\frac{1}{\mu_2}\right)$.

(C. U. 1950; '66)

17. What is meant by total reflection and critical angle? Describe two methods of measuring the refractive index of a material by determining the critical angle, one of which is suitable for a solid and the other for a liquid.

18. Describe and explain a method of measuring the refractive index of a transparent liquid using total internal reflection.

Calculate the angle of incidence of a ray on the face AB of an equilateral prism ABC in order that it may be totally internally reflected at the critical angle at the face BC, μ of the prism material being 1·5.

[*Ans.* 27°55′]

19. Derive an expression for the refractive index of the material of a prism in terms of the refracting angle of the prism and the angle of minimum deviation of a ray of light refracted by it.

20. Explain the terms *refractive index* and *critical angle* as used in optics and deduce the relation between them.

Describe how an air-cell is used for the determination of the refractive index of a transparent liquid and give the theory of the method.

21. A ray of light passes through a prism of refractive index $\sqrt{3}$ in such a way that the angle of incidence is twice the angle of emergence and the latter is equal to the angle of the prism. Find the angle of the prism. (Utk. U. 1951)

[*Ans.* 60°]

57. Refraction at a spherical boundary :—Let EP represents the principal section of a spherical boundary, concave in (a) and convex in (b), Fig. 94, between two media of refractive indices μ_1 and μ_2, the former being for the right-hand medium and the latter for the left-hand medium. Suppose μ_2 is greater than μ_1. Let C be the centre of curvature of the boundary surface. Let O be a point object situated on the principal axis OP, P being the pole of the surface. In the present treatment all the distances are measured with P as the origin. Let a ray OE be incident on the spherical surface at E, and let the refracted ray meet the axis, (a) when produced backwards, at I forming a virtual image there, in the case of the concave surface, (b) at I forming a real image, in the case of the convex surface. Let the angle ECO be θ and the angle of

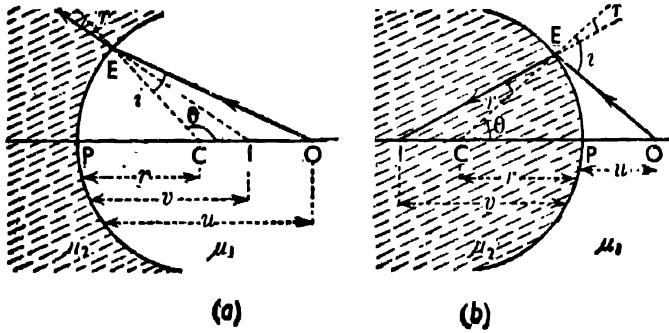


Fig. 94

incidence i, and that of refraction r, as shown in the figures. Then

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \text{ ; or, } \mu_1 \sin i = \mu_2 \sin r.$$

Dividing both the sides by $\sin\theta$, $\mu_1 \dfrac{\sin i}{\sin\theta} = \mu_2 \dfrac{\sin r}{\sin\theta}$ ... (X)

This can be re-written as

$$\mu_1 \frac{\sin(180^\circ - i)}{\sin\theta} = \mu_2 \frac{\sin r}{\sin(180 - \theta)} \quad \text{... (Y}$$

Applying (X) to the case of the concave surface and (Y) to the case of the convex surface,

we have, $\mu_1 \frac{CO}{EO} = \mu_2 \frac{CI}{EI}$, for both the cases.

The equation still holds good, when E coincides with P, *i.e*, for a surface of small aperture. Hence, $\mu_1 \frac{CO}{PO} = \mu_2 \frac{CI}{PI}$.

$\therefore$ For the concave surface,

$\mu_1 \frac{u-r}{u} = \mu_2 \frac{v-r}{v}$, with usual notations, where r stands for the radius of curvature.

Again, for the convex surface,

$$\mu_1 \frac{u+(-r)}{u} = \mu_2 \frac{\{-v-(-r)\}}{-v} = \mu_2 \frac{v-r}{v}.$$

Hence for either type of surface, $\mu_1 \frac{u-r}{u} = \mu_2 \frac{v-r}{v}$;

or, $\mu_1 \left(1-\frac{r}{u}\right) = \mu_2 \left(1+\frac{r}{v}\right)$; $\mu_2 \frac{r}{v} - \mu_1 \frac{r}{u} = \mu_2 - \mu_1$;

or, $r\left(\frac{\mu_2}{v} - \frac{\mu_1}{u}\right) = \mu_2 - \mu_1$; or, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{r}$ ... (1)

Eqn. (1) gives the *conjugate foci relation* for refraction at a single spherical surface. The expression $\frac{\mu_2 - \mu_1}{r}$ is known as the *power* of the spherical refracting surface. The power of the surface which converges the incident parallel light is positive and that of a surface which diverges it is negative.

Special Case.—When light passes from vacuum (or, air) to a medium of refractive index μ, we have $\mu_1 = 1$, and $\mu_2 = \mu$. In this case we have, therefore, $\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r}$ ... ... (2)

58. Principal Foci of a Concave or a Convex Refracting Surface ;—

For a spherical refracting surface $\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r}$, where μ is the *R. I.* of the second medium with respect to the first medium (air), and r = radius of curvature of the surface. The first principal

focal distance (f_1) **is defined to be the value of** u **for which** $v = \infty$. Therefore, a point F_1 on the principal axis at a distance f_1 from the pole P will be the **first principal focus** (Figs. 95 & 96).

The **second principal focal distance** f_2 **is defined to be the value of** v when u is infinite. Therefore, a point F_2 on the principal axis at a distance (f_2) from the pole P will be the **second principal focus.**

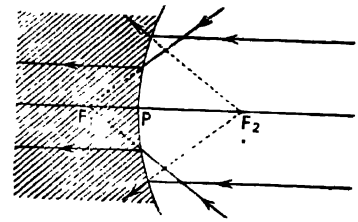


Fig. 95—Concave Refracting Surface.

(a) For a Concave Refractive Surface.—

Here r is positive (Fig. 95) and so, from the equation, $\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r}$, $f_1 = u$ when v is ∞ ; or, $f_1 = -\frac{r}{\mu - 1}$. Again, $f_2 = v$, when u is ∞ ; or, $f_2 = \frac{\mu r}{\mu - 1}$.

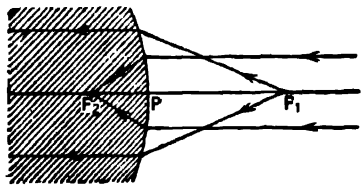


Fig. 96—Convex Refracting Surface.

(b) For a Convex Refracting Surface.—

Here r is negative **(Fig. 96)** and so, from the equation,

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r},$$

$f_1 = u$, when v is ∞ ; or, $f_1 = -\frac{\mu r}{\mu - 1}$

and $f_2 = v$, when u is ∞ ; or, $f_2 = -\frac{\mu r}{\mu - 1}$.

It is to be noted from the above that for concave and convex surfaces f_1 and f_2 bear opposite signs, and also that $f_2/f_1 = -\mu$.

(c) General Case.—

Using the general equation, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{r}$, for, say, a concave refracting surface for which r is positive, f_1 = the value of u when v is ∞ ; or, $f_1 = -\frac{\mu_1 r}{\mu_2 - \mu_1}$ ... (A), and f_2 = the value of v when u is ∞ ; or, $f_2 = \frac{\mu_2 r}{\mu_2 - \mu_1}$ ... (B)

From (A), $r = -\left(\frac{\mu_2 - \mu_1}{\mu_1}\right) f_1$, and from (B), $r = \left(\frac{\mu_2 - \mu_1}{\mu_2}\right) f_2$,

that is, $-\left(\frac{\mu_2 - \mu_1}{\mu_1}\right) f_1 = \left(\frac{\mu_2 - \mu_1}{\mu_2}\right) f_2$;

or, $\mu_2 f_1 + \mu_1 f_2 = 0$ ... ... (1)

59. Conjugate Foci Relation with the centre of curvature as the origin :—

From Art. 57, we know that for refraction at a concave or a convex spherical surface of small aperture,

$$\mu_1 \frac{CO}{PO} = \mu_2 \frac{CI}{PI} \qquad (1)$$

where CO, PO, CI and PI refer to Fig. 94(a) of Art. 57 When the centre of curvature is taken as the origin and p and q are considered as the object and image distances as measured from the new origin C, then for a concave surface (Fig. 94) we may write, $PC = -r$, $CO = p$, $PO = CO + PC = p - r$, $CI = q$, $PI = CI + PC = q - r$.

Hence from (1), $\mu_1 \frac{p}{p-r} = \mu_2 \frac{q}{q-r}$ ... (A)

Again for a convex surface [Fig. 94(b)], $CP = r$, $CO = p$, $PO = CO - CP = p - r$, $CI = -q$ and $PI = CP + CI = r - q$,

$\therefore$ From (1), $\mu_1 \frac{p}{p-r} = \mu_2 \frac{(-q)}{r-q} = \mu_2 \frac{q}{q-r}$ ... (B)

So in both the cases the general relations (A) and (B) are identical.

Hence, $\frac{\mu_1 p}{p-r} = \frac{\mu_2 q}{q-r}$;

or, $\mu_1 pq - \mu_1 pr = \mu_2 pq - \mu_2 qr$; or, $\mu_2 qr - \mu_1 pr = (\mu_2 - \mu_1) pq$.

Dividing throughout by pqr, we have, $\frac{\mu_2}{p} - \frac{\mu_1}{q} = \frac{\mu_2 - \mu_1}{r}$ ... (2)

This is the conjugate foci relation with the centre of curvature as the origin.

60. Conjugate Foci relation for refraction at a spherical surface with the principal foci as the origin :—

When the pole is taken as the origin, the conjugate foci relation for refraction at a single surface is given by,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{r} ;$$

or, $\frac{1}{v}\cdot\frac{\mu_2 r}{\mu_2-\mu_1}-\frac{1}{u}\cdot\frac{\mu_1 r}{\mu_2-\mu_1}=1$,

or, $\frac{f_2}{v}+\frac{f_1}{u}=1$, $\left(\because\ f_1=-\frac{\mu_1 r}{\mu_2-\mu_1}\text{ and }f_2=\frac{\mu_2 r}{\mu_2-\mu_1}\right)$

$\therefore\quad uf_2+vf_1=uv$;

or, $uv-uf_2-vf_1+f_1f_2=f_1f_2$, (adding f_1f_2 to both sides) .

or, $u(v-f_2)-f_1(v-f_2)=f_1f_2$,

or, $(u-f_1)(v-f_2)=f_1f_2$ ... ... (1)

If U and V be the object and image distances as measured from the first and second principal foci respectively,

then, $U=(u-f_1)$ and $V=(v-f_2)$.

$\therefore\quad UV=f_1f_2$ ... ... (2)

This is the required conjugate foci relation with the principal foci as the origin.

61. **Magnification of a spherical Refracting surface** :—In Fig. 97 a small object QB is in front of a refracting concave spherical surface EP separating the two media of refractive indices μ_1 and μ_2, the object being situated in the first medium. A ray BP along the axis is incident on the surface at P normally and is refracted without deviation. A second ray BE is incident at E at an angle i and is refracted in the denser medium of refractive index μ_2 at an angle r. Thus refracted ray *appears to proceed* along bE. Hence b is the point of intersection of these two directions of the refracted rays. So bq represents the virtual image of BQ. The magnification in this can be considered in three different ways : (*a*) linear (also called transverse or lateral) magnification, (*b*) angular magnification and (*c*) longitudinal magnification.

(a) **Transverse, Lateral or Linear magnification.**—Linear magnification is defined as the ratio of the size of the image to the size of the object. Hence,

$$m=\frac{bq}{BQ}$$

$$=\frac{cb}{CB}\quad\frac{Pb-PC}{PB-PC}=\frac{v-r}{u-r} \qquad (1)$$

For refraction at a spherical surface, we have,

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{r}\ ;\ =\frac{\mu_2}{r}-\frac{\mu_1}{r}.$$

$$\therefore\quad \mu_2\left(\frac{1}{v}-\frac{1}{r}\right)=\mu_1\left(\frac{1}{u}-\frac{1}{r}\right),$$

$$\text{or,}\quad \mu_2\frac{r-v}{rv}=\mu_1\frac{r-u}{ru};$$

$$\text{or,}\quad \frac{v-r}{u-r}=\frac{\mu_1}{u}\times\frac{v}{\mu_2}.$$

Hence substituting in (1), we get, $m=\dfrac{v-r}{u-r}=\dfrac{\mu_1 v}{\mu_2 u}$

(b) Angular magnification : Helmholtz's Magnification formula.— Let θ_1 be the angle made by the incident ray from B with the axis (Fig. 97). If the corresponding refracted ray appearing to diverge from b makes an angle θ_2 with the axis, the *angular magnification* is given by,

$$m_\theta=\frac{\tan\theta_2}{\tan\theta_1}$$

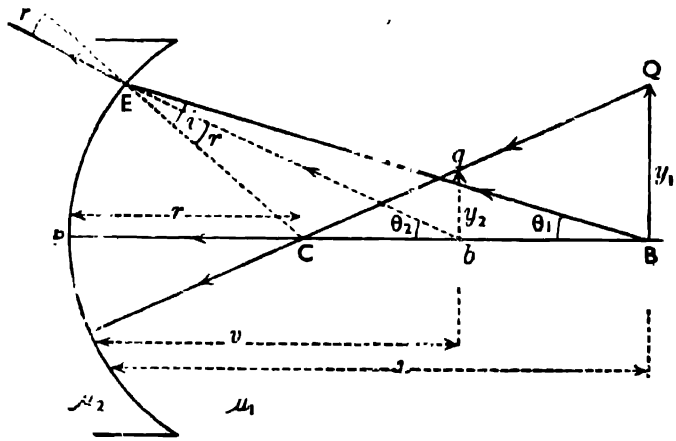


Fig. 97

But when θ_1 and θ_2 are small,

$\dfrac{\tan\theta_2}{\tan\theta_1}=\dfrac{EP/v}{EP/u}=\dfrac{u}{v}=\dfrac{\mu_1}{\mu_2}\times\dfrac{\mu_2 u}{\mu_1 v}=\dfrac{\mu_1}{\mu_2}\cdot\dfrac{1}{m}$, where m is the linear magnification.

Thus $m_\theta=\dfrac{\mu_1}{\mu_2}\cdot\dfrac{1}{m}$ ... ... (2)

So the angular and the linear magnifications are related to each other according to equation (2).

Again if $bq=y_2$ and $BQ=y_1$, we have,

$$\frac{y_2}{y_1}=m=\frac{\mu_1 v}{\mu_2 u} \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

But the angular magnification, $\dfrac{\tan\theta_2}{\tan\theta_1}=\dfrac{u}{v}$ ... ... (4)

$\therefore$ Multiplying (3) by (4), we get,

$$\frac{y_2 \tan \theta_2}{y_1 \tan \theta_1} = \frac{\mu_1}{\mu_2}.$$

$$\therefore \quad \mu_1 y_1 \tan \theta_1 = \mu_2 y_2 \tan \theta_2 \qquad \ldots \qquad \ldots \quad (5)$$

This is known as **Helmholtz's magnification formula or, Smith-Helmholtz relation.**

This formula can be extended for any number of refracting surfaces, because at each refraction the product of μ, y and $\tan \theta$ will be the same on both the sides of the surface. So we have, $\mu_1 y_1 \tan \theta_1 = \mu_2 y_2 \tan \theta_2 = \ldots = \mu_{n-1} y_{n-1} \tan \theta_{n-1} = \mu_n y_n \tan \theta_n$.

Thus from Helmholtz's formula, we find that the product of the linear and the angular magnifications is equal to $\frac{\mu_1}{\mu_2}$. If only the paraxial rays are considered such that θ_1 and θ_2 are very small. Helmholtz's formula reduces to,

$$\mu_1 y_1 \theta_1 = \mu_2 y_2 \theta_2.$$

This formula is commonly known as the **Helmholtz-Lagrange relation.** This relation can also be generalised for any number of refracting surfaces and the generalised form will be,

$$\mu_1 y_1 \theta_1 = \mu_2 y_2 \theta_2 = \ldots = \mu_{n-1} y_{n-1} \theta_{n-1} = \mu_n y_n \theta_n$$

With reference to Fig. 97 we have, $\mu_1 \sin i = \mu_2 \sin r$

$$\text{or,} \quad \frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \qquad \ldots \qquad (6)$$

But in $\triangle BEC$, $\frac{\sin i}{\sin \theta_1} = \frac{CB}{EC}$,

and in $\triangle bEC$, $\frac{\sin r}{\sin \theta_2} = \frac{Cb}{EC}$,

$$\therefore \quad \frac{\sin i}{\sin r} \cdot \frac{\sin \theta_2}{\sin \theta_1} = \frac{CB}{Cb}.$$

Substituting in eqn. (6), $\frac{\sin \theta_2}{\sin \theta_1} = \frac{CB}{Cb} \times \frac{\mu_1}{\mu_2}$. But $\frac{CB}{Cb} = \frac{y_1}{y_2}$.

$$\therefore \quad \frac{\sin \theta_2}{\sin \theta_1} = \frac{y_1 \mu_1}{y_2 \mu_2}; \quad \text{or,} \quad \mu_1 y_1 \sin \theta_1 = \mu_2 y_2 \sin \theta_2 \qquad (7)$$

Equation (7) is known as **Abbe's sine condition.** If the relation (7), $\frac{\sin \theta_1}{\sin \theta_2}$ remains constant, the linear magnification is also constant. So if rays proceeding from a point object on the axis pass through different zones of a refracting surface satisfying the condition $\frac{\sin \theta_1}{\sin \theta_2}$ = a constant, then they will produce image of the same size.

Moreover, Abbe's sine condition applies even when θ_1 and θ_2 are not small and is equally true for any system of refracting surfaces.

(c) **Longitudinal Magnification.**—If du and dv be the extension of the object and the image along the principal axis of the refracting surface, the longitudinal magnification is given by

$$m_l = \frac{dv}{du}.$$

Differentiating the relation $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{r}$ with respect to u, we get,

$$-\frac{\mu_2}{v^2}\frac{dv}{du} + \frac{\mu_1}{u^2} = 0$$

$$\therefore \quad \frac{dv}{du} = \frac{\mu_1}{\mu_2}\cdot\frac{v^2}{u^2} \qquad \ldots \quad \ldots \quad \ldots \quad (8)$$

This is the expression for the longitudinal magnification of a refracting spherical surface. m_l can also be correlated with the linear magnification m.

We know, $m = \frac{\mu_1 v}{\mu_2 u}$. $\therefore \quad \frac{dv}{du} = m^2 \times \frac{\mu_2}{\mu_1} \qquad \ldots \quad \ldots (9)$

Examples

(1) *When seen normally through the flat surface, the greatest thickness of a plano-convex lens appears to be 2·435, but when seen normally through the curved surface, it appears to be 2·910 cm. The actual thickness is 3·665 cm. Find the refractive index of the lens material and the radius of curvature of the curved surface.*

Ans. When viewed normally through the plane refracting face, we have,

$$\mu = \frac{\text{real thickness}}{\text{apparent thickness}} = \frac{3{\cdot}665}{2{\cdot}435} = 1{\cdot}505.$$

When viewed through the curved surface, the relation that applies is,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{r}.$$

Here, $\mu_2 = 1$, $\mu_1 = 1{\cdot}505$, $v = 2{\cdot}910$ and $u = 3{\cdot}665$.

$$\therefore \quad \frac{1}{2{\cdot}91} - \frac{1{\cdot}505}{3{\cdot}665} = \frac{1 - 1{\cdot}505}{r} = \frac{-0{\cdot}505}{r}.$$

Hence $r = 7{\cdot}52$ cm.

(2) *A small red mark on a glass sphere of radius 10 cm. is seen through the sphere along the axis. Find the position, magnitude and the nature of the image, (Given μ for glass = 3/2).*

Ans. **Since the red mark is seen along the axis through the spherical surface opposite the mark, the conjugate foci relation is,**

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{r}.$$

Here $\mu_2 = 1$, $\mu_1 = \frac{3}{2}$, $u = 2 \times 10 = +20$ cm., $r = +10$ cm.

$$\therefore \quad \frac{1}{v} - \frac{1{\cdot}5}{20} = \frac{1 - 1{\cdot}5}{10} = \frac{-0{\cdot}5}{10} = -\frac{1}{20}.$$

$$\therefore \quad v = \frac{20}{1{\cdot}5 - 1} = \frac{20}{0{\cdot}5} = +40 \text{ cm.}$$

So the image is formed on the same side as the red mark and is (40 − 20) = 20. cm. from the mark.

Its magnification $= \frac{\mu_1}{\mu_2} \cdot \frac{v}{u} = \frac{1\cdot5}{1} \times \frac{40}{20} = 3$.

So the image is 3 times magnified and is virtual and erect in nature, because m is $+ve$.

(3) *A glass hemisphere 10 cm. in diameter and of $\mu = 1\cdot5$ is silvered over its curved surface. There is an air bubble in the glass 3 cm. from the plane surface along the axis. Find the positions of the images of this bubble seen by an observer looking along the axis into the flat surface of the hemisphere.*

Ans. For the first image we apply the relation, $\mu = \frac{\text{real depth}}{\text{apparent depth}}$.

$$\therefore \quad 1\cdot5 = \frac{3}{\text{apparent depth}},$$

$$\text{or,} \quad \text{apparent depth} = \frac{3}{1\cdot5} = 2 \text{ cm.}$$

So the virtual first image is formed at a distance of 2 cm. from the flat surface.

Again for the reflection from the concave spherical mirror, we have,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}.$$

Here $u = 5 - 3 = 2$ cm.

$$\therefore \quad \frac{1}{2\cdot5} = \frac{1}{v} + \frac{1}{2}; \text{ or, } \frac{1}{v} = \frac{2 - 2\cdot5}{5} = -\frac{1}{10} \quad \therefore \quad v = -10 \text{ cms.}$$

Hence a virtual image is formed at a distance of 10 cms. from the curved surface on the side away from its centre. Hence its distance from the plane surface is $(10+5) = 15$ cms. When viewed from the flat surface normally this virtual image approaches this surface. We know,

$$\mu = \frac{\text{real depth}}{\text{app. depth}}; \quad \text{or,} \quad 1\cdot5 = \frac{15}{\text{app. depth}}.$$

$\therefore$ Apparent position of the second virtual image $= 15/1\cdot5 = 10$ cms. from the flat surface.

(4) *A small piece of paper is stuck on a glass sphere of 5 cm. radius and viewed through the glass from a position directly opposite. Find the position of the image. Find also the position of the image formed, by the sphere, of an object at infinity.*

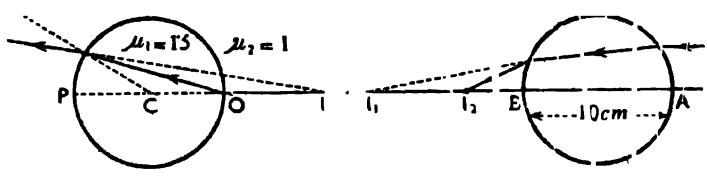


Fig. 98

Ans. **First Part :**

Let O be the piece of paper, Fig. 98(*a*), and OP be the direction in which the image is seen. We know,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{r}.$$

Here $\mu_2=1$, $u=+10$ cms., $r=+5$ cms. and $\mu_1=1{\cdot}5$.

$$\therefore \quad \frac{1}{v}=\frac{1-1{\cdot}5}{5}+\frac{1{\cdot}5}{10}=-\frac{{\cdot}5}{5}+\frac{1{\cdot}5}{10}=\frac{{\cdot}5}{10}=\frac{1}{20}.$$

$$\therefore \quad v=+20 \text{ cms.}$$

So the image is formed at 20 cms. from P on the same side as O.

Second Part :

Let I_1 be the position of the image formed by refraction at the surface at A [Fig. 98 (*b*)], when $u=\infty$, $\mu_2=1{\cdot}5$, $\mu_1=1$.

$$\therefore \quad \frac{1{\cdot}5}{v}-\frac{1}{\infty}=\frac{1{\cdot}5-1}{-5}. \quad (r=-5 \text{ cms.});$$

$$\text{or,} \quad \frac{1{\cdot}5}{v}=-\frac{1}{10}; \text{ or, } v=-15 \text{ cms.}=AI_1.$$

$$\therefore \quad BI_1=-15+10=-5 \text{ cms.}=\text{object distance for the refraction at } B$$

If $BI_2=v_1$ be the final image distance as measured from the pole of the second surface, then as in this case $\mu_2=1$ and $\mu_1=1{\cdot}5$ and $r=+5$ cms. we have,

$$\frac{1}{v_1}+\frac{1{\cdot}5}{5}=\frac{1-1{\cdot}5}{5}=-\frac{1}{10};$$

$$\text{or,} \quad \frac{1}{v_1}=-\frac{1}{10}-\frac{3}{10}=-\frac{2}{5}.$$

$$\therefore \quad v_1=-2{\cdot}5 \text{ cms.}$$

(5) *In a sphere of radius 10 cms. made of glass of refractive index 1·5, there is an air bubble situated 2 cms. from the centre and between the centre and an observer, where does the bubble appear to be ?*

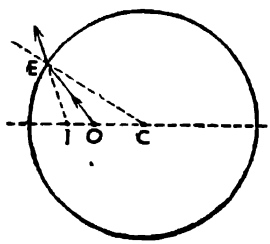


Fig. 99

Ans. Let O in Fig. 99 represent the position of the bubble and I that of its image as seen by the observer. Here the refraction occurs at E at a concave surface from glass to air.

Hence $\mu_2=1$, $\mu_1=1{\cdot}5$, $r=+10$ cms. and $u=10-2=8$ cms.

Hence from the relation, $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{r}$,

we have, $\frac{1}{v}-\frac{1{\cdot}5}{8}=\frac{1-1{\cdot}5}{10}=-\frac{1}{20}$;

or, $\frac{1}{v}=-\frac{1}{20}+\frac{3}{16}=\frac{-4+15}{80}=\frac{11}{80}$.

$v=7\frac{3}{11}$ cms. from the pole of the refracting surface inside the sphere.

LENSES

62. Lens :—A Lens is a transparent refracting medium bounded by two surfaces of regular geometrical form, such as spherical, cylindrical, etc., the spherical form being more common. One surface may, if required, be plane. A lens bounded by

cylindrical surfaces is called a cylindrical lens. Lenses are divided into two classes—

(1) **Convex** (or, **converging**).—These lenses are thicker in the middle than at the edges.

Such a lens may have any one of the following three forms :—

(*i*) *Double convex* or *Bi-convex* [Fig. 100(*a*)]— of which both the surfaces are convex.

(*ii*) *Plano-convex*—one side plane and one side convex [Fig. 100(*b*)]

(*iii*) *Concavo-convex* or *convex meniscus*—one side concave and the other convex [Fig. 100(*c*)].

(2) **Concave** (or, **diverging**).—These lenses are thinner in the middle than at the edges. They may be of the following three forms :—

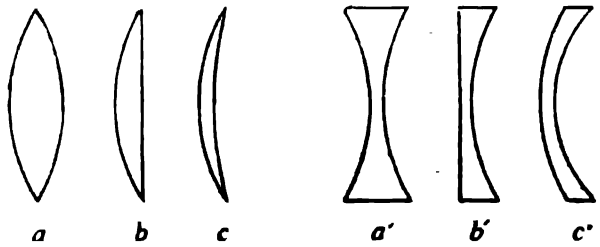


Fig. 100

(*i*) *Double-concave* or *Bi-concave* [Fig. 100(*a'*)], (*ii*) *Plano-concave* [Fig. 100(*b'*)], and (*iii*) *Convexo-concave* or *concave meniscus* [Fig. 100(*c'*)].

63. A lens acts like a Combination of a Number of Prisms :— The reason why convex lenses are described as converging and concave as diverging will be understood by reference to Figs. 101, (*a*) and (*b*). From Fig. 101 it will be seen that, though in reality a lens has a continuous curved surface, it may be looked upon as a combination of two sets of truncated prisms arranged symmetrically on the opposite sides of an axis *AB*. In a convex lens [Fig. 101(*a*)], the prisms have their bases turned towards the axial line and the angles of the successive

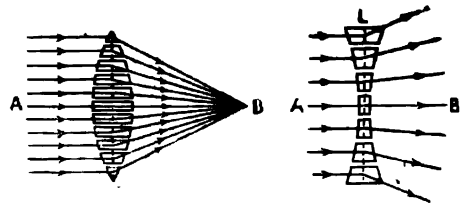


Fig. 101

prisms increase from the centre to the edge of the lens. A ray passing through such a prism will be deviated towards the base, provided the refractive index of the material relative to the surroundings is greater than unity, which is commonly the case. Thus the rays of a parallel bundle incident on such a lens will be brought nearer together after refraction and may be made to converge to a single point, and that is why a convex lens is termed a *convergent* lens.

In a concave lens [Fig. 101 (*b*)], the prisms have their refracting angles turned towards the axis AB. So in this case the rays after refraction will bend away from the axis being turned towards the base of the prisms, and so the emergent beam will be rendered *divergent*. This is why a concave lens is termed a *divergent* lens.

64. The optical centre of a Lens :—In order to find out a general expression for the position of the optical centre, let us consider a convexc-concave (or concave meniscus) lens. Since all the distances that come into the calculation are positive in this case, the relations obtained will be of a general nature. Let C_1 and C_2 be the centres of curvature of the two surfaces of the lens and t the thickness of the lens along the axis. Draw two radii C_1A and C_2B to the two surfaces parallel to each other. Since these radii are normal to the surfaces at A and B, the tangent planes at these points are also parallel. So a ray PAB incident at A emerges out at B in the parallel direction as in a parallel faced slab. The ray is laterally displaced and in a tin lens displacement is very small. The refracted ray through the lens appears to pass through the point O on the principal axis.

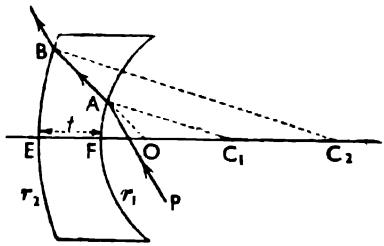


Fig. 102

In the similar $\triangle s$ OC_1A and OC_2B, (Fig. 102),

we have, $\frac{C_1A}{C_2B} = \frac{OC_1}{OC_2}$.

But $C_1A = C_1F = r_1$ and $C_2B = C_2E = r_2$.

$$\therefore \frac{r_1}{r_2} = \frac{C_1F}{C_2E} = \frac{OC_1}{OC_2} = \frac{C_1F - OC_1}{C_2E - OC_2} = \frac{FO}{EO}.$$

$$\therefore \frac{OF}{OE} = \frac{r_1}{r_2} = \text{a constant.}$$

Thus O divides EF (externally) in a fixed ratio, and O is, therefore, a fixed point. This point is called the **Optical centre** of the lens. It is thus found that any ray, which, after passing through

the lens, emerges out undeviated, will pass through the point O. Thus, *if a ray of light passes through a lens in such a way that the direction of the emergent ray is parallel to the direction of the incident ray, the path of the ray inside the lens intersects the axis* (C_1C_2), *called the principal axis, at a fixed point, which is called the* **Optical centre** *or the* **centre of the lens.**

Remember that a ray passing through the Optical centre is displaced but not deviated; and, in the case of a thin lens, the displacement is so small that it can be neglected. That is, *in the case of a thin lens, a ray passing through the optical centre* may be taken to pass straight without either deviation or displacement.

The position of the Optical centre :—

We know in a lens (Fig. 102), $\frac{OF}{OE}=\frac{r_1}{r_2}$.

$\therefore \quad \frac{OF}{OE-OF}=\frac{r_1}{r_2-r_1}$, or, $\frac{OF}{EF}=\frac{r_1}{r_2-r_1}$.

$\therefore \quad OF=t\,\frac{r_1}{r_2-r_1} \quad (\because \ t=EF) \qquad \ldots \qquad \ldots \qquad (1)$

Similarly, $OE=t\,\frac{r_2}{r_2-r_1} \qquad \ldots \qquad \ldots \qquad (2)$

Particular cases :—

(*a*) *Double convex lens* (Fig. 103).—
Here r_1 is $-ve$ and r_2 is $+ve$.

$\therefore \quad OF=-t\,\frac{r_1}{r_1+r_2}$ and $OE=t\frac{r_2}{r_1+r_2}$.

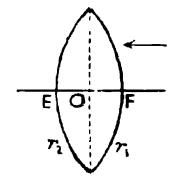


Fig. 103

When the lens is equiconvex $r_1=r_2$
$=r$ (numerically)

$\therefore \quad OF=-t/2$ and $OE=t/2$.

For a transparent sphere, $r_1=r_2=r$ (numerically) and $t=2r$. $\therefore \quad OF=-r$ and $OE=r$.

Hence the optical centre is the centre of the sphere.

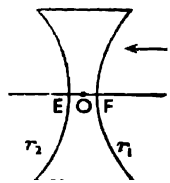


Fig. 104

(*b*) *Equiconcave lens* (Fig. 104).—
Here $r_1=+r$ and $r_2=-r$.

$\therefore \quad OF=\frac{tr}{-2r}=-t/2$ and $EO=\frac{t(-r)}{-2r}=\frac{t}{2}$.

So both in the equiconvex as well as in the equiconcave lenses, the optical centre is equidistant from the poles.

(*c*) *Plano-convex lens* (**Fig. 105**).—

Here $r_1 = -r$ and $r_2 = \infty$

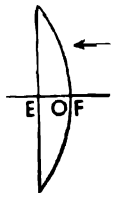


Fig. 105

$$\therefore \quad OF = \frac{t(-r)}{\infty + r} = 0 \text{ and } OE = t\,\frac{r_2}{r_2 - r_1}$$

$$= t - \frac{1}{1 - \frac{r_1}{r_2}} = t\,\frac{1}{1 - \frac{r}{\infty}} = t.$$

So the optical centre lies on the pole of the curved surface. Similar is the case with plano-concave lens.

(*d*) *Convexo-concave lens.*—

Here r_1 and r_2 are both $+ve$ but $r_2 > r_1$.

$$\therefore \quad OF = \frac{tr_1}{r_2 - r_1}, \text{ and } OE = \frac{tr_2}{r_2 - r_1}. \quad \text{So } OE > OF.$$

Here O is shifted towards the more curved surface.

65. Some Important Definitions :—

Principal Axis.—The line that joins the centres of curvature of the two bounding surfaces of a lens is termed the **principal axis** of the lens. In Fig. 102, where a double convex lens has been shown, the line C_1C_2 joining the centres C_1 and C_2 of the two surfaces AF and BE of the lens gives the principal axis of the lens.

Principal Section—The cut-away section of a lens made by a plane through the principal axis is termed its **principal section.**

Principal Focus.—A lens has two principal foci, the first principal focus and the second principal focus. The second principal focus is conventionally called the principal focus of a lens.

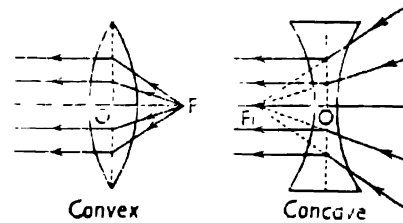


Fig. 106

The first principal focus of a lens is a point on its principal axis such that the rays diverging from it (in the case of a convergent lens), or tending to converge to it (in the case of a divergent lens), are rendered parallel to the axis after refraction through the lens (Fig. 106).

The second principal focus of a lens is a point on the principal axis such that the rays incident on the lens in a direction parallel to the axis either actually converge to it (in the case of convex lens), or appear to diverge from it (in the case of a divergent lens),

after refraction through the lens (Fig. 107). This second principal focus is conventionally referred to as the **principal focus** of a lens.

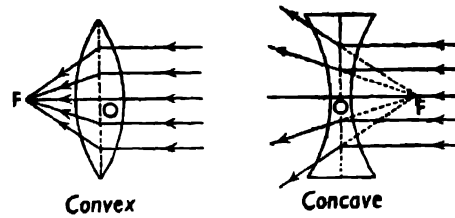


Fig. 107

Focal Length—The distance of either of the two principal foci of a lens, when measured from the optical centre, is the same if the lens is surrounded by the same medium on both the sides. The distance is called the focal length of a lens. Conventionally, the distance of the second principal focus from the optical centre is taken as the focal length of a lens.

Focal Plane.—A plane perpendicular to the principal axis of a lens at a focal point is known as the focal plane for a lens. A lens has two focal planes corresponding to its two focal points. The focal plane through the first focal point is called *the first focal plane*, and the focal plane through the second focal point is called the *second focal plane*.

The Aperture of a Lens.—The diameter of the circular boundary of the lens is known as its *linear aperture*. The *relative aperture of a lens* is defined as the ratio of the *diameter* to the *focal length* of the lens. This is a measure of the opening of the lens required for estimating the amount of light which will pass through it. As for example, if in a camera a lens requires twice as long an exposure as another of the same focal length, it is because the *fast* lens has a greater aperture than the *slow* lens. If two lenses are of the same diameter but one has twice the focal length of the other, the lens with the short focus will have the greater aperture. In books of photography, or on cameras, the lens is characterised by the term *f/number*, where any value, say 3·5, or 4, *etc* assigned to the *number*' means the focal length (f) of the lens divided by its diameter. The *number* indicates the speed of the lens, for the above described *number* determines the exposure on the film needed under certain lighting conditions. Thus a camera lens marked *f*/3·5, meaning that the focal length is 3·5 times the diameter, will be $(8/3{\cdot}5)^2$, *i.e*, about 5·25 times faster than one *f*/8 lens, and would need 5·25 times lesser exposure.

Thin Lens.—A lens is said to be *thin* when the thickness of the lens is negligible in comparison with the radii of curvature. The lenses used for ordinary experiments nearly satisfy this condition. For such a lens the points E, O and F (Fig. 102) may be regarded as coincident, and it may be said that the optical centre is the point at which the lens and the axis intersect.

The defects of an ordinary lens arise from the fact that all rays coming from a point object do not all converge to or appear to diverge from a single point after passing through the lens. The image thus appears to be diffuse and does not give a correct representation of the object.

66. (I) Graphical Construction for Images :—The position of an image formed by a thin lens may be traced by drawing the courses of any two of the following rays :—

(*a*) *A ray falling on the lens in a direction parallel to the principal axis passes through, or appears to diverge from, the principal focus after refraction.*

(*b*) *A ray passing through the optical centre of the lens emerges out undeviated and undisplaced.*

(*c*) *A ray passing through the first principal focus or appearing to pass through it, will, after refraction through the lens, emerge out in a direction parallel to the principal axis.*

The point where the above two emergent rays converge, as in the case of a *real* image, or from which they appear to diverge, as in the case of a *virtual* image, is the **image** of the luminous point.

(II) Rules of Sign :—These are the same as given for mirrors with the difference that the term *pole* in the case of mirrors should be exchanged for *optical centre* in the case of lenses.

So the focal length of a convex lens is negative and that of a concave lens is positive.

67. Refraction through Double Surfaces (i.e. a Lens) :—When a ray of light passes through a lens, it successively suffers refraction at the two surfaces of the lens ; it first crosses a surface, air to glass and then emerges through a surface, glass to air. Let O be a point object on the axis of a lens at a distance u from the pole P on the near surface, called the first surface, whose radius of curvature, say, is r_1 (Fig. 108). Let the image formed by this surface be I' at a distance v' from the pole P. For the refraction at this surface we have,

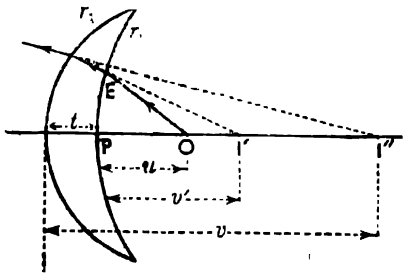


Fig. 108

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{r} \quad \ldots \quad \ldots \quad (1)$$

where, in this case, $\mu_1 = 1$ (air) ; $\mu_2 = \mu$ (glass), $r = r_1$ and $v = v'$, that is

$$\frac{\mu}{v'} - \frac{1}{v} = \frac{\mu - 1}{r_1} \quad \ldots \quad \ldots \quad (2)$$

Let the radius of curvature of the second surface be r_2 and the thickness of the lens t. The distance of the image I' measured from the pole of the second surface will be $(t+v')$. The image I' serves as a virtual object for the second surface which suppose, forms a final virtual image at I''. Let I'' be at a distance v from the pole of the second surface.

We have, for refraction at the second surface,

$u=t+v'$, $r=r_2$, $\mu_2=1$ and $\mu_1=\mu$, for here the refraction is from glass to air. Putting these values in equation (1) we have

$$\frac{1}{v}-\frac{\mu}{t+v'}=\frac{1-\mu}{r_2}.$$

In the case of a thin lens t is negligible compared to v'

$$\therefore \quad \frac{1}{v}-\frac{\mu}{v'}=\frac{1-\mu}{r_2} \qquad \ldots \quad \ldots \quad \ldots \quad (3)$$

Adding equations (2) and (3), we get,

$$\frac{1}{v}-\frac{1}{u}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \qquad \ldots \quad \ldots \quad \ldots \quad (4)$$

When $u=\infty$, $v=f$ (focal length of the lens). Therefore, putting these values in (4),

we have, $\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$ $\ldots \quad \ldots \quad (5)$

Substituting this value of f in (4), we get,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \qquad \ldots \quad \ldots \quad \ldots \quad (6)$$

This is the general *conjugate foci relation* for a lens of any type.

Principal Foci.—From the definition of the second principal focus, we have, $v=f$, when $u=\infty$. Hence the second focal length is given by, $\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$ from relation (4) above.

Again from the definition of the first principal focus, $u=f'$, when $v=\infty$, f' being the first focal length.

$$\therefore \quad \frac{1}{f'}=-(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right).$$

Hence $f=-f'$.

N B.—According to the new convention of signs the signs attributed to the *radii of curvature* of lens surfaces in air are as ollows : The radius of curvature of a surface convex to air, *i. e.* a fsurface which is convex to the incident light, is positive, and that which is concave to the incident light, is negative. Thus equation (1) above, when the lens is placed in air, becomes $\frac{\mu}{v}+\frac{1}{u}=\frac{\mu-1}{r_1}$

where $r_1 = r$. Again, r_2 in equation (5) above being a negative quantity, equation (5) becomes $\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$.

Examples

1. The radii of curvature of the surfaces of a double convex lens are 20 cm. and 40 cm. respectively, and its focal length is 20 cm. What is the refractive index of the material of the lens?

Ans. We have for a lens, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$.

Here, $f = -20$ cm., $r_1 = -20$ cm., $r_2 = +40$ cm., $\mu = ?$

That is, $-\frac{1}{20} = (\mu - 1)\left(-\frac{1}{20} - \frac{1}{40}\right)$.

or, $\frac{1}{20} = (\mu - 1)\frac{3}{40}$. or, $\mu - 1 = \frac{2}{3}$. $\mu = 1\frac{2}{3} = 1{\cdot}57$.

2. The plane side of a plano-convex lens is silvered and the lens then acts as a concave mirror of 30 cm. focal length. The refractive index of the lens is 1·5. calculate the radius of curvature of the convex lens.

Ans. The lens is acting as a concave mirror of 30 cm. focal length. Therefore, if an object be placed at the centre of curvature of such a mirror, *i.e.* at a distance of 60 cm. from the lens the image is also formed at the same place. Hence, here the focal length of the plano-convex lens is 60 cm. The plane side of the lens being silvered, the rays are reflected back and being again refracted through the lens along the same path meet at the focus.

The formula connecting the radii of curvature of the lens, the refractive index of the material of the lens, and its focal length is

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$. Here $\mu = 1{\cdot}5$, $r_1 = ?$, $r_2 = \infty$ (being plane), $f = -60$ cm.

$\therefore \quad -\frac{1}{60} = (1{\cdot}5 - 1) \times \frac{1}{r_1}$, whence $r_1 = -30$ cm. That is, the radius of curvature of the lens is 30 cm.

68. Conjugate Foci :—*These are any pair of points on the axes of a lens such that if one of them represents the object position, the other gives the corresponding image position.* For a thin lens their distances from the optical centre are given by the relation (with usual notations), $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, and so the relation is called the conjugate relation for a thin lens. The term *conjugate* means that the positions of these two points are interchangeable, *i.e.* if the object is placed at either point, the image is formed at the other point. This is what it should be from the *principle of reversibility of light.*

When the conjugate foci are both real, the rays of light diverging from the object, if placed at either point, will converge at the other point to form the image. But, if one of the foci is virtual, the rays diverging from the object placed at that focus will not so refract as to form an image at the other, but if actual rays from an object tend to converge at the virtual focus so as to give a virtual object there,

the rays after refraction will form a real image as the other conjugate point.

69. (i) Lateral or Linear Magnification :—The lateral or linear magnification of a lens producing an image of an object is defined as the ratio of any **linear dimension** of the image to the corresponding linear dimension of the object. In Fig. 109, PQ is a luminous object placed vertically on the axis of a thin convex lens RO of which O is the optical centre, F the principal focus and FOF the principal axis. To get the image, draw a ray PR parallel to the axis, which after refraction, passes through the principal focus F. Draw another ray PO through the optical centre of the lens, which passes out undeviated. The point P', where these rays intersect, is the image of P and let $P'Q'$ be drawn perpendicular to the axis. Thus $P'Q'$ is the real and inverted image of PQ.

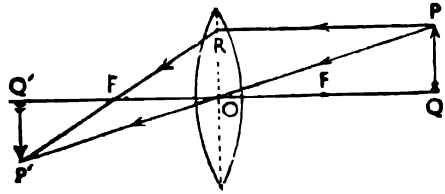


Fig. 109—Real Image formed by a Convex Lens.

$$\textbf{Linear Magnification (m)} = \frac{\textbf{size of image}}{\textbf{size of object}} = \frac{P'Q'}{PQ} = \frac{OQ'}{OQ} = \frac{v}{u}.$$

It should be noted that in Fig. 109 the image is *inverted* and v and u have opposite signs, v being negative ; so for lenses, when v/u (or, m) is *negative, the image is inverted.* Again, in Figs. 115 and 116 the image is erect and both v and u are positive ; so when v/u (or, m) *is positive, the image is erect.* Here note that lenses and mirrors differ in this respect.

The magnification may be expressed in terms of u, v and f as follows—

(a) We have $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$. Multiplying each terms by v, we get

$$1 - \frac{v}{u} = \frac{v}{f} \quad \therefore \quad m = \frac{v}{u} = 1 - \frac{v}{f} = \frac{f-v}{f} \qquad \ldots \qquad \ldots \quad (1)$$

(b) Multiplying each term of the general equation by u we get

$$\frac{u}{v} - 1 = \frac{u}{f} \text{ ; or, } \frac{u}{v} = \frac{u+f}{f} \quad \therefore \quad m = \frac{v}{u} = \frac{f}{u+f} \qquad \ldots \qquad \ldots \quad (2)$$

(ii) Longitudinal Magnifiction.—

Longitudinal magnification is the ratio of the extension of the image along the axis of the lens to that of the object along the axis.

$\therefore \quad m_1 = dv/du$ where dv and du are small extensions of image and object along the axis.

We know, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$.

Differentiating with respect to u,

$$-\frac{1}{v^2}\frac{dv}{du}+\frac{1}{u^2}=0$$

$$\therefore \quad \frac{dv}{du}=\frac{v^2}{u^2}=m^2.$$

$\therefore$ Longitudinal magnification $=m^2=$ (Linear magnification)^2.

***70. Formula according to New Convention of Signs :—**

(*i*) For a **convex lens producing real image—**

In deducing $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, both v and f were taken as negative.

According to the new conventions, v and f should be both positive, for they are *real* distances traversed by the rays. Hence the formula will suit the new conventions if v and f, in the above equation be made negative again.

That is, $-\frac{1}{v}-\frac{1}{u}=-\frac{1}{f}$; or, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$.

(*ii*) For a **convex lens producing virtual image—**

In deducing the equation $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, v is taken as positive and f negative. According to new conventions, they are reverse in this case.

$\therefore \quad -\frac{1}{v}-\frac{1}{u}=-\frac{1}{f}$. or, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$.

(*iii*) For a **concave lens, which always produces virtual image—**

In deducing the equation, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, v is taken as positive and f also positive. According to the new conventions, they being virtual distances are negative. Therefore, the formula should be given by, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$.

It is clear from the above that according to the new conventions, the general formula connecting the conjugate foci should be $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$; whereas, according to the old conventions it is, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$.

Thus, according to the new conventions, the same general formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, applies to the cases of both lenses and mirrors and a uniformity of practice is attained.

71. Position, Size and Nature of the Image in the case of Lenses :— The position, size and nature of the image formed by a lens depend on the position of the object with respect to the lens. A convex lens forms sometimes a real and sometimes a virtual image according to the position of the object ; but a concave lens always forms a virtual and erect image. Typical cases are shown below by diagrams drawn according to the rules of graphical construction of images. Here a linear object PQ is taken vertically on the axis of the lens. A ray from the point P is drawn parallel to the principal axis which after refraction passes through the second principal focus. A second ray from P passing through the optical centre O comes out undeviated and undisplaced and cuts the former ray at P'. Now the image $P'Q'$ is the perpendicular drawn from P' on to the axis.

(1) **Convex Lens.**—In the following diagrams the distance OF or OF' represents f, *i.e.* the focal length and that between O and $2F$ or $2F'$ represents $2f$ or twice the focal length of the lens

(*a*) *Object at Infinity* — Parallel rays inclined at a small angle to the principal axis proceeding from the object will converge at the focal plane of the lens, where the image is formed which is *real*, *inverted* and *extremely diminished* (Fig. 110).

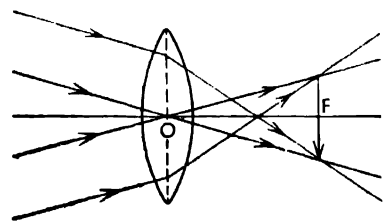


Fig. 110—Object at Infinity.

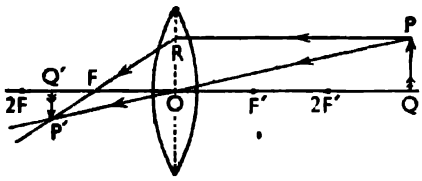


Fig. 111—Object between infinity and $2F$.

(*b*) *Object between Infinity and $2F'$, i.e.* twice the focal length.—When the image of the object is drawn according to the rules already stated, it is seen that the image is formed between F and $2F$, and is *real*, *inverted* and *diminished* (Fig. 111).

This case is applied in an ordinary *photographic camera.*

(*c*) *Object between $2F'$ and F'.*—The image in this case is formed between $2F$ and infinity (Fig. 112), and is *real*, *inverted* and *enlarged.* This case is applicable to the *magic lantern*, *enlarging camera*, etc.

(*d*) *Object at $2F'$* —In this case the image is also at a

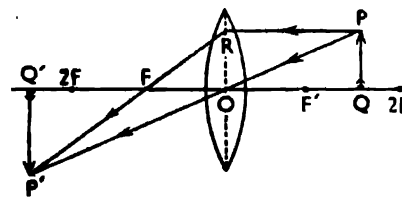


Fig. 112—Object between $2F$ and F.

distance of twice the focal length, *i.e.* at $2F$ (Fig. 113). This is also obtained mathematically by putting $u=2f$ in the general equation when $v=-2f$, which means that the image is formed on the other side at a distance equal to $2f$.

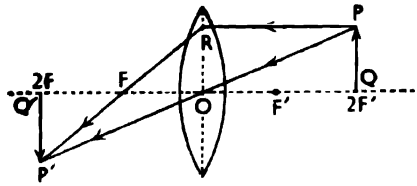


Fig. 113—Object at $2F$.

Again, magnification $m=\frac{v}{u}=\frac{-2f}{2f}=-1$, which means that this image is *real*, and is equal in size to the object. This principle is applied in the case of a *terrestrial telescope* fitted with an erecting eye-piece.

(*e*) *Object at F', i.e. at focus.*—This is the converse of case (*a*). The image is formed at infinity, and is *real*, *inverted*, and *infinitely enlarged* (Fig. 114).

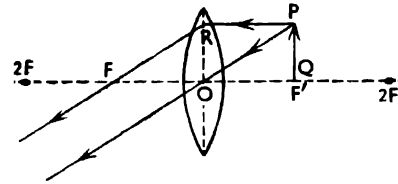


Fig. 114—Object at F

In the general equation, if $u=f$, then $\frac{1}{v}-\frac{1}{f}=-\frac{1}{f}$;

or, $\frac{1}{v}=0$, or, $v=\infty$. Again, magnification $m=\frac{v}{u}=\infty$

The principle is applied in the case of the *collimator* of a *spectrometer*.

(*f*) *Object between Focus and the Lens.*—The two rays taken from P diverge after refraction, and meet at P' when produced backwards (Fig. 115).

Draw $P'Q'$ perpendicular to the axis, which is the image of PQ. The image is *virtual*, *erect* and *enlarged*. This principle is applied in the case of *magnifying glasses and eye-pieces of telescopes and microscopes.*

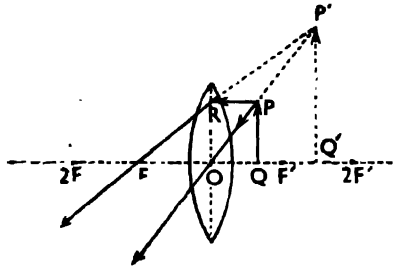


Fig. 115—Object between F' and the lens.

Conclusion.—Thus from the study of the above diagrams we find that as the object approaches the convex lens from infinity the real inverted image gradually recedes from the focus until it is formed at infinity when the object is at the first principal focus. On further approaching the lens, a virtual, erect and magnified image is formed on the same side as the object.

(2) Concave Lens—(i) For Real Objects—Here it may be noted that for all real values of u, the image lies between the second principal focus (F_2) and the optical centre (O), for when $u=\infty$, $v=f$ and when $u=0$, $v=0$ (Fig. 116). As a real object is brought from infinity upto the lens, the image travels from the second principal focus F_2 to the optical centre O. In Fig. 116, three real positions marked I, II and III

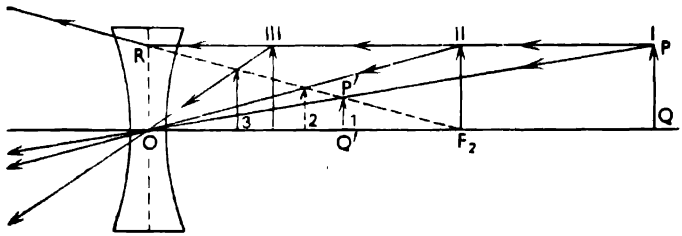


Fig. 116—Concave lens forming Virtual Image

of an object PQ have been shown giving corresponding virtual images $P'Q'$ marked by 1, 2 and 3. The images are always virtual and erect and increase in size as the object is brought nearer to the lens.

(ii) For Virtual Objects.—This is the only case where a concave lens forms a real image [Fig. 116 (a)]. Here a system of converging rays tending to form a virtual object PQ forms after refraction through the lens, a real image $P'Q'$ (erect and magnified). Between $u=0$ and $u=-f$, v changes from 0 to ∞, or in other words, the image moves from the lens to infinity along the negative side of the axis.

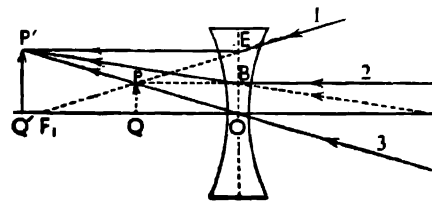


Fig. 116(a)

72. Identification of Lenses:—In identifying by means of the image formed whether a lens is convex or concave, (a) point it towards a distant object and move it towards a white wall or paper screen behind it. If any image is obtained, the lens is convex, otherwise it is concave; (b) hold the lens *close* to any object, and a virtual erect and enlarged image will be formed when the lens is convex, which may be viewed by means of an eye placed close to the other side of the lens. This is the principle of a magnifying glass or a simple microscope. If the image be erect and diminished the lens is concave.

73. Summary of Results :—

Figures	Real Position of Object	Image			Remark.
		Position	Size	Nature	
Convex Lens					
Fig. 110	At ∞	At F	Diminished	Real and Inverted	Image and object change their positions.
Fig. 111	Between $2F'$ and ∞	Between F and $2F$	,,	,,	,,
Fig. 112	Between F' and $2F'$	Between $2F$ and ∞	Enlarged	,,	,,
Fig. 113	At $2F'$	At $2F$	Same size	,,	,,
Fig. 114	At F	At ∞	Enlarged	,,	,,
Fig. 115	Between F' and Optical centre	On the same side as object	Enlarged	Virtual and Erect	Lens used as a magnifier.
Concave Lens	At ∞	At F_2	Diminished	Virtual and Erect	
Fig. 116	Between ∞ and Optical centre	Between F_2 and Optical centre	,,	,,	

74. Alternative Deviation of the conjugate foci relation of a Lens :— A thin lens may be supposed to consist of a number of thin prisms. So the principal section of a thin lens may be supposed to be made up of a large number of principal sections of thin prisms. The angles of these thin prisms will gradually increase as we approach the apex of the section of the lens. Let ABC represent the thin prism (shown extended in the figure) corresponding to the region of the lens section near its apex (Fig. 117). Let C_1 and C_2 be the centres of curvature of the two surfaces of radii of curvature r_1 and r_2 respectively. Let γ and δ be the inclinations of the radii of the two surfaces very

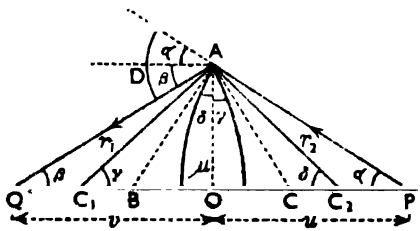


Fig. 117

near A with the principal axis. As apparent from the geometry of the figure, the $\angle A$ of the ΔABC is equal to $(\gamma+\delta)$, the line joining A with the optical centre, O, evidently dividing $\angle A$ into these two parts. Suppose P is a point object of the principal axis of the lens and Q is its image. Then the ray PA incident very near the apex passes along AQ after refraction. So the deviation due to refraction in the thin prism is $\angle D$ as shown in Fig. 117. If α and β be the inclinations of AP and AQ with the axis, then,

$$\Delta D=\alpha+\beta.$$

As the prism ABC is really a thin prism, the angle of deviation D is connected with the angle of the prism by the relation,

$$D=(\mu-1)A.$$

Substituting the values of D and A,

$$(\alpha+\beta)=(\mu-1)(\gamma+\delta) \quad \ldots \quad \ldots \quad (1)$$

Now since α, β, γ, δ are small angles and the linear aperture of the lens is also small, we have,

From (1), $\dfrac{OA}{OP}+\dfrac{OA}{OQ}=(\mu-1)\left(\dfrac{OA}{AC_1}+\dfrac{OA}{AC_2}\right)$,

But $OP=u$, $OQ=-v$, $AC_1=-r_1$ and $AC_2=r_2$.

$\therefore \quad \dfrac{1}{u}-\dfrac{1}{v}=(\mu-1)\left(\dfrac{1}{-r_1}+\dfrac{1}{r_2}\right)$;

or, $\dfrac{1}{v}-\dfrac{1}{u}=(\mu-1)\left(\dfrac{1}{r_1}-\dfrac{1}{r_2}\right) \quad \ldots \quad \ldots \quad (2)$

When $u=\infty$, $v=f$ (the second focal length)

$\therefore$ From (2), $\dfrac{1}{f}=(\mu-1)\left(\dfrac{1}{r_1}-\dfrac{1}{r_2}\right) \quad \ldots \quad \ldots \quad (3)$

Again, substituting in (2), $\dfrac{1}{v}-\dfrac{1}{u}=\dfrac{1}{f} \quad \ldots \quad \ldots \quad (4)$

75. Conjugate Foci relation with the Principal foci as origins :—The general conjugate foci relation for a lens with the optical centre as the origin is $\dfrac{1}{v}-\dfrac{1}{u}=\dfrac{1}{f} \quad \ldots \quad \ldots \quad (1)$

where f is the second focal length of the lens,

Rearranging (1), we have, $\dfrac{f}{v}-\dfrac{f}{u}=1$; or, $fu-fv=uv$ ... (2)

Since the first focal length $f'=-f$, the eqn, (2) may be rewritten as

$$fu+f'v=uv,$$

or, $uv-fu-f'v+ff'=ff'$:

or, $(u-f')(v-f)=ff' \quad \ldots \quad \ldots \quad (3)$

Let U be the distance of object from the first focal plane, *i.e.* $(u-f')$ and V be the distance of the image from the second focal plane *i.e.* $V=(v-f)$. Then relation (3) can be put in the form

$$UV=ff' \quad \ldots \quad \ldots \quad (4)$$

which is the required conjugate foci relation with respect to the first and the second principal foci.

76. Conjugate Foci Relation for a transparent sphere (or a spherical lens) :—In Fig. 118, P is a point object placed on the axis of the transparent sphere or the spherical lens. Let r be its radius and O its centre. If μ be the refractive index of the material of the sphere and if it be placed in air, then taking p and q_1 to be the object distance and the corresponding image distance *as measured from O*, we have for the refraction at the right hand surface,

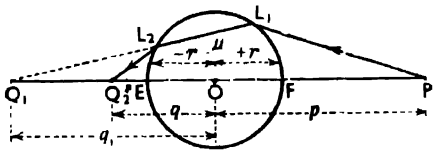


Fig. 113

$$\frac{\mu}{p}-\frac{1}{(-q_1)}=\frac{\mu-1}{r}, \quad \text{(Vide, Art. 59)}$$

because here p and r are positive and q_1 is negative.

When refraction at the left hand surface is considered, the distance OQ_1 serves as the object distance and the final image is formed at Q_2 and the corresponding image distance is $OQ_2=q$. This q may be positive or negative depending on the position of P. So in the treatment we do not consider the sign of q. In this case object distance $=(q_1)$, r is $-ve$ and the light passes from the medium of the sphere to air.

Hence, $\dfrac{1}{(-q_1)}-\dfrac{\mu}{q}=\dfrac{1-\mu}{-r}=\dfrac{\mu-1}{r}$.

$\therefore$ Adding up these two conjugate foci relations,

$$\mu\left(\frac{1}{p}-\frac{1}{q}\right)=\frac{2(\mu-1)}{r};$$

$$\text{or,} \quad \frac{1}{p}-\frac{1}{q}=\frac{2(\mu-1)}{\mu r} \quad \ldots \quad \ldots \quad (1)$$

When $p=\infty$, $q=f$ (the second focal length),

$$\therefore \quad -\frac{1}{f}=\frac{2(\mu-1)}{\mu r}; \text{ or, } f=-\frac{\mu r}{2(\mu-1)} \quad \ldots \quad \ldots \quad (2)$$

Substituting this value of $\dfrac{1}{f}$ in (1),

$$\frac{1}{p}-\frac{1}{q}=-\frac{1}{f}; \text{ or, } \frac{1}{q}-\frac{1}{p}=\frac{1}{f} \quad \ldots \quad (3)$$

Relation (3) is the required conjugate foci relation for a refracting sphere.

77. Power of a Lens :—The power of a lens means its power of convergence in the case of a convex lens, or its power of divergence in the case of a concave lens. That is, it indicates to what degree the lens can converge or diverge (as the case may be) an incident beam of light. The convergence or divergence, as the case may be, produced by a lens, will be greater, the shorter the focal length is. So the reciprocal of the focal length ($1/f$) is taken to indicate the power of a lens. A lens having a focal length of one metre is said to possess **Unit Power**, called a **Dioptre.** According to the opticians, *the power of a convex lens is positive and that of a concave lens is negative.*

So to express the power of a lens in Dioptres, express the focal length in metres, get the reciprocal and change the sign.

Thus, a concave lens of 100 cms. focal length has a power of $-1D$, and a convex lens of -50 cms. focal length has a power of $1/\frac{1}{2}=2D$.

Note that according to the new *convention of signs* the change of sign is not necessary.

Example

What is the power of a lens which, when combined with a converging lens of focal length 25 cms, will produce a combination power of 2 dioptres ?

Ans. The power of the converging lens is $\frac{100}{25}=4$ dioptres.

$\therefore$ $2=4+p$, where p is the power of the other lens. $\therefore$ $p=-2$.

The negative sign shows it that the lens is concave and has a focal length f, where $2=\frac{100}{f}$, *i.e.* $f=50$ cms.

78. The Dioptremeter (Lensometer) :—It is a handy instrument commonly used by all opticians for quick measurement of the optical power of the surface of a lens. Its shape resembles that of a pocket watch. It has a dial at its front on which a *hand* rotates in anti-clockwise direction (Fig. 119).

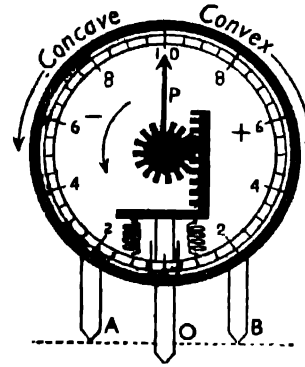


Fig. 119—The Dioptremeter

Two pins A and B of equal length and whose tips are at the same level are fixed at its bottom, and centrally between them lies a third pin O of greater length protruding below the common level of the tips of A and B. This third pin O, which can be pushed in, has a

carriage mounted on it provided with a toothed vertical arm engaging with a miniature toothed wheel placed at the centre. The rotating *hand* P of the instrument, which is attached to the wheel, turns in an anti-clockwise fashion when the central pin O is pushed in against the two springs which control the vertical movement of the carriage.

When not in use, the pointer is vertical and reads 10 as shown in the dial. By pushing in, as the pin is raised to the level of the tips of A and B, the pointer moves to the zero mark at the bottom. Thus up to the stage the central leg lies below the level of the tips of A and B, the instrument corresponds to a spherometer when placed on a concave surface. The only difference between them is that here in this instrument there are two legs instead of three as in a spherometer. If the central pin is pushed still further, *i.e.*, above the level of the tips of A and B (by placing the instrument on a convex surface), the pointer will swing to the right half of the dial from 0 to 10. When the three pins will lie on a plane surface, the pointer will read zero.

Thus, what is to be noted is that this instrument acts as a spherometer, the function of the screw being done by a rack and pinion arrangement and the springs make the instrument automatic in action.

Theory —The working formula of a spherometer is given by,

$$R=\frac{S^2}{2h}+\frac{h}{2},$$

where R = radius of curvature, S = distance between the central pin and any of the outer pins, and h = the vertical distance through which the central pin is above or below the plane of the tips of the outer pins. When $\frac{h}{2}$ is negligible compared to S, $R=\frac{S^2}{2h}$, or, $\frac{1}{R}=\frac{2h}{S^2}$.

That is, $\frac{1}{R}\propto h$, since $\frac{2}{S^2}$ is a constant of the instrument.

Again, the dioptric power D of a surface is given by $100/f$, where f, the focal length for the refracting surface is measured in centimetres, f being given by $\frac{1}{f}=\frac{\mu-1}{r}$ (with usual notations).

For a glass surface, $\mu=1{\cdot}5$ and so,

$$D=\frac{100}{f}=100\left(\frac{1{\cdot}5-1}{r}\right)=\frac{50}{r}.$$

or, $D=50\times\frac{2h}{S^2}$ ($\because r=R$), where $\frac{2}{S^2}$ is a constant for the instrument. Thus the contribution to the dioptric power of a lens by each of its surfaces can be obtained from the reading of the instrument, and from this the dioptric power of the lens can be calculated.

79. Equivalent lens :—Where a number of lenses is used to form the image of an object, a single lens which can produce the image at the same position and of the same magnification, may be termed an *equivalent lens* for the actual system of lenses used. The focal length of the single equivalent lens is called the *equivalent focal length* of the lens system. When the lenses used are thin and are in contact with one after another, the pewer of the equivalent lens is the algebraic sum of the powers of the component lenses, as is shown in the next article.

80. Two Thin Lenses in Contact :—(*a*) Let two thin lenses L_1, L_2 of focal lengths f_1 and f_2 be in contact on the same axis. Let O be a point source of light on the axis and I' be its image formed after refraction through the first lens L_1 (Fig. 120). Then, if u and v_1 be the distances of O and I' respectively from the optical centre of the concave lens L_1, we have, $\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}$ (1)

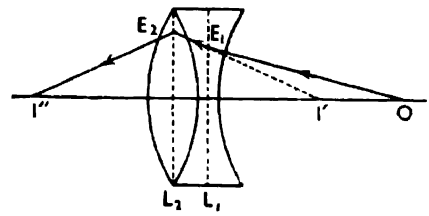


Fig. 120

I' now serves as the object with respect to the second lens L_2 (convex) which, suppose, produces the image at I''. The lenses being thin, the distance of I' from the optical centre of L_2 can be taken to be v_1. If v be the distance of I'' from the optical centre of lens L_2, we have,

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2} \qquad (2)$$

Now, considering the two lenses L_1 and L_2 together as equivalent to a single lens of focal length F, we have,

$\frac{1}{v} - \frac{1}{u} = \frac{1}{F}$, where u is the distance of the object O, and v that of the image I'' with respect to the optical centre of the equivalent lens.

But on adding (1) and (2) we get, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}$.

That is, $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$ ... (3)

or, $F = \frac{f_1 f_2}{f_1 + f_2}$ (4)

(*b*) In Fig. 121, L_1 and L_2 are two concave lenses in contact having focal lengths f_1 and f_2 respectively. Suppose for the object O, I' is the virtual formed by the lens L_1. This virtual image serves as object so far as the lens L_2 is concerned by which the final virtual image I'' is produced. Proceeding as in (*a*), it can be shown that if F be the focal length of the equivalent lens,

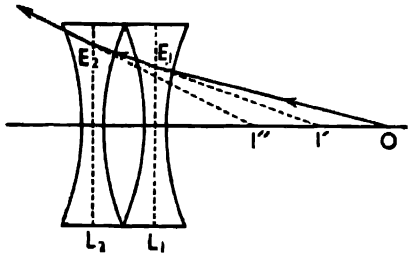


Fig. 121

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}, \quad \text{or,} \quad F=\frac{f_1 f_2}{f_1+f_2}$$

(*c*) Proceeding as in (*a*) or (*b*) above, it can be shown that for the case of two thin convex lenses L_1 and L_2 (Fig. 122) as well the equivalent focal length F of the combination of the lenses will be expressed in terms of their focal lengths f_1 and f_2 by the same equation, $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$, as in (*a*) or (*b*) above.

Extending the above result to a number of thin lenses in contact we may write in general,

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+\ldots \quad (5)$$

That is, the power of an equivalent lens is equal to the algebraic sum of the powers of the component lenses.

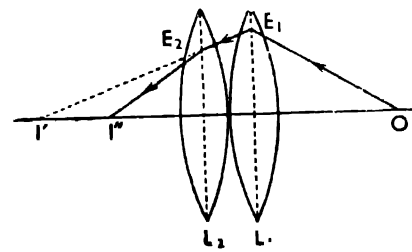


Fig. 122

If P be the power of the equivalent lens and P_1, P_2, P_3, etc. be the powers of the component lenses, then $P=P_1+P_2+P_3+\ldots\ldots$

81. Two thin lenses separated by a distance :—In the case of two thin coaxial lenses separated by a distance, it is not possible to find one single lens which when placed in any particular position would produce an image of the same size and exactly in the same position as that formed by the combination of the above two lenses. A single lens may, however, be placed in a suitable position on the axis of the above lens combination such that the magnification produced by this lens is exactly the same as the overall magnification of the combination. Though the size of the image is the same,

the position where it is formed is different from that due to the combination. This single lens producing an image of same magnification as the combination is known as the **equivalent lens** of the above two lenses of the combination. The term equivalent lens is, however, used here only *in a restricted sense* because *the positional exactitude is not attained* in this case.

Let L_1 and L_2 be two thin lenses separated by a distance a and let f_1 and f_2 be their respective focal lengths (Fig. 123).

Let u be the distance of an object O from the centre of the lens L_1 and v_1 the distance of the corresponding image at I_1.

So due to the refraction in the first lens, we get,

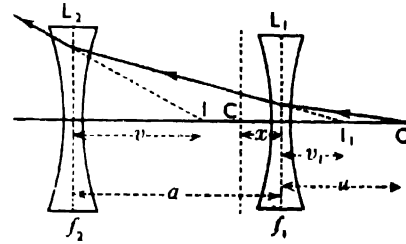


Fig 123

$$\frac{1}{v_1}-\frac{1}{u}=\frac{1}{f_1};$$

$$\text{or, } \frac{1}{v_1}=\frac{u+f_1}{uf_1}.$$

$\therefore$ The magnification of I_1 is, $m_1=\dfrac{v_1}{u}=\dfrac{f_1}{u+f_1}$ ... (1)

For the lens L_2, I_1 serves as the object and the corresponding final image is formed at I where v_1+a and v are the distances of I_1 and I as measured from the centre of L_2.

$$\therefore \quad \frac{1}{v}-\frac{1}{v_1+a}=\frac{1}{f_2}; \text{ or, } \frac{1}{v}=\frac{f_2+v_1+a}{f_2(v_1+a)}.$$

$\therefore$ The magnification of L_2 is $m_2=\dfrac{v}{v_1+a}=\dfrac{f_2}{f_2+v_1+a}$.

But $v_1=\dfrac{uf_1}{u+f_1}$. $\therefore$ $m_2=\dfrac{f_2}{f_2+a+\dfrac{uf_1}{u+f_1}}=\dfrac{f_2(u+f_1)}{u(f_1+f_2+a)+f_1(f_2+a)}$

... (2)

Now if this combination of two lenses be replaced by a single lens placed at the position C at a distance x behind L_1 and if V be the distance of the final image of same magnification from the centre of this lens, then for this equivalent lens of focal length F, we get,

$$\frac{1}{V}-\frac{1}{u+x}=\frac{1}{F},$$

where $u+x$ is the new object distance.

$$\therefore \quad \frac{1}{V} = \frac{F+u+x}{F(u+x)}.$$

$\therefore$ The magnification of this single lens is,

$$M = \frac{V}{u+x} = \frac{F}{F+u+x} \qquad \ldots \qquad \ldots \quad (3)$$

Now since according to definition the magnification produced by the lens-combination is equal to that produced by the corresponding equivalent lens,

$$M = m_1 \times m_2 \ , \text{ or, } \frac{1}{M} = \frac{1}{m_1 \times m_2}.$$

$$\therefore \quad \frac{F+u+x}{F} = \frac{(u+f_1)}{f_1} \times \frac{u(f_1+f_2+a)+f_1(f_2+a)}{f_2(u+f_1)}$$

$$= \frac{u(f_1+f_2+a)+f_1f_2+af_1}{f_1f_2}.$$

$$\therefore \quad 1+\frac{u}{F}+\frac{x}{F} = \frac{u(f_1+f_2+a)}{f_1f_2}+1+\frac{a}{f_2};$$

$$\text{or,} \quad \frac{u}{F}+\frac{x}{F} = \frac{u(f_1+f_2+a)}{f_1f_2}+\frac{a}{f_2} \qquad \ldots \qquad \ldots \quad (4)$$

This relation (4) is valid for all values of u. So when $u=0$,

$$\frac{x}{F} = \frac{a}{f_2} \qquad \ldots \qquad \ldots \quad (5)$$

$$\therefore \quad \frac{1}{F} = \frac{f_1+f_2+a}{f_1f_2} = \frac{1}{f_1}+\frac{1}{f_2}+\frac{a}{f_1f_2} \qquad \ldots \qquad \ldots \quad (6)$$

This is the relation connecting f_1, f_2, a and F. If the component lenses are convex, *i.e.*, if f_1 and f_2 are negative, F will also be negative. Hence relation (6) will be modified as,

$$\frac{1}{F} = \frac{1}{f_1}+\frac{1}{f_2}-\frac{a}{f_1f_2}.$$

The position of the equivalent lens is x *behind the first lens* L_1 where x as obtained from relation (5) is given by,

$$x = \frac{aF}{f_2} = \frac{a(f_1f_2)}{f_2(f_1+f_2+a)} = \frac{af_1}{f_1+f_2+a}.$$

Alternative treatment.—The focal length and the position of the equivalent lens corresponding to two thin lenses separated by a distance can also be deduced from the simple principle of image formation of a finite object bearing in mind the fact that the size of the final image formed by the equivalent lens must be the same as that of the image formed by the combination. But the positional correspondence of the two images cannot be attained in this case

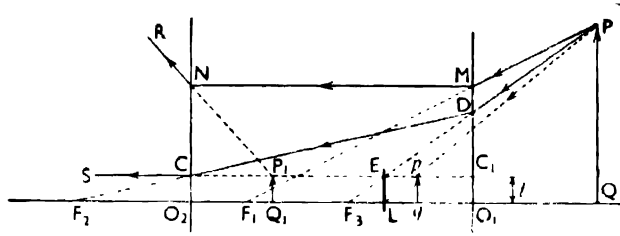


Fig 123(a)

In Fig. 123(*a*) O_1M and O_2N represent the positions of the two thin lenses mounted coaxially, O_1O_2 being the axis of the system. PQ is an extended object placed on this axis. From P a ray PM, starting in the direction of the principal focus F_1 of the first lens O_1M, will be parallel to the axis after refraction. It is incident on the lens O_2N at N, where it undergoes a second refraction and comes out along NR. A second ray PD falls on O_1M and after refraction proceeds along the direction of the principal focus F_2 of O_2N and meets the second lens at C. After refraction it emerges along CS parallel to O_1O_2. Produce SC to C_1 and produce RN to meet SC produced at P_1. Then P_1Q_1 represents the final image of PQ as cast by the combination. If this combination be replaced by a single equivalent lens, the size of the image formed by it will be equal to P_1Q_1. But this image will not be formed at the same position Now if PD be produced to F_3 on the axis intersecting CC_1 at E and if a single lens (the equivalent lens) of focal length F_3L be now placed at EL, replacing O_1M and O_2N, then since the ray PE proceeds along the focus, after refraction it passes along ES. Another ray passing along the optical centre of EL proceeds along PL. The point of intersection of PL and CC_1 gives the position of the image formed by EL. It is evidently of the same size as P_1Q_1. This image is shown in the figure as pq. So the lens EL satisfies the condition of the equivalent lens and F_3L is its focal length, which can be evaluated from the geometry of the figure.

Let a = separation O_1O_2 between the component lenses,

f_1 = focal length of $O_1M = O_1F_1$,

f_2 = focal length of $O_2N = O_2F_2$,

and F = focal length of the equivalent lens = F_3L

Now in similar $\triangle s$ F_2EL and F_2DO_1,

$$\frac{F_2L}{F_2O_1}=\frac{EL}{DO_1}=\frac{l}{DO_1}, \quad (l=EL=pq=P_1Q_1=CO_2) \quad \ldots \quad (1)$$

Again in similar $\triangle s$ F_2CO_2 and F_2DO_1,

$$\frac{F_2O_2}{F_2O_1}=\frac{l}{DO_1} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

Equating L.H. sides of (1) and (2),

$$\frac{F}{F_2O_1}=\frac{f_2}{f_2+a}; \text{ or, } F=\frac{f_2}{f_2+a}\times F_2O_1 \quad \ldots \quad (3)$$

As is evident from figure, F_2 is the point image of the point object F_2 formed by the first lens.

$$\therefore \quad \frac{1}{f_1}=\frac{1}{v}-\frac{1}{u}=-\frac{1}{(f_2+a)}+\frac{1}{F_2O_1}, \quad \left[\begin{array}{l}\text{since } u \text{ and } v \text{ are}\\ \text{both negative here}\end{array}\right]$$

$$\therefore \quad \frac{1}{F_2O_1}=\frac{1}{f_1}+\frac{1}{f_2+a}=\frac{f_1+f_2+a}{f_1(f_2+a)} \quad \ldots \quad (4)$$

Substituting in (3) $F=\frac{f_2}{(f_2+a)}\times\frac{f_1(f_2+a)}{f_1+f_2+a}=\frac{f_1f_2}{f_1+f_2+a}$.

$$\therefore \quad \frac{1}{F}=\frac{1}{f_2}+\frac{1}{f_2}+\frac{a}{f_1f_2} \quad \ldots \quad \ldots \quad (5)$$

When both lenses are convex, f_1 and f_2 are both negative and F is also $-ve$. Hence the relation (5) takes the form,

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{a}{f_2f_2}.$$

The *position of the equivalent lens* is evidently at a distance LO_1 behind the first lens.

Now, $LO_1=F_2O_1-F_2L=\frac{f_1(f_2+a)}{f_1+f_2+a}-\frac{f_1f_2}{f_1+f_2+a}=\frac{f_1a}{f_1+f_2+a}$.

Hence the equivalent lens must be placed at a distance $\frac{f_1a}{f_1+f_2+a}$ behind the first lens facing the incident light.

82 Problems on Lenses :—

(a) The minimum distance between the object and its real image formed by a lens.—Suppose a convex lens is taken. If u and v be the distances of an object and its real image from the optical centre of the lens, then the numerical value of the distance between the object and its image is $D=u+v$ ... ... (1)

$\therefore$ D is minimum, when $\frac{dD}{du}=0$.

Hence differentiating (1) we get,

$$1+\frac{dv}{du}=0\ ;\quad \text{or,}\quad \frac{dv}{du}=-1 \qquad \ldots \qquad \ldots \qquad (2)$$

Again for a convex lens producing a real image the conjugate foci relation should be written as

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \qquad \ldots \qquad \ldots \qquad (3)$$

($\because$ v and f are both $-ve$). Differentiating (3) with respect to u,

$$-\frac{1}{v^2}\frac{dv}{du}-\frac{1}{u^2}=0\ ;\quad \text{or,}\quad \frac{dv}{du}=-\frac{v^2}{u^2} \qquad \ldots \qquad \ldots \qquad (4)$$

So from (2) and (4) we have,

$$-1=\frac{-v^2}{u^2},\ \text{or,}\ v=u.$$

Putting $v=u$ in (3) $\frac{2}{u}=\frac{1}{f}$, or, $u=2f=v$.

This is the condition for D to be minimum.

$\therefore$ Substituting in (1), the required value of the minimum distance between the object and its real image is $D=2u=4f$.

The same value can be had also from the theory of the real image formation.

(b) **The theory of the formation of Real Images**—If a convex lens of focal length f produces a real image of an object, the said image can be cast on a screen placed at a suitable position. Suppose for the sharpest possible image the screen is placed at a distance D from the object. Let u be the object distance and v the image distance. Evidently $u=D-v$ and $v=D-u$. For a convex lens producing a real image, f and v are negative. So the conjugate foci relation can be put in the form,

$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$$

$$\text{or,}\quad \frac{1}{f}=\frac{1}{v}+\frac{1}{D-v}=\frac{D}{Dv-v^2},$$

$$\text{or,}\quad Df=Dv-v^2\ ;$$

$$\text{or,}\quad v^2-Dv+Df=0 \qquad \ldots \qquad \ldots \qquad (1)$$

This is a quadratic equation in v and so v shall have two possible values for a particular value of D. These two values are :

$$v_1=\frac{D+\sqrt{D^2-4Df}}{2}\quad \text{and}\quad v_2=\frac{D-\sqrt{D^2-4Df}}{2}.$$

Hence $v_1+v_2=D$ and $v_1v_2=Df$.

$\therefore$ For a particular position of the object and the screen, since v may have two values, two images may be formed on the screen

when the centre of the lens is at distances $v_1 = \frac{D+\sqrt{D^2-4Df}}{2}$ and $v_2 = \frac{D-\sqrt{D^2-4Df}}{2}$ from the screen *i.e.*, if the lens be displaced through a distance $v_1 - v_2$. These two images are real and different when $D^2 > 4Df$, or when $D > 4f$. They are real and identical when $D^2 = 4Df$ or $D = 4f$. But if $4Df > D^2$ or $4f > D$, these two images will be imaginary and have no real existence. So the limiting condition for formation of real images is that the distance between screen and the source must be either equal to or greater than $4f$.

(c) **Expression for the focal length of a convex lens in terms of the displacement of the lens.**—As shown in the above theory, if D be the distance between the source and the screen and if a convex lens of focal length f be placed in between, the two sharp real images will be cast on the screen for two different positions of the lens, provided the distance D is greater than $4f$. Here the first sharp image will be formed when the lens is at a distance $v_1 = \frac{D+\sqrt{D^2-4Df}}{2}$ from the screen. Now the lens is displaced through a distance d and a second sharp image will be formed on the screen and the distance of the lens from the screen becomes $v_2 = \frac{D-\sqrt{D^2-4Df}}{2}$.

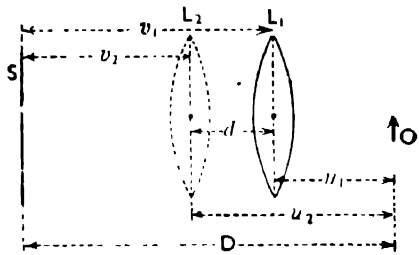


Fig. 121.

$\therefore \quad d = v_1 - v_2 = \sqrt{D^2-4Df}.$

Hence $d^2 = D^2 - 4Df$,

or, $4Df = D^2 - d^2$;

or, $f = \frac{D^2-d^2}{4D}$.

This is the expression for the focal length of a convex lens in terms of the displacement of the lens. Focal length of a convex lens or a converging lens-combination can be determined with the help of the above relation. The method of this determination is known as the **displacement method**.

(d) **Relation between the size of the object and the sizes of the two images in the Displacement method.**—In the displacement method when the lens is at a distance v_1 from the screen, the magnification is given by $m_1 = \frac{v_1}{u_1} = \frac{I_1}{O}$ where u_1 is the distance

of the object from the lens and I_1 and O the linear dimensions of the image and the object respectively.

Similarly in the second position, $m_2 = \frac{v_2}{u_2} = \frac{I_2}{O}$ where u_2, v_2 and I_2 respectively refer to the object distance, the image distance and the length of the image in the new setting.

But $u_1 = D - v_1$ and $u_2 = D - v_2$, where, D=distance between the screen and the object. From the property of the quadratic equation for v, we know that $v_1 + v_2 = D$,

$\therefore \quad u_1 = D - v_1 = v_2$ and $u_2 = D - v_2 = v_1$,

$\therefore \quad m_1 \times m_2 = \frac{v_1}{D - v_1} \times \frac{v_2}{D - v_2} = \frac{v_1}{v_2} \times \frac{v_2}{v_1} = 1.$

But $m_1 \times m_2 = \frac{I_1}{O} \times \frac{I_2}{O} = \frac{I_1 I_2}{O^2}.$

$\therefore \quad \frac{O^2}{I_1 I_2} = 1, \quad \text{or,} \quad O = \sqrt{I_1 I_2}.$

(e) Relations between the focal length and the magnifications in the Displacement method.—In the displacement method when the lens is at a distance v_1 from the screen, the magnification $m_1 = \frac{v_1}{u_1} = \frac{v_1}{-(D - v_1)} = \frac{v_1}{v_2}$ [$\because \quad u_1 = (D - v_1)$ and $v_1 + v_2 = D$, D being the distance between the screen and the object.]

Again, when the lens is displaced keeping the position of the screen fixed at a distance $D > 4f$, the new magnification is

$$m_2 = \frac{v_2}{u_2} = \frac{v_2}{D - v_2} = \frac{v_2}{v_1}.$$

$$\therefore \quad m_1 - m_2 = \frac{v_1}{v_2} - \frac{v_2}{v_1} = \frac{(v_1 + v_2)(v_1 - v_2)}{v_1 v_2}.$$

But $v_1 + v_2 = D$, $v_1 - v_2 = d$, the displacement, and $v_1 v_2 = fD$.

[$\because$ When the image is real, $\frac{1}{v_1} + \frac{1}{u_1} = \frac{1}{f}$, or, $u_1 v_1 = f(u_1 + v_1)$ $= fD$.]

$$\therefore \quad m_1 - m_2 = \frac{D \times d}{fD},$$

$$\text{or,} \quad f = \frac{d}{m_1 - m_2}.$$

(f) Relation between focal length of a lens and the magnification when the lens is fixed and the object and the screen are displaced :—

(*i*) *Expression in terms of* m_1, m_2 *and* d, *the displacement of the screen.*—

Let u_1, u_2 and v_1, v_2 be the object distances and image distances in the original and the displaced positions. Let d be the displacement of the screen such that $v_1 - v_2 = d$. If f be the focal length of the convex lens, then as f is negative for a convex lens and so is v for a real image, we have with proper sign correction the conjugate foci relation for the original setting of the screen as,

$$\frac{1}{f} = \frac{1}{v_1} + \frac{1}{u_1} \text{ ; or, } \frac{v_1}{f} = 1 + \frac{v_1}{u_1} = 1 + m_1.$$

Similarly for the second setting of the screen,

$$\frac{1}{f} = \frac{1}{v_2} + \frac{1}{u_2} \text{ ; or, } \frac{v_2}{f} = 1 + m_2 \text{ ;}$$

$$m_1 - m_2 = \frac{v_1}{f} - \frac{v_2}{f} = \frac{v_1 - v_2}{f} = \frac{d}{f} \text{ ;}$$

$$\text{or, } f = \frac{d}{m_1 - m_2}$$

(*ii*) *Expression in terms of* m_1, m_2 *and the displacement of the object.*—

Let u_1, u_2 and v_1, v_2 be the object distances and image distances in the two positions. Let d be the displacement of the object when $u_1 - u_2 = d$. If f be the focal length of the convex lens producing a real image, we have for the first setting,

$$\frac{1}{f} = \frac{1}{v_1} + \frac{1}{u_1} \text{ ; or, } \frac{u_1}{f} = \frac{u_1}{v_1} + 1 = \frac{1}{m_1} + 1 \quad \ldots \quad \ldots \quad (1)$$

Similarly for the second setting,

$$\frac{1}{f} = \frac{1}{v_2} + \frac{1}{u_2} \text{ ; or, } \frac{u_2}{f} = \frac{u_2}{v_2} + 1 = \frac{1}{m_2} + 1 \quad \ldots \quad \ldots \quad (2)$$

$\therefore$ From (1) and (2), we have,

$$\frac{1}{m_1} - \frac{1}{m_2} = \frac{u_1}{f} - \frac{u_2}{f} = \frac{u_1 - u_2}{f} = \frac{d}{f}$$

$$\therefore \quad f = \frac{d}{\frac{1}{m_1} - \frac{1}{m_2}}$$

83. Determination of Focal Length of a Lens :—

(A) Convex Lens

(a) By the $u-v$ method (conjugate foci method) :—The determination of the focal length of a lens by this method is usually done with the help of an optical bench. Fig. 125 shows a form of the optical bench, which is used in different lens measurements. It consists of a horizontal base-board of wood or iron to which is fixed a scale graduated in centimetres and millimetres.

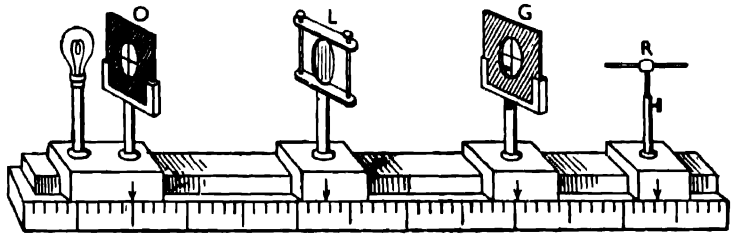


Fig. 125

Lenses, object, a ground glass screen or a pin, as the case may be, are mounted when required on vertical uprights fitted to sliders each of which bears an index mark. These sliders can slide along the axis of the bench and their positions with reference to the index mark can be noted from the scale. The object O may be either a pin or a hole in an opaque screen fitted with a cross-wire which is illuminated by an electric lamp.

The convex lens L is mounted on a vertical upright. The image of the object referred to above when sharpest is received on the ground glass screen (G) which is provided with a cross-mark. Parallax should be avoided between the image of the cross-wire and cross-mark on the ground glass screen. When this is secured, the object distance and the image distance are noted from the readings of the index marks on the different sliders. As these index marks are not necessarily in the same vertical planes as the object, the lens or the screen, *index corrections* are to be introduced. This is done by taking a straight rod (index rod) R which is placed horizontally parallel to the axis of the optical bench and is mounted on one of the uprights. The rod is so placed that its two ends just touch the object and the lens. Hence its apparent length as given by the index mark readings is determined. Then its actual length is directly measured by placing the rod against a metre scale. The difference gives the index correction. If the actual length be greater, the correction is to be added to the apparent object distance

measured. If the apparent length is greater, the correction is to be subtracted from the observed value of the object distance. Thus the corrected value of u is obtained. Similarly finding out the index correction for image distance, the corrected value for v is obtained. To these corrected values of u and v another correction term for the thickness of the lens equal to $\frac{t}{3}$ is to be added where t is the thickness of the lens. This is so because the lenses ordinarily used in the laboratory are really thick lenses. Repeating the experiment with different values of u and v, f is obtained from the relation, $\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$ which is the form after sign correction).

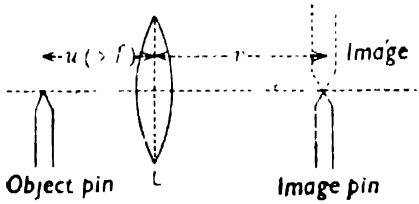


Fig 126

[*The experiment can also be performed by exchanging the positions of the object and the screen by two pins fixed on two stands.* Looking at the image of the object-pin from the other side of the lens, the position of the image can be located by **avoiding parallax** between the second pin and the image of the first pin (Fig. 126

Focal Length from Graph—(*a*) The graph obtained by plotting u and v (*using equal scales along the two axes*) is shown in Fig. 127. This is a branch of a hyperbola. A line OP drawn from the origin O making an angle of 45° with the axis of X or Y will meet the curve at the point P, suppose. This is a straight line for which $u=v$. The point P will be equidistant from the two axes (X and Y). The value of the perpendicular distance (y or x) of P from X or Y-axis will be equal to $2f$ of the lens, from which f can be calculated (to be done in the same way as in the case of a concave mirror).

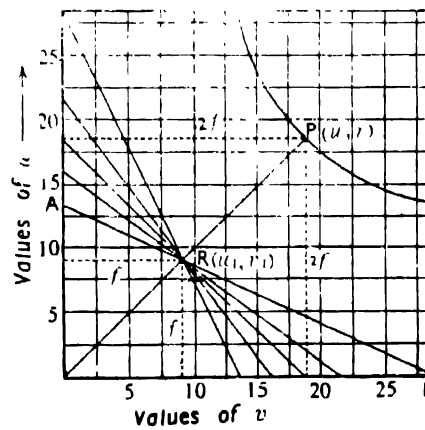


Fig 127

Theory.—The graph represents the relation, $\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$ **(for the convex lens) which is a hyperbola and the line** OP

satisfies the relation $u = v$. So putting $u = v$, in the first relation we get, $\frac{1}{f} = \frac{2}{u}$ or, $\frac{2}{v}$,

$$\therefore \quad \text{At } P,\ u = v = 2f.$$

(*ii*) *Another method* of obtaining the value of f from the graph is to make off the value of u along one axis and the values of v along the other axis. The corresponding points on the axes are then joined by straight lines, as done in Fig. 127, and if the construction is carried out accurately, all these straight lines should intersect at a single point R such that OR is inclined at 45° to the axes. The perpendicular distances of this point from each of the axes, *i.e.*, the co-ordinates of the common point, give the focal length f.

Theory :—Let u_1 and v_1 be the co-ordinates of R when $u_1 = v_1$.
Now in the $\triangle ABO$,

$$\text{area } ABO = \text{area } ARO + \text{area } BRO.$$

Here $OA = u$ and $OB = v$.

$$\therefore \quad \tfrac{1}{2}uv = \tfrac{1}{2}uv_1 + \tfrac{1}{2}vu_1,$$

or, $\tfrac{1}{2}uv = \tfrac{1}{2}ux + \tfrac{1}{2}vx$ (where $x = u_1 = v_1$).

$$\frac{1}{x} = \frac{1}{v} + \frac{1}{u}.$$

But we know for a convex lens producing a real image,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}.$$

$$\therefore \quad x = f = v_1 = u_1.$$

Hence each co-ordinate of R gives the focal length of the lens.

(b) Plane Mirror Method.—Another simple method is to place a plane mirror vertically behind the lens L mounted on a stand (Fig. 128). A pin PQ is clamped in front of the lens with its tip very nearly on the principal axis of the lens. The pin is then moved backwards and forwards in front of the lens until (say, at PQ) there is no *parallax* between the pin and its inverted image $P'Q'$.

The position PQ gives the position of the focus of the lens. For, the rays diverging from the object at this position will be rendered parallel to the axis after emergence from the lens, and will be incident normally on the mirror. They will be reflected along the same path and, after refraction by the lens, will converge to the same position.

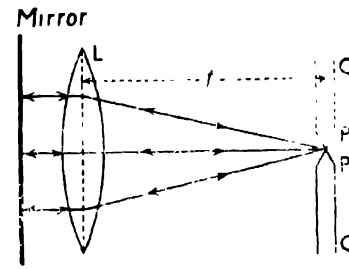


Fig. 128

The distance of the pin PQ at this position from the optical centre of the lens will give the focal length of the lens. For an equi-con-

vex lens, this distance is equal to the distance of the pin from the first surface of the lens plus $\frac{t}{3}$, where t = thickness of the lens.

The distance of the lens from the mirror is immaterial. So the plane mirror may also be placed on a horizontal platform and the lens placed on it, and a pin attached horizontally to a vertical stand may be adjusted in height until a real inverted image coincident with the object pin is found.

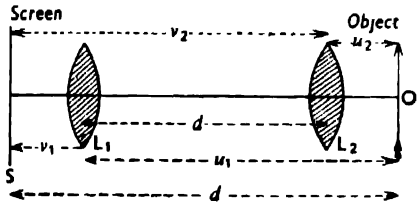


Fig. 129

(c) **Displacement method.**—If in the u–v method *the screen and the object are placed apart at any fixed distance greater than 4 times the focal length of the lens*, it will be found that there are two positions L_1, L_2, of the lens (Fig. 129) for each one of which a sharp, real (inverted) image is obtained on the screen. If D be the distance between the object and the screen, and d the distance between the first and the second position of the lens, called the *displacement of the lens*, the focal length of the lens is given by the relation,

$$f = \frac{D^2 - d^2}{4D}.$$

In carrying out this experiment with an optical bench, an index correction is to be introduced for D whereas no such correction is needed for the displacement d.

(d) Magnification Method :—

(*i*) *Object and screen fixed.*—Here the object should be a fine ink line of the length l_o drawn on a ground glass plate illuminated from behind. The object and the screen are to be placed apart at a distance greater than $4f$. Now placing the lens at a particular position a sharp image of length l_1 is obtained on the screen. Hence $m_1 = \frac{l_1}{l_o}$. Then displacing the lens through a distance d, a second sharp image is again cast on the screen. Here $m_2 = \frac{l_2}{l_o}$, where l_2 is the length of the image. The focal length f of the lens is then given by the relation

$$f = \frac{d}{m_1 - m_2}.$$

(*ii*) *Lens fixed*—Here keeping the lens fixed, the two sharp images are produced on the screen by shifting either the object or

the screen. If d be the shift of the screen and m_1 and m_2 the magnifications in the two positions, then the focal length is given by

$$f = \frac{d}{m_1 - m_2}.$$

Again if d_1 be the shift of the object, then the screen being fixed

$$f = \frac{d_1}{\frac{1}{m_1} - \frac{1}{m_2}}.$$

(e) **Direct method with a telescope.**—A telescope (Fig. 130) is first focussed for parallel rays. The convex lens is then placed between this focussed telescope and the object. The object position is then adjusted by trial till it is clearly seen through the telescope. The distance between the object and the lens plus one-third the thickness of it gives the focal length of the lens.

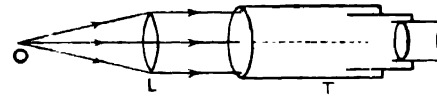


Fig 130

(f) **Lenses of very long focal length.**—In this case an auxiliary convergent lens of short focal length f_1 is to be placed in contact with the long focus lens of focal length f_2 and the focal length f of the combination is determined by any convenient method. Then,

$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$. From this relation f_2 is calculated.

The determination can also be made by taking the short focus lens C_1 in front of the object O so as to cast an image I_1 at a distance v_1 from C_1 (Fig. 131). The long focus lens C_2 is then placed at a distance

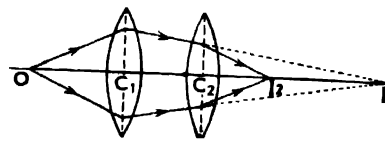


Fig. 131

d from C_1 to form the final image at I_2. So for the second lens C_2, $u = -(v_1 - d)$ (virtual object) and $v = -C_2I_2$.

Hence substituting in the general relation, $\frac{1}{f_2} = \frac{1}{v} - \frac{1}{u}$ we get f_2.

(B) CONCAVE LENS

(a) Combination Method.—The image in the case of a concave lens being always virtual, cannot be received on a screen. In order to get a real image, the given concave lens is combined in this method with a suitable convex lens of greater power, to render the whole system converging. After this, proceeding just as in the case of a convex lens, the focal length F of the combined lens can be determined.

Now, knowing the focal length of the convex lens used in the combination, the focal length of the concave lens can be determined from the relation, $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$, where f_1 is the focal length of the convex lens, and f_2 that of the concave lens. With proper signs it becomes,

$$-\frac{1}{F}=-\frac{1}{f_1}+\frac{1}{f_2} \quad \text{or.} \quad \frac{1}{f_2}=\frac{1}{f_1}-\frac{1}{F}.$$

Note—To get a real image with the combination, the focal length of the convex lens should be less than that of the concave lens *i.e.* the power of the convex lens should be greater.

(b) Auxiliary Lens Method.—(*i*) When the power of the convex lens is less than that of the concave lens, it will not be possible to receive an image on the screen when they are combined as in the above method. In this case a sharp image Q_1 of the object P (Fig. 132) is first of all obtained on the screen by the convex lens L_1 (auxiliary lens) alone, or the position of the image is located by placing a pin at Q_1 and avoiding parallax as explained before. Keeping the position of L_1 and L_1 fixed, the given concave lens L_2 is then interposed between L_1 and Q_1, and a new position of the image is obtained at a somewhat greater distance, say at Q_2. This is due to the fact that the converging rays after passing through the lens L_1 fall on the lens L_2 by which they are a little diverged.

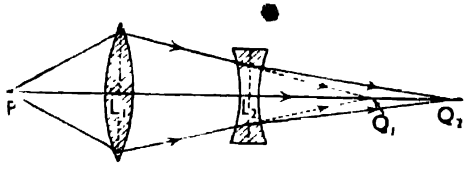


Fig. 132

Reversibly, rays of light from Q_2 will appear to diverge from Q_1. hence Q_1 may be considered as the virtual image of Q_2 with reference to the lens L_2, where $L_2Q_2=u$, and $L_2Q_1=v$, both u and v being positive. Hence f is calculated from the general formula for a lens, and will be positive in value.

Note that the image Q_2 will be obtained only when the distance L_2Q_1 is less than the focal length of the concave lens L_2, otherwise the light, after passing through L_2, will diverge from a point behind L_2.

(*ii*) Here a plane mirror M (Fig. 133) is set normally to the axis of the lenses C_1 and C_2 and the position of the object is so adjusted that the inverted real image of O is formed at the same position as that of the object. In this setting the rays are normally reflected back from M in a direction parallel to the principal axis.

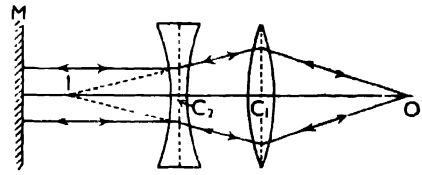


Fig. 133

Hence IC_2 is the focal length of the concave lens C_2. Position of C_2 is noted on the optical bench and removing the concave lens, the position of I due to the convex lens alone is ascertained with the help of a screen. Thus C_2I, *i.e.* the focal length of C_2, is determined.

(c) **Conjugate Foci Method** :—A pin with its tip on the axis of the lens is taken as the object O (Fig. 134). It is placed in front of the concave lens L and a virtual image appears to have been formed at I, whose position can be located by using a pin P and a plane mirror M covering half the aperture of L, the image P' of P being made coincident with I. Distances $MP=d$ and $ML=x$ are measured. Then, $u=+OL$ and $v=IL=IM-LM=+(d-x)$ (virtual image).

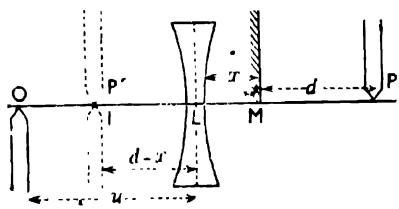


Fig 134

$\therefore$ Substituting in $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$, we can find f.

(d) **With the help of a concave mirror** :—The object O is placed in front of the lens behind which is kept an adjustable concave mirror M. The principal axes of the lens and the mirror should coincide. The rays incident on the lens after refraction proceed in directions shown in the figure and a virtual image is formed at I (Fig. 135). Now if by adjusting the position of M, it is found

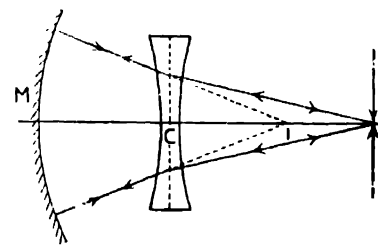


Fig. 135

that *a real inverted image* is formed at O, the distance MI shall be equal to the radius of curvature (r) of the mirror, which is known.

Now measuring the distance MC and CO we can get u and v, because, $u = +CO$ and $v = CI = MI - MC = +(r - MC)$.

$\therefore$ Substituting in the relation, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get f.

83(a). Determination of the radii of curvature of the surfaces of a convex lens by Boys' method :—In this method the lens is floated on the surface of a pool of mercury in order to increase the internal reflection of the lens surface B whose radius of curvature is to be determined. A strongly illuminated pin O, is held horizontally above the lens on a vertical stand. The position of the pin is adjusted till it coincides with its inverted real image formed by the rays reflected from the mercury-glass interface. The light from O is now reflected back along its own path and as such it must meet the surface B normally So the refracted rays inside the lens appear to diverge from C, the centre of curvature of the lower surface B (Fig. 136). Since the incidence is normal at the surface B, the absence of the mercury pool would not alter the situation, the emergent rays still appearing to diverge from C as usual.

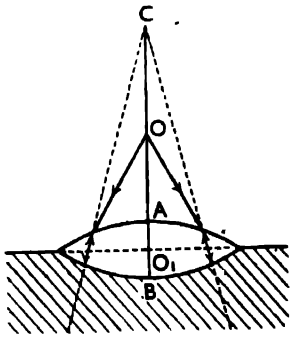


Fig. 136

Thus the lens in the adjusted position of the pin forms a virtual image of the pin at C. Let f be the focal length of the lens previously determined. Now,

object distance, $u = +O_1O$

and the image distance, $v = O_1C = r$, the radius of curvature of the surface B (to be determined).

$\therefore$ Putting these values in the general formula of a lens,

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$, we have,

$-\frac{1}{f} = \frac{1}{r} - \frac{1}{u}$; or, $\frac{1}{r} = -\frac{1}{f} + \frac{1}{u}$

Hence, $r = \frac{uf}{f-u}$.

Thus r is determined for the surface B. Similarly the radius of curvature of the other surface can also be determined.

Examples

1. Two convex lenses of focal lengths 3 inches and 4 inches respectively are placed at a distance of 6 inches apart and a luminous object 1 inch high is situated on the common axis of the lenses at a distance of 4 inches in front of the lens of smaller focal length. Find the position, nature and size of the image.

Ans. The general formula for a lens is, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$.

Here, for the lens of smaller focal length, $u=4, f=-3$; $v=$?

Substituting the values, we have $\frac{1}{v}-\frac{1}{4}=-\frac{1}{3}$, whence $v=-12$ inches.

This image serves as an object for the second lens. As the distance between the lenses is 6 inches, the object distance u' for the second lens $=v+6=-12+6=-6$ inches. So, $\frac{1}{v'}-\frac{1}{-6}=-\frac{1}{4}$; whence $v'=-2\frac{2}{5}$ inches.

That is, the final image is formed at a distance of $2\frac{2}{5}$ inches beyond the second lens, and is real and inverted.

Magnification by the first lens $=\frac{v}{u}=\frac{-12}{4}=-3$, and that by the second $=\frac{-2\frac{2}{5}}{-6}=\frac{2}{5}$.

Therefore, the total magnification $=-3\times\frac{2}{5}=-\frac{6}{5}=-1\frac{1}{5}$.

So, the length of the image is $(1\times 1\frac{1}{5})=1\frac{1}{5}$".

2. Rays of light from a luminous objects are brought to a focus at a point A. A convex lens of 12 in. focal length is then placed 12 in. from A so as to intercept the rays before they meet at A. If they now meet at B, find the distance AB. Where else could the lens be placed so that after passing through it, the rays might appear to diverge from B ?

Ans. In the first case, A serves as a virtual object with respect to the lens. Draw the diagram. Here $u=-12$, $f=-12$; $v=$?

Hence, $\frac{1}{v}-\frac{1}{-12}=\frac{1}{-12}$; whence $v=-6$.

That is, the image is formed (*i.e.* the rays meet) at B on the same side as A at a distance of 6 inches from the lens. $AB=12-6=6$ in.

Now let the lens be placed at a distance of x inches from A on the side opposite to B.

Hence $u=x$ in. (*i.e.* the distance of A from the lens) ; $v=(x+6)$ in. (*i.e.* distance of B from the lens) ; $f=-12$.

$$\therefore\quad \frac{1}{x+6}-\frac{1}{x}=-\frac{1}{12}\text{, or, }\frac{x-(x+6)}{x(x+6)}=-\frac{1}{12}\text{ ;}$$

$$\text{or, }\quad x^2+6x-72=0\text{ , or, }\quad (x-6)(x+12)=0\text{ ; }\therefore\quad x=6,\text{ or, }-12.$$

The negative value of x is not admissible here ; so the lens must be placed at a distance of 6 inches from A on the side opposite to B in order that the rays might appear to diverge from B.

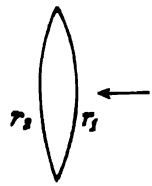
3. Four lenses are each made of glass of $\mu = 1.6$ and each is bounded by surfaces whose radii of curvature are 20 and 30 cm. respectively. If the lenses are (a) bi-convex, (b) convex meniscus, (c) double concave and (d) concave meniscus, calculate their focal lengths.

Ans. (i) Bi-convex [Fig. 137 (*a*)].—

Here $r_1 = -20$ cm., $r_2 = +30$ cm.

$$\therefore \quad \frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = (1{\cdot}6 - 1)\left(-\frac{1}{20} - \frac{1}{30}\right) = -0{\cdot}6 \times \frac{5}{60} = -\frac{1}{20}.$$

$\therefore$ $f = -20$ cm.



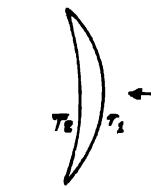
(a)

(ii) Convex meniscus [Fig. 137 (*b*)].—

Here $r_1 = -20$ cm., $r_2 = -30$ cm.

$$\therefore \quad \frac{1}{f} = (1{\cdot}6 - 1)\left(-\frac{1}{20} + \frac{1}{30}\right) = 0{\cdot}6 \times -\frac{1}{60},$$

$$\therefore \quad f = -100 \text{ cm.}$$



(b)

(iii) Double concave [Fig. 137 (*c*)] —

Here $r_1 = +20$ cm. $r_2 = -30$ cm.

$$\therefore \quad \frac{1}{f} = (1{\cdot}6 - 1)\left(\frac{1}{20} + \frac{1}{30}\right) = 0{\cdot}6 \times \frac{5}{60} = \frac{1}{20}.$$

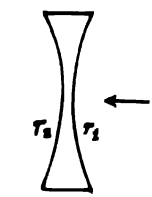
(iv) Concave meniscus [Fig. 137 (*d*)].—

Here $r_1 = +20$ cm., $r_2 = +30$ cm.

$$\therefore \quad \frac{1}{f} = (1{\cdot}6 - 1)\left(\frac{1}{20} - \frac{1}{30}\right) = 0{\cdot}6 \times \frac{1}{60}.$$

$\therefore$ $f = 100$ cm.

4. Show that the radius of curvature of an equi-concave or equi-convex lens is equal to its focal length if the refractive index of the material of the lens is 1·5.



(c)

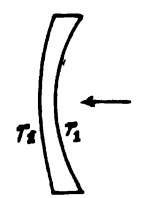
Ans. Let f and r be the focal length and the radius of curvature.

For an equi-concave lens, where f is +ve, $r_1 = +r$ and $r_2 = -r$, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = (1{\cdot}5 - 1)\left(\frac{1}{r} + \frac{1}{r}\right) = 0{\cdot}5 \times \frac{2}{r} = \frac{1}{r}$.

$\therefore$ $f = r$.

Again for an equi-convex lens, where $r_1 = -r$, $r_2 = +r$, and f is $-$ve, $-\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = (1{\cdot}5 - 1)\left(-\frac{1}{r} - \frac{1}{r}\right)$

$$= -0{\cdot}5 \times \frac{2}{r} = -\frac{1}{r}.$$



(d)

Fig. 137

5. A thin plano-convex lens of focal length f and refractive index μ is silvered on its spherical surface. Calculate the focal length of the mirror-lens combination.

Ans. Rays of light, PL and QL_1, are incident on the plane surface of the lens in a direction parallel to the principal axis. They suffer no deviation at the plane

surface and are reflected from the silvered curved surface LOL_1 along the directions LN and L_1M respectively, LN and L_1M appear to converge to the point A where OA is the focal length $\left(=\frac{r}{2}\right)$ of the concave mirror LOL_1, r being the radius of curvature of the surface. But due to refraction at N and M these rays actually converge to the point F (Fig. 138).

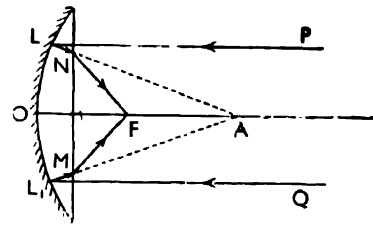


Fig. 138

$\therefore \quad OF = \frac{OA}{\mu} = \frac{r}{2\mu}$ where μ is the refractive index of the material of the lens.

This OF is evidently the focal length f_1 of the mirror-lens combination since the incident rays are parallel.

$$\therefore \quad f_1 = \frac{r}{2\mu}; \text{ or, } \frac{1}{r} = \frac{1}{2\mu f_1}$$

But for plano-convex lens the focal length f is given by,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{\infty} - \frac{1}{r}\right) = -\frac{\mu - 1}{r} = -\frac{\mu - 1}{2\mu f_1}.$$

$$f_1 = -\frac{\mu - 1}{2\mu} f.$$

As f is actually negative for a plano-convex lens, the focal length of the mirror-lens combination f_1 is positive.

6. A thin plano convex lens of focal length f and of refractive index μ is silvered on the plane side. Calculate the focal length of the lens-mirror combination so formed.

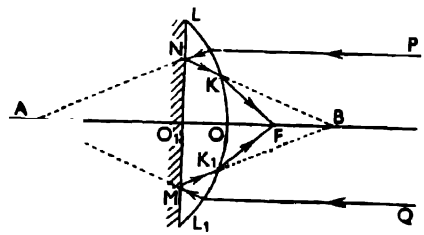


Fig. 139

Ans. Parallel rays of light PL, QL_1 are incident on the curved surface LOL_1 of the lens (Fig. 139). They suffer refraction at LL_1, reflection at NM and second refraction at KK_1 and finally converge to F.

$\therefore$ OF is the focal length of the lens-mirror combination, f_1 (say).

Due to refraction at L and L_1, $\frac{\mu}{AO} - \frac{1}{\infty} = \frac{(\mu - 1)}{-r}$, where r is the radius of curvature of the surface LOL_1.

$$\therefore \quad AO = -\frac{\mu r}{\mu - 1}.$$

Due to reflection at N and M, $-AO_1 = O_1B = -AO$ approximately, since the lens is thin.

$$O_1B = \frac{\mu r}{\mu - 1} = OB \text{ (approx.)}.$$

Due to the second refraction at K and K_1, $\left(\because\ \mu \text{ is to be replaced by } \frac{1}{\mu}\right)$

$$\frac{1}{OF}-\frac{\mu}{OB}=\frac{1-\mu}{-r}=\frac{\mu-1}{r};$$

or, $$\frac{1}{f_1}=\frac{\mu-1}{r}+\frac{\mu(\mu-1)}{\mu r}=\frac{2(\mu-1)}{r}.$$

But for a plano-convex lens, $\frac{1}{f}=-\frac{\mu-1}{r}$.

$$\therefore\ f_1=\tfrac{1}{2}f \text{ (numerically)}.$$

7. A plano-convex lens is silvered on its plane side and then it acts like a concave mirror of 20 cm. focal length. When the convex surface is silvered, it acts as a concave mirror of 7 cm. focal length. Calculate the refractive index of the lens material.

Ans. When the curved surface is silvered, the focal length of the concave mirror so formed is $f_1=-\frac{\mu-1}{2\mu}$ (vide, Ex. 5)

When the plane surface is silvered, the focal length of the concave mirror so formed is $f_2=-\frac{f}{2}$. (vide, Ex. 6)

Here, $20=-\frac{f}{2}$ and $7=-\frac{\mu-1}{2\mu}f$.

$$\therefore\ \frac{20}{7}=\frac{f}{2}\cdot\frac{2\mu}{\mu-1}\cdot$$

or, $20\mu-20=7\mu$, or, $13\mu=20$; or, $\mu=1{\cdot}54$.

8. If the radii of curvature of a double convex lens be 30 and 45 cms. what will be its focal length in air and in water? The refractive indices of glass and water are given as $\frac{3}{2}$ and $\frac{4}{3}$ respectively. (Agra U. 1945)

Ans. (a) In air,—

Here $r_1=-30$ cm., $r_2=+45$ cm. and $\mu=\frac{3}{2}=1{\cdot}5$.

$$\therefore\ \frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=(1{\cdot}5-1)\left(-\frac{1}{30}-\frac{1}{45}\right)=-0{\cdot}5\times\frac{5}{90}=-\frac{1}{36}.$$

$$\therefore\ f=-36 \text{ cm}.$$

(b) In water,—

Here the relative refractive index of glass with respect to water

$$=\frac{a\mu_g}{a\mu_w}=\frac{3}{2}\Big/\frac{4}{3}=\frac{9}{8}$$

$$\therefore\ \frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=\left(\frac{9}{8}-1\right)\left(-\frac{1}{30}-\frac{1}{45}\right)$$

$$=-\frac{1}{8}\times\frac{5}{90}=-\frac{1}{8}\times\frac{1}{18}=-\frac{1}{144}.$$

$$\therefore\ f=-144 \text{ cm}.$$

9. In a bi-convex lens of refractive index 1·5 if the radius of curvature be changed from 5 to 6 cm. find the change in the power of the lens.

Ans. In a bi-convex lens, $r_1=-r$ and $r_2=r$.

$$\therefore\ \frac{1}{f}=(1{\cdot}5-1)\left(-\frac{1}{r}-\frac{1}{r}\right)=-0{\cdot}5\times\frac{2}{r}=-\frac{1}{r}.$$

If $r=5$ cm., $P_1=+\frac{100}{5}=20$ dioptres and if $r=6$ cm., $P_2=+\frac{100}{6}=16{\cdot}66$ dioptres.

$\therefore$ Change in power $=P_1-P_2=20-16{\cdot}66=3{\cdot}34$ dioptres.

10. A convex lens of focal length 30 cm, is placed 20 cm. away from a concave lens of focal length 4 cm. An object is placed 6 metres distant from the convex lens (which is nearer to it) and on the common axis of the system. Find the position, and the magnification of the image.

Ans. For the convex lens, $u=600$ cm. and $f=-30$ cm.

$\frac{1}{v_1}-\frac{1}{u}=\frac{1}{f}$, or, $\frac{1}{v_1}-\frac{1}{600}=-\frac{1}{30}$. Hence $v_1=\frac{-600}{19}=-31\frac{11}{19}$ cm.

Since the lenses are separated by a distance of 20 cm. the virtual object distance for the concave lens is $u_1=-\left(31\frac{11}{19}-20\right)=-11\frac{11}{19}$ cm $=-\frac{220}{19}$ cm

The focal length of the concave lens $=+5$ cm.

$\therefore\ \frac{1}{5}=\frac{1}{v}-\left(-\frac{19}{220}\right)$, or, $\frac{1}{v}=\frac{1}{5}-\frac{19}{220}=\frac{25}{220}$.

$\therefore\ v=\frac{220}{25}=8{\cdot}8$ cm.

Hence a virtual image is formed at a distance of 8·8 cm. from the concave lens in between the two lenses.

The magnification $m=m_1\times m_2$, where m_1 and m_2 are the magnifications of the two component lenses.

Now, $m_1=-\frac{600}{19}\Big/600=-\frac{1}{19}$, and $m_2=\frac{8{\cdot}8}{\frac{220}{19}}=\frac{220}{25}\times\frac{19}{220}=\frac{19}{25}$.

$=\frac{1}{19}\times\frac{19}{25}=\frac{1}{25}$.

11. An equi-convex lens is placed just below the surface of water in a tank which is 40 cm. deep. A point object at the bottom of the tank is normally viewed through the lens and water. Find the position of the final image assuming $\mu_w=\frac{4}{3}$, $\mu_g=\frac{3}{2}$ *and the radius of curvature of the lens surfaces* $r=3$ *cm.*

Ans. The relative refractive index of glass with respect to water,

$=\frac{3}{2}\Big/\frac{4}{3}=\frac{9}{8}$

$\therefore$ Effective focal length of the lens (f) under water is given by

$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$, where $\mu=\frac{9}{8}$ and $r_1=-3$ cm. and $r_2=3$ cm.

$\therefore\ \frac{1}{f}=\left(\frac{9}{8}-1\right)\left(-\frac{1}{3}-\frac{1}{3}\right)=-\frac{1}{8}\times\frac{2}{3}=-\frac{1}{12}$.

$\therefore\ f=-12$ cm.

If v be the image distance in water,

$\frac{1}{v}-\frac{1}{40}=-\frac{1}{12}$, or, $v=-\frac{120}{7}$ cm.

Due to refraction at the water surface of the tank, there is an apparent change of this distance, which serves as the object distance.

Since $\mu=\frac{\text{real distance}}{\text{apparent distance}}=\frac{-120}{7}\Big/\text{apparent distance}$.

$\therefore$ The apparent distance of the final image from the surface of water in the tank $=\frac{120}{7}\times\frac{3}{4}=\frac{90}{7}=12\frac{6}{7}$ cm.

12. An object and a screen are placed on an optical bench and a convex lens is placed between them so as to throw a sharp image on the screen. The linear magnification is found to be 2·5. The lens is then moved 30 cm. nearer to the screen and a sharp image is formed again. Calculate the focal length of lens.

Ans. Let x be the distance of the object from the lens in the undisplaced position. If D be the distance between the object and the screen, the image distance is $D-x$. When the lens is displaced through 30 cm, the image distance (between the screen and the lens) becomes x and the object distance becomes $x+30$.

$\therefore$ $D=2x+30$, and the image distance in the first case is $D-x=x+30=v$.

$\therefore$ $m=\frac{v}{u}$, or, $2{\cdot}5=\frac{x+30}{x}$, or, $1{\cdot}5x=30$, or, $x=20$ cm.

$D=2\times 20+30=70$

$f=\frac{D^2-d^2}{4D}=\frac{70^2-30^2}{4\times 70}$. ($\because$ $d=30$ cm.)

$\frac{100\times 40}{4\times 70}=\frac{100}{7}=14\frac{2}{7}$ cm. (numerical value).

13. The tip of a pointer is made to move along the principal axis of a plano-convex lens and is finally so adjusted that the image of the tip formed by refraction at the front surface and the internal reflection at the back surface, coincides with the object. If the convex surface faces the pointer this adjusted distance is 30 cm. from the lens and when the plane surface faces the pointer it is only 10 cms. Calculate the focal length, the radius of curvature of the convex surface and the refractive index of the lens material.

Ans. When the plane surface faces the pointer the rays refracted from the plane surface appear to converge to the centre of curvature of the curved surface such that the rays are normally incident and are reflected back in the same direction. Hence the object and the final image are coincident. Since the final image of the first image at 10 cm. is formed due to refraction at a plane surface, we have, $\mu=\frac{r}{10}$; or, $\frac{\mu}{r}=\frac{1}{10}$.

Again when the convex surface faces the object, the refracted ray should be parallel to the axis so that it is normally incident on the plane surface and is reflected back. Here $v=\infty$ for the refraction at the curved surface.

$\therefore$ $\frac{\mu}{\infty}-\frac{1}{30}=\frac{\mu-1}{-r}=-\frac{\mu}{r}+\frac{1}{r}$,

or, $\frac{1}{30}=\frac{\mu}{r}-\frac{1}{r}$, or, $\frac{1}{30}=\frac{1}{10}-\frac{1}{r}$ $\therefore$ $r=15$ cm.

Hence $\mu=r\times\frac{1}{10}=\frac{15}{10}=1{\cdot}5$

Again for the plano-convex lens

$\frac{1}{f}=-\frac{\mu-1}{r}=-\frac{(1{\cdot}5-1)}{15}=-\frac{0{\cdot}5}{15}=-\frac{1}{30}$ $\therefore$ $f=-30$ cm.

14. A thin equi-convex lens is placed on a horizontal plane mirror, and a pin held 20 cm. vertically above the lens coincides in position with its own image. The space between the bottom surface of the lens and the mirror is filled with water ($\mu=1{\cdot}33$) and the pin has to be raised to a distance of 27·5 cm. from the lens to coincide with its image. Find the radius of curvature of the lens surfaces.

Ans. f_1 of the lens $=-20$ cm.; F of the water and glass-lens combination $=-27{\cdot}5$ cm. (the combined lens being convex).

$\therefore$ The focal length, f_2, of the water lens is given by, $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$.

$\therefore$ $-\frac{1}{27{\cdot}5}=-\frac{1}{20}+\frac{1}{f_2}$.

Hence, $f_2=+\frac{220}{3}$, (concave lens).

Since it is a plano-concave lens,

$$\frac{1}{f_2}=\frac{\mu-1}{r}\text{ ; or, }\quad r=\frac{220}{3}(\mu-1)=\frac{220}{3}(1{\cdot}33-1).$$

Hence, $r=24{\cdot}2$ cm.

Questions (Chapter IV)

1. In the case of refraction at a spherical surface of refractive index μ, derive the formula connecting the distances of the image and the object from the pole. (P. U. 1952, S.)

2. Deduce the relation $\frac{\mu}{v}+\frac{1}{u}=\frac{\mu-1}{R}$ when an image is formed by refraction through a single spherical surface. Apply the formula to derive $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ for refraction through spherical lenses. (Pat. U. 1947)

3. A convex lens of focal length 15 cm is made of glass of R. I. 1·52. What will be its focal length if it is totally immersed in a liquid R. I. 1·35 ?

[*Ans.* 61·9 cm.]

4. Deduce the relation $\frac{\mu}{v}-\frac{1}{u}=\frac{\mu-1}{r}$ for refraction at a single spherical surface and hence deduce the formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ for refraction through a lens. (C. U. 1940)

5. Prove Helmholtz's formula for angular magnification.

[C. U. (H) 1950, '57, '65]

What do you mean by linear and angular magnification of an optical system.

[C. U. (H) 1950]

6. A small air-bubble in a sphere of glass 4 inches in diameter, appears, when looked along a diameter, to be 1 inch from the surface. What is its true distance ? Refractive index of glass is 3/2.

[*Ans.* 1·2 inches from the surface.]

7. A ray of light is incident on a transparent sphere of refractive index 1·32. After refraction, it is once internally reflected and is then refracted out. Find the minimum deviation and the angle of incidence at which deviation is minimum. (C. U. 1949)

[*Ans.* Min. deviation = 136° ; Angle of incidence = 60°]

8. A small object is enclosed in a sphere of solid glass 7 cms. in radius. It is situated 1 cm. from the centre and is viewed from the side to which it is nearest. Where will it appear to be, if the refractive index of glass is 1·4 ?

[*Ans.* 5·67 cms. from the viewing side]

9. A small air-bubble in a sphere of glass of diameter 4″ is looked at so that the bubble and the centre of the sphere are in a line with the eye. If the bubble appears to be 1″ from the surface, find its true distance ($\mu_g=1{\cdot}54$).

[*Ans.* 1·21″ from surface]

10. (*a*) Derive the conjugate foci relation for a thin lens with the optical centre as the origin and hence find out the conjugate foci relation with the principal foci as the origins. (C. U. 1966)

(*b*) Prove from the first principle that for a lens,

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$ (P. U. 1954)

What approximations have been used and what limitations do they impose on the use of this formula ? (P. U. 1937)

11. A thin plano-convex lens acts like a concave mirror of 28 cms. focal length when its plane surface is silvered and like a concave mirror of 10 cms. focal length when its curved surface is silvered. Calculate the refractive index of the material of the lens.

[*Ans.* 1·55] (Utk. U.1955)

12. If the refractive index from air to glass is 3/2 and that from air to water 4/3, find the ratio of the focal lengths of a glass lens in water and in air.
[*Ans.* 4 : 1] (P. U. 1952, S)

13. The radii of curvature of the surfaces of a double convex lens are 20 cms. and 40 cms. and its focal length is 20 cms. What is the refractive index of glass ?
[*Ans.* 1·667]

14. Investigate with the help of diagrams, the conditions for the formation of (*a*) real image, (*b*) virtual image, by a plano-convex lens.

15. The image of a pin is found to coincide with the pin itself when it is placed at a distance of 25 cm. from a concave mirror. Then a concave lens is placed between the pin and the mirror at a distance of 20 cms. from the mirror, the image again coincides with the pin, when it is moved to a distance 35 cms. from the mirror. Find the power of the lens.

[*Ans.* $13\frac{1}{3}D$]

16. An object is placed at a distance of $2f$ from a convex lens of focal length The rays, after traversing the lens, are reflected from a convex mirror and again refracted by the lens forming a real inverted image coincident with the object. If the distance between the lens and the mirror is a. show that the radius of curvature of the mirror is $2f-a$.

17. An electric light is distant 6 ft. from a wall. A convex lens gives a sharp image of the light on the wall, when it is held 2 ft. away from the light. A second sharp image is obtained on the wall, when the lens is held 2 ft. away from the wall. Determine the focal length of the lens and compare the magnifications produced in the two positions of the lens.

[*Ans.* $f=\frac{4}{3}$ ft. ; $I_1 : I_2 = 4 : 1$]

18. A convex lens of focal length 12 cms. and a concave lens of focal length 10 cms. are placed co-axially at a distance of 10 cms. from each other. An object which is nearer to the convex lens on the common axis of the two lenses, is placed 48 cms. away from the convex lens. Find the position, magnification and nature of the final image formed.
[*Ans.* 15 cms. from the concave lens on the side away from the convex lens ; $m=-\frac{5}{8}$].

19. Two co-axial lenses of focal length f_1 and f_2 are separated by a distance a'. State clearly what you understand by the lens equivalent to the combination and in what sense it is equivalent. Deduce the focal length of the equivalent lens. (Ag. U. 1941)

20. Find out the equivalent focal length of two thin lenses separated by a distance.
Two convex lenses of focal lengths 10 cms. and 15 cms. are separated by a distance of 2 cms. An object is placed at a distance of 12 cms. upon the common axis in front of the 10 cms. lens. Calculate the position of the final image. (C. U. 1956)

[*Ans.* 11·9 cms. beyond 15 cm. lens]

21. Deduce an expression for the focal length of a thin lens in terms of the refractive index and the radii of curvature of the two faces. (C. U. 1953)

Two thin lenses of focal lengths f_1 and f_2 are separated by a distance d. Show that their equivalent focal length F is given by

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}.$$ (C. U. 1942, '53, '56 ; C. U. Hon. '49, '57 ; G. U. 1957)

22.(*a*) A convex lens of focal length f is placed between an object and a screen, which are d distance apart. If m be the magnification of the image produced by the lens on the screen, then prove that, $f=\frac{md}{(1+m)^2}$.

(*b*) Find the minimum distance between the object and the image due to a convex lens for the image to be a real one. (C. U. 1965)

23. An object is placed at a distance u from a convex lens and a plane mirror is held behind the lens at a distance equal to the focal length f of the lens. The image of the object, after reflection at the plane mirror and a second refraction through the lens, is at a distance v in front of the lens. Show that, $u+v=2f$

24. Find the limiting distance between a fixed object and a screen, so that for two positions of the convex lens placed between them, sharp real images are formed on the screen (C. U. 1944, '50)

If m_1 and m_2 be the magnifications of the image for the two positions of the lens, then show that the focal length f of the lens is given by $f = \frac{d}{m_1 - m_2}$ where d is the displacement of the lens. (P. U. 1946)

25. (*a*) Show that the minimum distance between an object and its real image in a convex lens is four times the focal length of the lens. (P. U. 1946)

(*b*) A convex lens is adjusted to form the image of a candle flame on a wall. It is found that by moving the lens 25 cms. nearer to the wall a sharp image is again obtained exactly one quarter of the length of the first. Find the focal length of the lens. (Pat. U. 1946)

[*Ans.* $16\frac{2}{3}$ cm.]

26. Mention the various methods for determining the focal length of a converging lens and describe one of them giving reasons for your choice.

27. A luminous source and a screen are placed at a fixed distance apart along a scale, and a convex lens can be moved between them. Explain how you will utilise this arrangement to determine the focal length of the lens. If a and b be the sizes of the image for the two positions of the lens, show that the size of the object is equal to $\sqrt{ab}$.

28. Explain with diagrams how you would find the focal length f of a convergent lens by the displacement method.

Established the formula, $f = \frac{d^2 - x^2}{4d}$, where d = distance of the source of light from the screen, and x = distance between two positions of the lens.

29. Two equiconvex lenses of focal length 20 cms. are placed in contact and the space between them is filled up with water. Find the focal length of combination ($\mu_g = 3/2$ and $\mu_w = 4/3$).

[*Ans.* 15 cm.]

30. Two narrow slits are in a plane perpendicular to the axis of a converging lens. The lens produces real images of the slits 3 cm. apart. If the lens is moved 60 cm. along its axis, real images of the slits 1/3 cm. apart are again produced in the same plane as before. Find the distance between the slit and the focal length of the lens.

[*Ans.* 1 cm., 22·5 cm.]

31. Explain how you can measure the focal length of a very long focus lens such as spectacle lens.

32. Describe an optical method of finding the radius of curvature of the surface of a convex lens.

Describe the principle of working of a lensometer.

84. **Thick Lens** :—In thin lenses the thickness of a lens is negligible compared to its focal length. But this is not the case with a thick lens or a system of lenses. Here the thickness of the lens is to be considered. Due to the introduction of this thickness, the relation between the object and the image distances becomes much more complicated than that of a thin lens. The case of a thick lens is similar to that of a system of two thin lenses separated by a distance equal to thickness of the thick lens under consideration, the intervening space being filled up with the material of the lens. As most of the optical instruments consist of a system of lenses, the components of which are neither necessarily thin, some easy and quick method of determining the focal lengths and conjugate foci positions for such systems must be found out. Gauss showed that simplified relations between object distance, image distance, and focal length can be found for a compound lens or a thick lens if these distances are measured from two definite points on the principal axis. These two definite points of a lens are known as its *principal points* or Gaussian points. If two planes are imagined to pass through these points in a direction at right angles to the principal axis of the lens system, they will be termed as the principal planes or Gaussian planes. Over and above the principal points, four other fixed points on the principal axis are usually defined in order to draw the geometrical images formed by a thick lens in an elegant way. Two of these four points are known as the *focal points* and the other two as the *nodal points*. The planes through these points perpendicular to the axis are known respectively as the **focal planes** and the **nodal planes**. The above six fixed points for a thick lens or a compound lens are generally known as the **Cardinal points** of the lens.

85. **Definitions of the cardinal points** :—

(i) **Focal Points (or Principal Foci)**—On the principal axis of a thick lens or a system of lenses there is a fixed point such that rays actually diverging from that point or appearing to converge to that point will, after refraction through the lens or the system, form a beam parallel to the axis. This point is known as the *first focal point*. In Fig. 140, F_1 is the first focal point of the thick lenses O_1O_2. The ray F_1M is refracted along MN in lens and emerges out in the direction NT parallel to the axis.

Again a beam of rays incident parallel to the principal axis will, after refraction, actually converge to or appear to diverge from another fixed point on the principal axis. This point is known as the *second focal point*. In Fig. 140, the ray PQ which is parallel to the axis is refracted in the lens along QR and emerges out in the direction RF_2 where F_2 is the second focal point of the thick lens.

In the case of a system of two lenses separated by a distance, the positions of the two lenses can be represented by two straight lines O_1M and O_2N perpendicular to the principal axis and an exactly similar drawing as in Fig. 140 will serve the purpose.

(ii) **Principal Points.**—For a thick lens or a system there are two conjugate planes transverse to the principal axis of the lens or

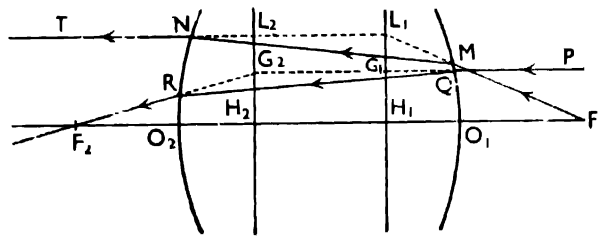


Fig 140

the system for which the lateral magnification produced by the lens or the system is unity and positive. Such planes are called *unit planes*, *principal planes* or *Gaussian planes*. The points where they cut the axis are known as the *unit points* or *principal points*. From the property of the principal planes, it follows that a ray of light (like PQ or F_1M in Fig. 140) in the object space striking the first principal plane ($H_1G_1L_1$) at a point (like G_1 or L_1) will appear to come from a point (like G_2 or L_2) on the second principal plane ($H_2\ G_2\ L_2$) at the same height from the axis. It should, however, be realised that the above way of representation gives us only an easy method of drawing the image of an object as will be shown in the next article. The actual path of the ray PQ or F_1M will be $PQRF_2$ or F_1MNT as shown by the arrow marks in Fig 140. Another importance of the principal points is that if the object and image distances are respectively measured from the first and the second principal points, the simple conjugate foci relation $\frac{1}{V}-\frac{1}{U}=\frac{1}{F}$ holds also in the case of a thick lens, where F, the focal length of the lens is also measured from the second principal point.

Here also the above modification of drawing as indicated in (*i*) is sufficient for a system of lenses.

(iii) Nodal Points :—There are two points on the principal axis of a thick lens or a system such that any ray in the object space directed towards one of them, emerges from the other in the image space in a direction parallel to the incident ray. These points are respectively known as the *first* and the *second nodal points*. The angular magnification referred to these points is unity and positive. In Fig. 141, PQ is the ray incident on a thick lens O_1O_2 in a direction PQN_1, cutting the axis at N_1. It cuts the first principal plane at L_1 and second principal plane at L_2 so that $L_1H_1 = L_2H_2$. Then it emerges out along L_2N_2RS cutting the axis at N_2 and in a direction parallel to PQ. Here N_1 and N_2 are the two nodal points and $PQRS$ represented by the arrow marks is the actual path of rays.

In the case of a system of two lenses separated by a distance, an exactly similar drawing will serve the purpose provided O_1Q and O_2R are replaced by two straight lines representing the position of the lenses.

Over and above the above six points and planes, in some cases the introduction of four other points or planes often simplifies the treatment. They are : (1) *The antiprincipal planes* or *negative principal planes*. These are conjugate planes of unit negative lateral magnification. They lie as far from the focal points as the corresponding principal points, but are on the other sides. (2) *The antinodal points*. These are conjugate points for which the angular magnification is unity and negative. They lie as far away from the focal points as the corresponding nodal points, but are on the opposite sides.

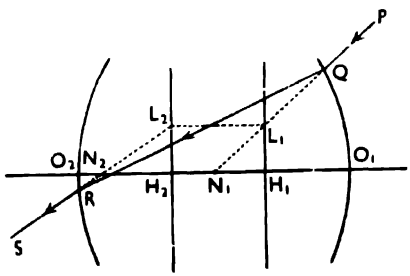


Fig. 141

Note.—It must be realised that the positions of the cardinal points depend on the shape of the lens. None of them need necessarily be inside the lens material.

86. Geometrical construction of an image formed by a thick lens :—For the geometrical construction of an image formed by a thick lens the positions of any four of the six cardinal points are essential. In the method of construction described here the actual path of rays are not shown The drawing is based purely on the properties of the cardinal points.

In Fig. 142 AB represents the thick lens. H_1 and H_2 are the first and the second principal points. F_1 and F_2 are the first and second focal points. N_1 and N_2 are the first and second nodal points. PQ is an extended object on the principal axis F_1F_2 of the lens. In order to get the image of PQ draw any two of the following geometrical paths of rays.

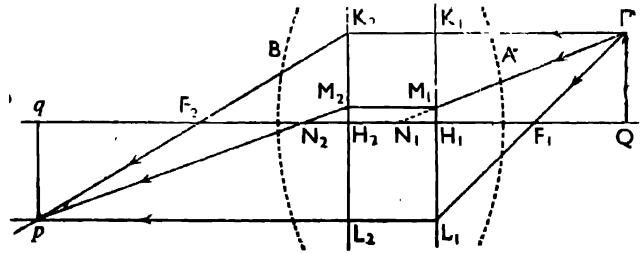


Fig. 142

(1) A ray from P is incident parallel to the axis, It cuts the first principal plane at K_1 and appears to emerge from the second principal plane at the same height at K_2 and finally passes through the second focal point F_2. The geometrical path of the ray is shown as $PK_1K_2F_2p$ in Fig. 142.

(2) A second ray from P passing through the first focal point F_1 falls on the first principal plane at L_1 and emerges out from the second principal plane at the same height at L_2 in a direction parallel to the axis. The geometrical path is evidently $FF_1L_1L_2p$.

(3) A third ray PM_1 appearing to cut the axis at the first nodal point N_1 meets the first principal plane at M_1, proceeds along M_1M_2 parallel to axis and finally from the second principal plane it emerges out along M_2N_2p through the second nodal point N_2 such that M_2p is parallel to PM_1.

All the three rays meet at p which is the image of P. So the perpendicular pq drawn from p on to the axis gives the image of PQ.

Ordinarily the medium on the two sides of the lens is the same (air in general) and it can be shown that the nodal points coincide with the principal points under such condition. Hence the first two rays are commonly used for drawing the images of extended objects formed by a thick lens.

For a system consisting of two lenses separated by a distance, an exactly similar figure is to be drawn only replacing the dotted curved surfaces by two straight lines perpendicular to the axis, which would represent the positions of the two lenses.

86(a). Helmholtz's equation relating the lateral and the angular magnifications of an optical system (or a thick lens) :—

As shown in the attached figure O_1H_1 and O_2H_2 are the principal planes of the system, F_1R_1 and F_2R_2 are the focal planes and μ_1 and μ_2, the refractive indices of the media on the two sides of the systems.

Let P_1Q_1 and P_2Q_2 be the conjugate elements (object and image) of heights h_1 and h_2. Suppose that a ray from P_1 starts at an angle

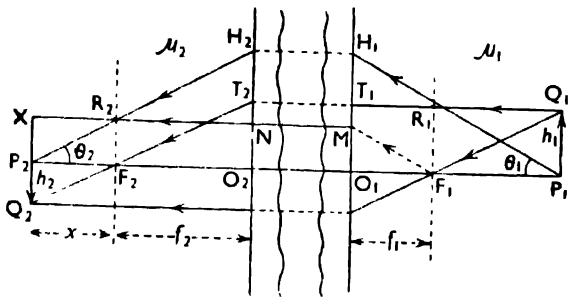


Fig. 142(*a*)

θ_1 with the axis and meets the axis at P_2 in the image space at an angle θ_2, and that the image plane is at a distance x from the second focal plane.

Now from similar Δs $P_2Q_2F_2$ and $O_2T_2F_2$, $\dfrac{x}{f_2}=-\dfrac{h_2}{h_1}$ (1)

Now considering an imaginary ray starting from F_1 at θ_1 with the axis as shown by F_1MNX, we get,

$$O_1M=f_1\tan\theta_1=O_2N=F_2R_2=x\tan\theta_2.$$

$$\therefore\quad \frac{\tan\theta_2}{\tan\theta_1}=\frac{f_1}{x} \qquad \ldots \qquad (2)$$

From (1) and (2), $\dfrac{h_2}{h_1}\times\dfrac{\tan\theta_2}{\tan\theta_1}=-\dfrac{x}{f_2}\times\dfrac{f_1}{x}=-\dfrac{f_1}{f_2}=\dfrac{\mu_1}{\mu_2}$

(Vide Art. 66 *I*)

$$\therefore\quad \mu_1h_1\tan\theta_1=\mu_2h_2\tan\theta_2 \qquad \ldots \quad (3)$$

This is Helmholtz's relation and for small angles it reduces to

$$\mu_1 h_1 \theta_1 = \mu_2 h_2 \theta_2$$

When media on the two sides are identical (*i.e.* $\mu_1 = \mu_2$)

$$h_1 \tan \theta_1 = h_2 \tan \theta_2$$

$$\text{or,} \quad \frac{h_2}{h_1} = 1 \Big/ \frac{\tan \theta_2}{\tan \theta_1} \quad \text{or,} \quad m_l = \frac{1}{m_\theta}$$

where m_l = lateral magnification $= \frac{h_2}{h_1}$ and m_θ = angular magnification $= \frac{\tan \theta_2}{\tan \theta_1} = \frac{\theta_2}{\theta_1}$, when θ's are small.

***Modifications of Helmholtz's relation :—**

(a) **When one of the Conjugate planes is at Infinity** :—Suppose that the object plane is at infinity. The corresponding image plane will be the second focal plane of the optical system. For the object space, then,

$\theta_1 = O$ and $h_1 = \infty$,

so that $\mu_1 h_1 \tan \theta_1$ in Helmholtz's relation becomes indeterminate.

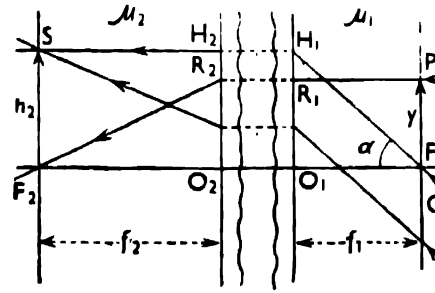


Fig. 142 (*b*)

To express the relation in a workable form, let us consider a beam of parallel rays from one end of the object at infinity proceeding in a direction parallel to the axis. $PR_1R_2F_2$ is such a ray meeting the axis at F_2 at an angle θ_2. Let the height of the ray PR_1 above the axis in the object space be y. Let us consider a second set of rays coming from the other end of the object and proceeding in the direction parallel to QH_1, which will, after refraction meet at S in the second focal plane. So the final image formed in the second focal plane is of height $SF_2 = h_2$. Let α be the angle subtended by QH_1 at the axis in the object space. It is evident from Fig. 142(*b*), that $f_2 \tan \theta_2 = O_2R_2 = y$ and $f_1 \tan \alpha = O_1H_1 = h_2$

$$\therefore \quad y f_1 \tan \alpha = h_2 f_2 \tan \theta_2$$

$$\text{or,} \quad y \tan \alpha = \frac{f_2}{f_1} \cdot h_2 \tan \theta_2 = -\frac{\mu_2}{\mu_1} h_2 \tan \theta_2 \quad \left(\because \frac{f_2}{f_1} = -\frac{\mu_2}{\mu_1} \right)$$

Hence, $-\mu_1 y \tan \alpha = \mu_2 h_2 \tan \theta_2$.

This is the modified form of Helmholtz's relation when the first conjugate is at infinity.

(b) When the system is Telescopic :—We know that for a truly telescopic system both the object and the image formed are at infinity and that the refractive indices of the image and object spaces are identical (*i.e.* $\mu_1 = \mu_2$).

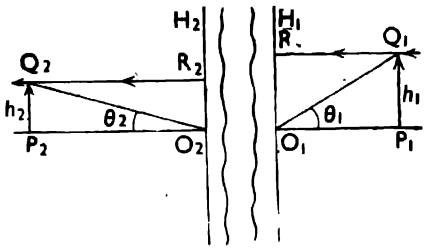


Fig. 142 (c)

Let Q_1R_1 be a ray incident on the telescopic system at a height h_1 from the axis and parallel to the axis. This ray comes from the object at infinity. Since the second focal plane is also at infinity, the emergent ray in the image space is also parallel to the axis, but at a different height h_2. Consequently the image height is independent of the position of the object.

Hence the lateral magnification is $m_l = \frac{h_2}{h_1}$, which is same for all pairs of conjugate planes.

From Helmholtz's relation,

$$\mu_1 h_1 \tan\theta_1 = \mu_2 h_2 \tan\theta_2$$

$$\frac{h_2}{h_1} = \frac{\mu_1 \tan\theta_1}{\mu_2 \tan\theta_2} = \frac{\tan\theta_1}{\tan\theta_2} \quad (\because \mu_1 = \mu_2)$$

When θ's are small, $m_l = \frac{h_2}{h_1} = \frac{1}{\tan\theta_2/\tan\theta_1} = \frac{1}{\theta_2/\theta_1} = \frac{1}{m_\theta}$.

That is, in a telescopic system lateral magnification is equal to the reciprocal of the angular magnification. Hence angular or lateral magnification for any pair of conjugate planes of a telescopic system remains constant.

87. (i) Location of the Cardinal Points of a thick lens in air.—In the following treatments, for the simplification of calculations, we shall assume that the medium on the two sides of the lens is air. In Fig. 143, A_1A_2 is the lens ; A_1 and A_2 are

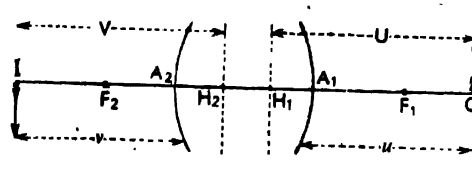


Fig. 143

the poles of the refracting surfaces ; F_1 and F_2 are the focal points ; H_1 and H_2 are the unit (or principal) points ; O and I are respectively the object and the image on the axis.

(a) **Focal Points.**—Due to refraction at the first surface, if v_1 be the image distance corresponding to the object distance u, both being measured from the pole A_1, the magnification due to the first refraction is $m_1 = \frac{v_1}{\mu u}$ (since v_1 is formed inside the medium of the lens of refractive index μ).

Again if due to the refraction at the second surface the final image is formed at a distance v from A_2, the corresponding magnification, $m_2 = \frac{\mu v}{v_1 + t}$ where t is the thickness of the lens.

$\therefore$ Total magnification, $m = m_1 \times m_2$.

If U and V be the object and image distances measured from the first and the second principal points, the overall magnification $m = \frac{V}{U}$.

We know that for the first refraction, $\frac{\mu}{v_1} - \frac{1}{u} = \frac{\mu - 1}{r_1}$ ... (1

where r_1 is the radius of curvature of the first surface.

If the incident rays are parallel, *i.e.* if the object is at infinity, $u \to \infty$ and from (1), we get, $v_1 = \frac{\mu r_1}{\mu - 1}$, ... (2)

and $v_1 + t = \frac{\mu r_1 + t(\mu - 1)}{\mu - 1}$ ... ... (3)

For second refraction,

$$\frac{1}{v} - \frac{\mu}{v_1 + t} = \frac{1 - \mu}{r_2},$$ (where r_2 is the radius of curvature of the second surface). That is,

$$\frac{1}{v} = \frac{(1-\mu)(v_1 + t) + \mu r_2}{r(v_1 + t)}$$

or, $$v = \frac{r_2(v_1 + t)}{(1-\mu)(v_1 + t) + \mu r_2}$$ ... ... (4)

Supposing $u \to \infty$ we have from (3) and (4) $v = A_2F_2$ (the focal distance from the pole of the second surface)

$$\frac{r_2 \cdot \frac{[\mu r_1 + t(\mu - 1)]}{\mu - 1}}{\mu r_2 - (\mu - 1)\frac{[\mu r_1 + t(\mu - 1)]}{(\mu - 1)}} = \frac{\mu r_1 r_2 + t r_2(\mu - 1)}{(\mu - 1)[\mu(r_2 - r_1) - (\mu - 1)t]} \quad (5)$$

When $u \to \infty$, U also tends to infinity and in the limit $u = U$. Hence V becomes the focal length as measured from the second principal point and $V = f_2 = H_2F_2$.

Since $m = \frac{V}{U} = \frac{v_1}{\mu v} \times \frac{\mu v}{v_1 + t}$ and $u = U$.

we have, $f_2 = H_2F_2 = V = \frac{v_1}{\mu v} \times \frac{\mu v}{v_1 + t} \times U = v_1 \cdot \frac{v}{v_1 + t}$

$$= \frac{\mu r_1}{(\mu - 1)} \times \frac{r_2[\mu r_1 + t(\mu - 1)]}{(\mu - 1)[\mu(r_2 - r_1) - (\mu - 1)t]} \times \frac{(\mu - 1)}{[\mu r_1 + t(\mu - 1)]}$$

[from (2), (3) & (5)]

$$= \frac{\mu r_1 r_2}{(\mu - 1)[\mu(r_2 - r_1) - (\mu - 1)t]} \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

$$\therefore \quad \frac{1}{f_1} = (\mu - 1)\left[\frac{1}{r_1} - \frac{1}{r_2} - \frac{(\mu - 1)t}{\mu r_1 r_2}\right] \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

Thus relations (5) and (6) give the distance of the second focal point F_2 respectively from the pole and the second principal point. Relation (7) gives an expression for the power of the lens.

It can be shown in a similar way that the expression for the distance f_1 of the first focal point from the first principal point is also exactly identical with that for f_2 in magnitude. According to our sign convention f_2 is taken as negative and the first focal length f_1 as positive.

Special Cases.—

(i) *For a transparent sphere* (Fig. 144) used as a thick lens $r_1 = -r$ $r_2 = r$ and $t = 2r$, where r is the radius of the sphere. Hence the second focal length f_2 is given by the relation,

$$\frac{1}{f_2} = (\mu - 1)\left[-\frac{1}{r} - \frac{1}{r} - \frac{2r(\mu - 1)}{-\mu r^2}\right]$$

$$= -(\mu - 1)\left[\frac{2}{r} - \frac{2(\mu - 1)}{\mu r}\right]$$

$$= -\frac{2(\mu - 1)}{\mu r}.$$

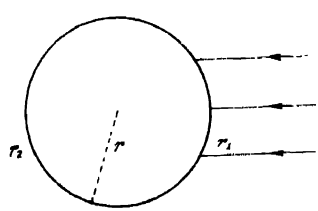


Fig. 144

(ii) In the case of a thin lens since $t \to 0$, the focal length, f_2, is given by the relation,

$$\frac{1}{f_2} = (\mu - 1)\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

(b) Principal Points —

With reference to Fig. 143, the distance of the second principal point (H_2) from the pole (A_2) of the second surface is given by $h_2 = A_2H_2$

$= (F_2H_2 - A_2F_2)$(numerically) $= A_2F_2 - F_2H_2$ ($\because$ both A_2F_2 and F_2H_2 are $-$ve).

$\therefore$ From relations (5) and (6) above, we have,

$$h_2 = A_2F_2 - F_2H_2 = \frac{\mu r_1 r_2 + tr_2(\mu-1)}{(\mu-1)[\mu(r_2-r_1)-(\mu-1)t]} - \frac{\mu r_1 r_2}{(\mu-1)[\mu(r_2-r_1)-(\mu-1)t]}$$

$$= \frac{tr_2}{\mu(r_2-r_1)-(\mu-1)t} \quad \ldots \quad \ldots \quad (8)$$

Since the principal planes are planes of unit positive magnification, if an object be at the first principal point, an image of the same size is formed at the second principal point, *i e.* the overall magnification is $+1$. So if $u = h_1$ (the distance of the first principal point from the pole of the first surface), the final image distance $v = h_2$. Let v_1 be the image distance for refraction at the first surface. Then $v_1 + t$ will be the object distance for the second refraction.

For the first refraction, $\frac{\mu}{v_1} - \frac{1}{u} = \frac{\mu-1}{r_1}$; or, $\frac{\mu}{v_1} = \frac{\mu-1}{r_1} + \frac{1}{h_1}$.

$$\therefore \quad v_1 = \frac{\mu r_1 h_1}{r_1 + (\mu-1)h_1}.$$

Hence the magnification due to the first refraction is,

$$m_1 = \frac{v_1}{\mu u} = \frac{v_1}{\mu h_1}.$$

For the second refraction, the magnification is,

$$m_2 = \frac{\mu v}{v_1 + t} = \frac{\mu h_2}{v_1 + t}.$$

Where $(v_1 + t)$, the object distance for second refraction is derived exactly in a similar way as v_1.

It is found that $v_1 + t = \frac{\mu r_2 h_2}{r_2 + (\mu-1)h_2}$.

Now since $m = 1 = m_1 \times m_2 = \frac{v_1}{\mu h_1} \times \frac{\mu h_2}{v_1 + t}$,

$$\frac{h_2}{h_1} = \frac{v_1 + t}{v_1} = \frac{\mu r_2 h_2}{\{r_1 + (\mu-1)h_2} \times \frac{\{r_1 + (\mu-1)h_1\}}{\mu r_1 h_1}$$

or, $r_1r_2+(\mu-1)r_1h_2=r_1r_2+(\mu-1)r_2h_1$;

or, $\frac{h_2}{h_1}=\frac{r_2}{r_1}$; or, $h_1=\frac{r_1h_2}{r_2}$.

$\therefore$ From relation (8),

$$h_1=\frac{tr_1}{\mu(r_2-r_1)-(\mu-1)t} \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

Special Cases.—

(*i*) *Principal Points for a transparent sphere.—*

Here $-r_1=r=r_2$ and $t=2r$

$$\therefore \quad h_1=\frac{-tr}{2\mu r-2r(\mu-1)}=\frac{-2r^2}{2r}=-r.$$

So the first principal point is at the centre of the sphere. Similarly it can be shown that the second principal point also coincides with the first at the centre of the sphere.

(*ii*) *Principal points of an ordinary double convex lens of small thickness.—*

Here r_1 is $-ve$ and r_2 is $+ve$.

$$\therefore \quad h_1=\frac{-tr_1}{\mu(r_1+r_2)-t(\mu-1)} \text{ and } h_2=\frac{tr_2}{\mu(r_1+r_2)-t(\mu-1)}.$$

Here r_1 and r_2 are simple numerical values. When $r_1=r_2=r$ (for equi-convex lenses) and $\mu=1{\cdot}5$ and t is sufficiently small compared to r,

$$h_1=\frac{-tr}{1{\cdot}5\times 2r-t(1{\cdot}5-1)}=\frac{-tr}{3r-t/2}=-\frac{t}{3}\left(\because 3r>>\frac{t}{2}\right).$$

Similarly $h_2=t/3$.

So while measuring u and v with an ordinary laboratory lens of equi-convex nature, a correction term $\frac{t}{3}$ is to be added to these distances.

(*iii*) *Principal Points of a plano-convex lens.*

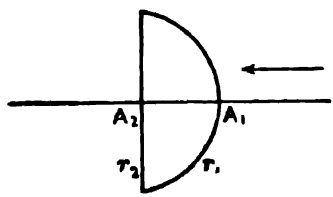


Fig. 145

Here $-r_1=r$ and $r_2=\infty$.

$$\therefore \quad k_1=\frac{-tr}{\mu(\infty+r)-t(\mu-1)}=0$$

We know, $h_1=A_1H_1$.

Hence the first principal point coincides with A_1.

$$h_2=\frac{tr_2}{\mu(r_2+r_1)-t(\mu-1)}$$

$$\frac{t}{\mu\left(1+\frac{r_1}{r_2}\right)-\frac{t}{r_2}(\mu-1)} \quad \mu\left(1+\frac{r}{\infty}\right)-\frac{t}{\infty}(\mu-1) \quad \mu$$

(c) Nodal Points.—As in the above treatments we have supposed the surrounding medium of the lens to be air (on both sides) it can be shown from the following simple geometrical considerations that the nodal points coincide with the principal points. Since the positions of these principal points have been determined above, those of the nodal points will exactly be the same.

To establish the exact coincidence of the nodal points and the principal points in the case of a thick lens surrounded by air, let us consider the principal planes H_1R_1 and H_2R_2 of the lens. Let F_1 and F_2 be the focal points and N_1 and N_2 the nodal points. Suppose a point object P is taken on the focal plane F_1P. Taking two rays, one parallel to the axis and the other through the first nodal point N_1, let us trace the directions of the emergent rays (Fig. 146). These are R_2F_2 and M_2Q parallel to each other. The drawings are evident from the figure itself. Here, R_2F_2, M_2Q and PM_1 are mutually parallel and hence N_1M_1 is $\parallel$ to N_2M_2. Moreover N_1N_2 is $\parallel$ to M_1M_2. Hence in the parallelogram $N_1N_2M_2M_1$, $N_1N_2 = M_1M_2$ = distance between the principal planes which is a constant for a lens.

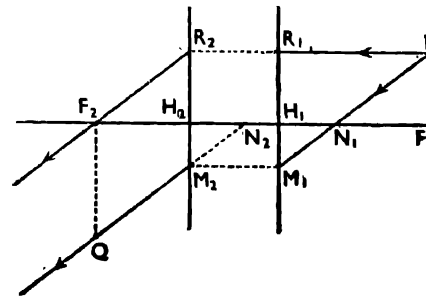


Fig. 146

Again, as $F_2Q = R_2M_2 = R_1M_1$, Δs F_2QN_2 and R_1M_1P are identical (the angles being also equal).

$\therefore$ $F_2N_2 = R_1P = H_1F_1 = f_1$.

$\therefore$ N_2 is a fixed point and so also is N_1 as can be shown from the Δs $H_2M_2N_2$ and $H_1M_1N_1$ which are identical.

Since $F_2H_2 = f_2$, we can write, $\dfrac{F_2H_2}{F_2N_2} = \dfrac{f_2}{f_1}$.

When the medium is air on both sides, we know, $f_1 = f_2$ (numerically), $\therefore$ $F_2H_2 = F_2N_2$. So the points H_2 and N_2 are coincident.

Similarly it can be shown that N_1 and H_1 are also coincident.

87(ii). Alternative General Treatment of the Location of cardinal points :—

Let us here consider the general case when the thick lens OO_1 is bounded by two different media on two sides. Let r_1 and r_2 be the radii of curvature of the right hand and the left hand surfaces of the lens respectively and μ_0, μ_1 and μ_2 be the refractive indices of the three different media concerned as shown in Fig. 146 (*a*). Let t be the thickness of the lens measured along the axis. Let O, O_1 be the points of intersection of the lens surfaces with the axis and P, Q_1, Q_2 be a point object and its first and second images.

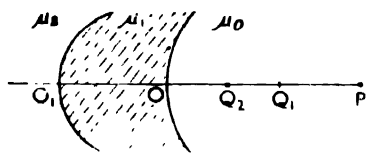


Fig. 146 (*a*)

Let $OP = u$, $OQ_1 = v_1$ and $O_1Q_2 = v$ all taken positive towards the object. r_1 and r_2 are also positive as the surfaces are concave towards the object.

For first refraction at the right hand surface we get,

$$\frac{\mu_1}{v_1} - \frac{\mu_0}{u} = \frac{\mu_1 - \mu_0}{r_1} \quad \ldots \quad (1)$$

For the second refraction at the left hand surface we get,

$$\frac{\mu_2}{v} - \frac{\mu_1}{v_1 + t} = \frac{\mu_2 - \mu_1}{r_2} \quad \ldots \quad \ldots \quad (2)$$

Let us now put $= u = \mu_0 x$, $v_1 = \mu_1 y_1$ and $v = \mu_2 y$ and $t = \mu_1 T$, where x, y_1, y are the reduced distances and T, the reduced thickness* and $k_1 = \frac{\mu_1 - \mu_0}{r_1}$, $k_2 = \frac{\mu_2 - \mu_1}{r_2}$ where k_1 and k_2 are the powers of the refracting surfaces. Then equations (1) and (2) can be rewritten as,

$$\frac{1}{y_1} - \frac{1}{x} = k_1 \quad \ldots \quad (3)$$

and $$\frac{1}{y} - \frac{1}{y_1 + T} = k_2 \quad \ldots \quad (4)$$

From (3), $y_1 = \frac{x}{k_1 x + 1}$ and substituting in (4), we have,

$$\frac{1}{y} = k_2 + \frac{1}{T + x/(k_1 x + 1)} = k_2 + \frac{k_1 x + 1}{x(Tk_1 + 1) + T}$$

On simplification we get,

$$y = \frac{x(Tk_1 + 1) + T}{x(Tk_1 k_2 + k_1 + k_2) + Tk_2 + 1} = \frac{Cx + D}{Ax + B} \quad (5)$$

*Note.—These reduced distances are not to be confused with the optical or reduced path. These reduced distances are rather similar to the vertically viewed apparent thickness of a transparent medium and is defined as the actual distance divided by the refractive index.

Here $A=(Tk_1k_2+k_1+k_2)$, $B=(Tk_2+1)$, $C=(Tk_1+1)$ and $D=T$.

On actual multiplication it is found that $BC-AD=1$; so the coefficients A, B, C, D are not independent.

(A) Principal Foci.—

When Q_2 is at infinity, the corresponding position of P is the *first principal focus* F_1.

$\therefore$ Putting $y=\infty$, we get, $Ax+B=0$, or, $x=-\frac{B}{A}$.

$OF_1=[u]$, when v or y tends to infinity

$$=\mu_0 x=-\mu_0\frac{B}{A} \quad \ldots \quad \ldots \quad \ldots \quad (\alpha)$$

Again when u or x is infinite, the corresponding position of Q_2 gives the *second principal focus*.

When $x=\infty$, $y=\dfrac{C+\frac{D}{\infty}}{A+\frac{B}{\infty}}=\frac{C}{A}$.

$$\therefore \quad O_1F_2=\mu_2 y=\mu_2\frac{C}{A} \quad \ldots \quad \ldots \quad \ldots \quad (\beta)$$

(B) Linear Magnification and Unit Points.—

The linear magnification of the first refracting surface is

$$m_1=\frac{v_1}{\mu_1}\Big/\frac{u}{\mu_0}=\frac{y_1}{x}.$$

and the linear magnification of the second refracting surface is

$$m_2=\frac{v}{\mu_2}\Big/\frac{v_1+t}{\mu_1}=\frac{y}{y_1+T}.$$

$\therefore$ Overall magnification of the lens is $m=m_1\times m_2=\frac{y_1}{x}\cdot\frac{y}{y_1+T}$

$$\therefore \quad \frac{1}{m}=\frac{x}{y_1}\cdot\frac{y_1+T}{y}.$$

From equation (4), $\frac{y_1+T}{y}=k_2(y_1+T)+1$.

Again from equation (3), $\frac{x}{y_1}=k_1x+1$.

$$\therefore \quad \frac{1}{m}=\frac{x}{y_1}\times(k_2y_1+k_2T+1)=\frac{x}{y_1}(k_2T+1)+xk_2$$
$$=(k_1x+1)\,k_2T+1)+xk_2=x(Tk_1k_2+k_1+k_2)+(Tk_2+1)$$
$$=Ax+B \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

Again from relation (5),

$$Axy+By-Cx-D=0$$

and we have, $BC-AD=1$.

Hence eliminating D,

$$A^2xy+ABy-ACx-BC+1=0;$$

or, $(Ax+B)(C-Ay)=1$.

$$m=\frac{1}{Ax+B}=C-Ay \tag{7}$$

Relations (6) and (7) give us expressions for the linear magnification of the thick lens.

The *unit points* H_1 and H_2 of the lens can be obtained by putting $m=1$ in (6) and (7). For unit magnification, the values of u and v will give the distances of these points from the respective surfaces (*i.e.*, from O and O_1).

$$\therefore \quad OH_1=u=\mu_0 x \text{ (when } m=1)=\mu_0\frac{1-B}{A} \tag{8}$$

$$\text{and,} \quad O_1H_2=v=\mu_2 y \text{ (when } m=1)=\mu_2\frac{C-1}{A} \tag{9}$$

Combining the relations (8) and (9) with (α) and (β), we get the *distances of the principal foci from the principal points* as,

$$F_1H_1=OF_1-OH_1=-\frac{\mu_0}{A}=f_1$$

$$\text{and } F_2H_2=O_1F_2-O_1H_2=\frac{\mu_2}{A}=f_2.$$

These distances are the **focal lengths** of the lens. If the media on both the sides are identical, f_1 and f_2 are numerically equal but of opposite signs. Otherwise, $-\mu_2 f_1=\mu_0 f_2$.

The distance between the principal points can also be deduced from the following simple consideration when $\mu_2=\mu_0=1$.

We have, $H_2H_1=t+OH_1-O_1H_2=t+(2-B-C)/A$.

If the thick lens placed in air be of refractive index $\mu(=\mu_1)$

$$H_2H_1=t-\frac{T(k_1+k_2)}{Tk_1k_2+k_1+k_2}=t-\frac{T(r_2-r_1)}{(r_2-r_1)-T(\mu-1)}.$$

Now if thickness $t=\mu T$ is small compared to (r_2-r_1), we have,

$$H_2H_1=t-T=t\left(1-\frac{1}{\mu}\right).$$

So for a comparatively thin lens of $\mu=1{\cdot}5$, $H_2H_1=\frac{t}{3}$.

When a thick lens of refractive index μ is placed in air, its focal

length and the position of its principal foci and unit points expressed in terms of μ, t, r_1 and r_2 can be written as,

$$\frac{1}{f}=\left(\mu-1\right)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)-\frac{t}{\mu}\frac{(\mu-1)^2}{r_1r_2} \quad \therefore \quad \frac{1}{f}=A)$$

(Here f is numerically same as f_1 and f_2).

$$OH_1=h_1=\frac{r_1t}{\mu(r_2-r_1)-(\mu-1)t}\ ;\ O_1H_2=h_2=\frac{r_2t}{\mu(r_2-r_1)-(\mu-1)t}$$

$$OF_1=\frac{-r_1\{\mu r_2-t(\mu-1)\}}{(\mu-1)\{\mu(r_2-r_1)-(\mu-1)t\}} \qquad O_1F_2=\frac{r_2\{\mu r_1+t(\mu-1)\}}{(\mu-1)\{\mu(r_2-r_1)-(\mu-1)t\}}$$

$$H_1H_2=\frac{(\mu-1)(r_2-r_1-t)t}{\mu(r_2-r_1)-(\mu-1)t}.$$

(c) **Nodal Points.—**

When the medium on both the sides of lens is air, the nodal points and principal points of thick lens conicide as have been shown in Art. 87(i) (c).

For a thick lens which is bounded by two different media on two sides, the positions of nodal points can be found by using Helmholtz's formula for linear magnification, If l_0, l_1, l_2, be the linear dimensions of the object and its successive images and θ_0, θ_1, θ_2, the angles of divergence of the rays cutting the axis at the positions of l_0, l_1, and l_2 we have,

$$\mu_2l_2\theta_2=\mu_1l_1\theta_1=\mu_0l_0\theta_0.$$

For nodal points $\theta_2=\theta_0$,

$$\therefore \quad \mu_2l_2=\mu_0l_0 \quad \text{or,} \quad \mu_2=\frac{l_0}{l_2}\mu_0.$$

But $$\frac{l_2}{l_0}=m=\frac{1}{Ax+B}=C-Ay.$$

$$\therefore \quad \mu_2=\mu_0(Ax+B) \text{ and } \mu_2\ (C-Ay)=\mu_0.$$

Under the above condition of equal angular magnification, the first and second nodal points respectively indicate the object and image distance u and v.

$$ON_1=u=\mu_0x=\frac{\mu_2-\mu_0B}{A}.$$

We know $OF_1=-\frac{\mu_0B}{A}$; hence $F_1N_1=-\mu_2=H_2F_2$ (10)

Again $O_1N_2=v=-\mu_2y=\frac{\mu_2C-\mu_0}{A}$; but $O_1F_2=\mu_2\frac{C}{A}$

$$F_2N_2=-\frac{\mu_0}{A}=H_1F_1 \tag{11}$$

Hence from (10) and (11), $N_1N_2=H_1H_2$ and if the media on both the sides of the lens are the same, as $H_1F_1=H_2F_2=f$, and N_1, N_2 coincide with H_1, H_2.

88. Conjugate Foci relations in the case of a thick lens :

(*a*) *When the distances are measured from the focal points.*—

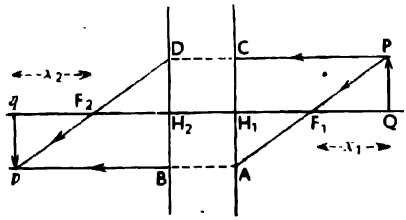


Fig. 147

Here the object distance is measured from F_1 and the image distance is measured from F_2.

In Fig. 147, $H_1F_1 = f_1$, $H_2F_2 = -f_2$, $F_1Q = x_1$ and $F_2q = -x_2$, where x_1 and x_2 are the object and image distances respectively as measured from the focal points, the object and the image both being real.

Considering the magnification,

$$m = \frac{pq}{PQ} = \frac{AH_1}{PQ} = \frac{H_1F_1}{F_1Q} = \frac{f_1}{x_1}$$

Again, $m = \frac{pq}{PQ} = \frac{pq}{DH_2} = \frac{F_2q}{F_2H_2} = \frac{-x_2}{-f_2} = \frac{x_2}{f_2}$.

$$\frac{f_1}{x_1} = \frac{x_2}{f_2} \quad \text{or,} \quad x_1x_2 = f_1f_2 = f^2 \qquad \ldots \qquad (1)$$

(when the medium on both the sides is air, *i.e.* when $f_1 = f_2 = f$)

(*b*) *When distances are measured from the principal points.*—

Suppose $U = H_1Q$ = object distance from the first principal point and $V = H_2q$ = image distance from the second principal point.

From (1) we know, $x_1x_2 = f^2$.

$\therefore \quad (U - f)(V - f) = f^2 \quad UV - f(U + V) + f^2 = f^2$.

Hence, $f(U + V) = UV$; or, $\frac{1}{V} + \frac{1}{U} = \frac{1}{f}$.

This is the conjugate foci relation that holds for a real image of a real object formed by a thick lens. This relation can also be deduced by a different process without assuming the property of unit magnification of the principal points but the process is laborious.

Linear Magnification of a thick lens : Magnification of principal planes (alternative method).—We know that for a thick lens the conjugate foci relation is given by,

$$\frac{1}{V} + \frac{1}{U} = \frac{1}{f} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (2)$$

when V, U and f are measured from the principal points. Hence the linear magnification is,

$$m = \frac{H_2q/\mu_2}{H_1Q/\mu_0} \text{ (general case)} = \frac{H_2q}{H_1Q} \text{ (in air)} = \frac{V}{U}.$$

From (1), $V = \frac{Uf}{U-f}$ and $U = \frac{Vf}{V-f}$.

$$\therefore \quad m = V \times \frac{V-f}{Vf} = \frac{V-f}{f} = \frac{x_2}{f}.$$

Similarly, $m = \frac{Uf}{U-f} \times \frac{1}{U} = \frac{f}{U-f} = \frac{f}{x_1}$.

Here x_1 and x_2 refer to the object and image distances measured from the respective focal points.

$$\therefore \quad m = \frac{x_2}{f} = \frac{f}{x_1} = \frac{x_2+f}{f+x_1}.$$

When the two principal planes are conjugate, then $x_2 = f$ and $x_1 = f$.

$$\therefore \quad m = \frac{2f}{2f} = 1.$$

89. Combination of two thin lenses separated by a distance as a thick lens :—Let L_1 and L_2 represent the positions of two thin lenses on the common principal axis of the system. Let f_1 and f_2 be their respective focal lengths and d be their separation. Suppose the system is placed in air and H_1 and H_2 are the principal points of the thick lens formed by the system. If O be an object on the principal axis and I its image, then the object and image distances as measured from the lens positions are u and v respectively and those measured from the respective principal points (H_1 and H_2) are U and V as shown in Fig. 148. If due to refraction in the first lens L_1, the image is formed at a distance v_1 from L_1, this would serve as the object for the second lens and the corresponding object distance would be $v_1 + d$.

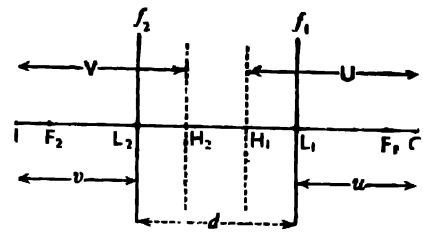


Fig. 148

$\therefore$ The magnification of the first lens is, $m_1 = \frac{v_1}{u}$ and that of the second lens is, $m_2 = \frac{v}{v_1+d}$

So the overall magnification of the system $= m_1 \times m_2 = \frac{v_1}{u} \times \frac{v}{v_1+d}$

This overall magnification must be equal to the magnification $m = \frac{V}{U}$ when we consider the system as a thick lens and V and U are measured from the principal points.

$$\therefore \quad \frac{V}{U} = \frac{v_1}{u} \times \frac{v}{v_1+d} \qquad \ldots \qquad \ldots \qquad (1)$$

For refraction through L_1, $\frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u}$.

If the object is at infinity, $u = \infty$ $\therefore$ $v_1 = f_1$ and $v_1 + d = f_1 + d$.

Again for refraction through L_2, $\frac{1}{f_2} = \frac{1}{v} - \frac{1}{v_1+d}$.

So when, $u \to \infty$, $\frac{1}{f_2} = \frac{1}{v} - \frac{1}{f_1+d}$;

or, $\frac{1}{v} = \frac{1}{f_2} + \frac{1}{f_1+d} = \frac{f_1+f_2+d}{f_2(f_1+d)}$;

or, $v = \frac{f_2(f_1+d)}{f_1+f_2+d}$ $\qquad \ldots \qquad \ldots \qquad \ldots \qquad (2)$

When $u \to \infty$, U also tends to infinity and in the limit $u = U$. Again also when $U \to \infty$, $V = F_2$, the equivalent focal length or more correctly the second focal length.

$$\therefore \quad \text{From (1)} \quad \frac{F_2}{U} = \frac{f_1}{u} \times \frac{f_2(f_1+d)}{(f_1+d)(f_1+f_2+d)}$$

$$\text{or,} \quad F_2 = \frac{f_1 f_2}{f_1+f_2+d} \quad (\because \ u = U) \qquad \ldots \qquad \ldots \qquad (3)$$

$$\therefore \quad \frac{1}{F_2} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{d}{f_1 f_2} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (4)$$

Relations (3) and (4) represent the expressions for the second focal length of the equivalent thick lens measured from the second principal point. The expression for the first focal length F_1 is exactly similar.

Positions of the principal points from the lenses.—Since when $u \to \infty$, the expression (2) for v gives the focal length measured from L_2 and F_2 gives the second focal length measured from H_2.

$$\therefore \quad L_2H_2 = -F_2 - (-v) = \frac{-f_1f_2}{f_1+f_2+d} + \frac{f_2(f_1+d)}{f_1+f_2+d} = \frac{f_2d}{f_1+f_2+d}.$$

Similarly it can be shown that $L_1H_1 = -\frac{f_1d}{f_1+f_2+d}$.

Now since the magnitude of $(L_1H_1+L_2H_2) = \frac{d(f_1+f_2)}{f_1+f_2+d}$.

$$H_1H_2 = d-(L_1H_1+L_2H_2) = d-\frac{d(f_1+f_2)}{f_1+f_2+d} = \frac{d^2}{f_1+f_2+d}.$$

If, in the expression for F above, we put $f_1+f_2+d=0$, F_2 becomes infinite. L_1H_1 and L_2H_2 and H_1H_2 also assume infinite values and d, under the condition, is given by $d=-(f_1+f_2)$.

Thus the system of two coaxial lenses has a length equal to the sum of their focal lengths. Its principal foci and principal points are situated at infinity. Such a system is known as the **telescopic system.**

89(A). Alternative treatment of a coaxial system of two thin lenses separated by a distance :—

Let the two lenses of focal lengths f_1 and f_2 placed at L_1 and L_2 on the same axis be separated by a distance d as shown in Fig. 148(a).

Let O be a point object on the axis and I be its final image after refraction through both the lenses. Let the first image formed due to refraction through the first lens be at I_1 which evidently serves as the object for the refraction through the second lens.

Let $L_1O=u$, $L_1I_1=v_1$ and $L_2I=v$, then we have for the refraction through the first lens,

$$\frac{1}{v_1}-\frac{1}{u}=\frac{1}{f_1}=k_1 \text{ (where } k_1 \text{ is the power of the first lens).} \quad \ldots \quad (1)$$

Similarly for the refraction through the second lens, as object distance here is v_1+d, we have,

$$\frac{1}{v}-\frac{1}{v_1+d}=\frac{1}{f_2}=k_2 \text{ (where } k_2 \text{ is the power of the second lens)} \quad \ldots \quad \ldots \quad (2)$$

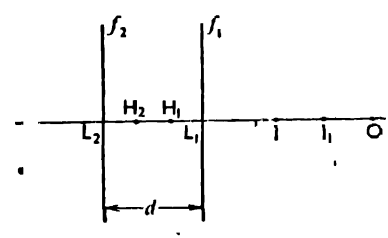


Fig. 148(a)

From (1), $v_1=\frac{u}{k_1u+1}$

and substituting this value in (2),

$$\frac{1}{v}=k_2+\frac{1}{d+u/(k_1u+1)}.$$

On simplification, $v=\dfrac{u(dk_1+1)+d}{u(dk_1k_2+k_1+k_2)+dk_2+1}$ ... (3)

This relation may be written as, $v=\dfrac{Cu+D}{Au+B}$ ... (4)

Where $A=dk_1k_2+k_1+k_2$, $B=dk_2+1$, $C=dk_1+1$ and $D=d$, and A, B, C, D are inter-related by the relation, $BC-AD=1$, which can be verified by actual multiplication. Here the different terms f_1, f_2, u, v etc. are taken algebraically, their proper signs are to be considered in numerical calculations.

(i) Principal Foci.—

The principal foci F_1 and F_2 of the system can be found out from the definition of these focal points, *i.e.* when the image I is at infinity, the object position O gives F_1 and when O is at infinity I gives the position of F_2 [Fig. 148(*a*)].

So when $v\to\infty$, $Au+B=0$.

Hence, $L_1F_1=u=-\dfrac{B}{A}$ ... . (5)

(In the figure F_1 lies to the left of L_1).

Again when $u\to\infty$, $v=\dfrac{C+D/u}{A+B/u}=\dfrac{C}{A}$.

$\therefore\quad L_2F_2=v=\dfrac{C}{A}$ ... ... (6)

(F_2 lies to the right of L_2).

Now O and I be the object and image positions at distances x_1 and x_2 from the respective principal foci F_1 and F_2 then,

$$x_1.\,x_2=(u-L_1F_1)(v-L_2F_2)$$

$$=\left(u+\frac{B}{A}\right)\left(v-\frac{C}{A}\right)=\left(\frac{Au+B}{A}\right)\left(\frac{Cu+D}{Au+B}-\frac{C}{A}\right) \quad \text{[from (4)]}$$

$$=\frac{ACu+AD-ACu-BC}{A^2}=-\frac{BC-AD}{A^2}=-\frac{1}{A^2} \quad \ldots. (7)$$

(ii) Linear magnification : Unit Points : Nodal Points.—

Linear magnification due to the first lens is $m_1=\dfrac{v_1}{u}$.

and that due to the second lens is $m_2=\dfrac{v}{v_1+d}$.

$\therefore$ Overall linear magnification, $m = m_1 \times m_2 = \frac{v_1}{u} \cdot \frac{v}{v_1 + d}$

$\therefore \quad \frac{1}{m} = \frac{u}{v_1} \cdot \frac{v_1 + d}{v}$

But $\frac{v_1 + d}{v} = 1 + k_2(v_1 + d)$, from relation (2).

Hence $\frac{1}{m} = \frac{u}{v_1}[1 + k_2(v_1 + d)] = \frac{u}{v_1}(1 + k_2 d) + u k_2$.

From relation (1), we have, $\frac{u}{v_1} = 1 + k_1 u$

$\therefore \quad \frac{1}{m} = (1 + k_1 u)(1 + k_2 d) + u k_2 = u(d k_1 k_2 + k_1 + k_2) + (1 + k_2 d)$

$= Au + B$, [where $A = d k_1 k_2 + k_1 + k_2$ and $B = 1 + k_2 d$] ... (8)

Again, since $v = \frac{Cu + D}{Au + B}$ we have $Auv + Bv - Cu - D = 0$.

Moreover, $BC - AD = 1$ (by actual multiplication)

$\therefore$ Eliminating D, we get,

$A^2uv + ABv - ACu - BC + 1 = 0$;

or, $(Au + B)(C - Av) = 1$,

$\therefore \quad m = \frac{1}{Au + B} = C - Av$ ... ... (9)

Expressions (8) and (9) give the lateral magnification of a system of lenses as described above.

To find the **unit points** H_1 and H_2, let us make O coincide with H_1 and I with H_2 when the magnification m will be 1.

$\therefore \quad L_1H_1 = u$ (when $m = 1$) $= \frac{1 - B}{A}$ [from (8)]

and $L_2H_2 = v$ (when $m = 1$) $= C - 1$ [from (9)]

Thus the positions of the principal points are determined. The **nodal points** will also coincide with H_1 and H_2 as the medium surrounding the lenses is supposed to be air. This is so because we know that the inclinations of the rays passing through the first and the second nodal points are identical. According to Helmholtz-Lagrange formula,

$\mu_1 y_1 \theta_1 = \mu_2 y_2 \theta_2$, where symbols have their usual significance. When the system is in air $\mu_2 = \mu_1 = 1$,

and $\theta_1 = \theta_2$ for nodal points,

$\therefore$ $y_1 = y_1$ and the lateral magnification is unity. Hence the nodal points of a system in air coincide with the unit points.

(iii) Focal length.—

$$F_1H_1 = L_1F_1 - H_1L_1 = -\frac{1}{A} = F_1(\text{say})$$

and $$F_2H_2 = L_2F_2 - F_2L_2 = \frac{1}{A} = F_2(\text{say})$$

If F denotes the focal length which is numerically the same as F_1H_1 or F_2H_2, we get,

$$\frac{1}{F} = A = k_1 + k_2 + d\,k_1\,k_2 = \frac{1}{f_1} + \frac{1}{f_2} + \frac{d}{f_1 f_2}.$$

(iv) Formula in terms of distances from principal points.—

Let $H_1O = U$ and $H_2I = V$ where O and I are object and image positions.

From (7) above, we know, $x_1.x_2 = -\frac{1}{A^2}$.

Here $x_1 = F_1O$ and $x_2 = F_2I$.

$$\therefore \quad F_1O.F_2I = (OH_1 - H_1F_1)(H_2I - H_2F_2)$$

$$= \left(U + \frac{1}{A}\right)\left(V - \frac{1}{A}\right) = -\frac{1}{A^2};$$

or, $$UV - \frac{1}{A}U + \frac{1}{A}V - \frac{1}{A^2} = -\frac{1}{A^2};$$

or, $$UV = \frac{1}{A}U - \frac{1}{A}V;$$

or, $$A = \frac{1}{V} - \frac{1}{U};$$

or, $$\frac{1}{F} = \frac{1}{V} - \frac{1}{U}.$$

(v) Telescopic System—

In an actual optical instrument, the system of lenses used are generally more than two. But even in the case of system consisting of a number of coaxially mounted lenses separated by distances chosen according to the purpose to be served, it can be shown that the general relation, $v = \frac{Cu + D}{Au + B}$ holds good where A, B, C, D

will be expressed in more complicated forms. So to deal with telescopic system we shall start with the above general relation. A telescopic system consists of a coaxial system of lenses so adjusted that the rays, which are parallel at incidence, emerge also as parallel rays.

So the focal length is infinite in such a system.

$$\therefore \quad F=\frac{1}{A}=\infty \ ; \text{ or, } A=0 \ ;$$

$$\text{or,} \quad dk_1k_2+k_1+k_2=0 \ ;$$

$$\text{or,} \quad \frac{d+f_1+f_2}{f_1f_2}=0 \ ; \text{ or } d=-(f_1+f_2).$$

So when the system has only two lenses, in order to make it telescopic, the separation between them must be equal to the sum of their focal lengths.

Moreover, the positions of the unit points are also at infinity as evident from the following calculations,

$$L_1H_1=\frac{1-B}{A}=\frac{1-B}{0}=\infty$$

$$\text{and} \quad L_2H_2=\frac{C-1}{A}=\frac{C-1}{0}=\infty$$

The linear magnification (or **lateral magnification**) **is given by,**

$$m=\frac{1}{Au+B}=\frac{1}{0\times u+B}=\frac{1}{B}=1/(dk_2+1)=\frac{1}{\frac{-(f_1+f_2)}{f_2}+1}=-\frac{f_2}{f_1}.$$

But the angular magnification m_θ (Art. 121)$=f_1/f_2$. $\therefore \quad m=\frac{1}{m_\theta}$

numerically.

90. Lateral magnification of a telescopic system :—Suppose that the two component lenses of the telescopic system be of focal lengths f_1 and f_2 and are separated by a distance d such that

$$d=-(f_1+f_2) \ ; \text{ or } d+f_1+f_2=0.$$

If V and U be the image and object distances measured from the principal points of the system, the lateral magnification, $m=\frac{V}{U}$.

$$\text{But} \quad V=v+L_2H_2=v+\frac{df_2}{f_1+f_2+d},$$

$$\text{and} \quad U=u+L_1H_1=u+\frac{df_1}{f_1+f_2+d},$$

where v and u are measured from the lenses.

$$\therefore \quad m=\frac{v(f_1+f_2+d)+df_2}{u(f_1+f_2+d)+df_1}=\frac{f_2}{f_1}.$$

But the angular magnification of a telescopic system is $m_\theta = \frac{f_1}{f_2}$

(See Art. 121).

So the angular magnification of a telescopic system is the reciprocal of the lateral magnification.

91. Determination of the Cardinal Points of a lens system or a thick lens (by a Nodal Slide):—Assuming the lens system or the thick lens to be convergent, the distances of the principal foci (*i.e.* the focal points) from the two surfaces of the thick lens or from the constituent lenses of the system can be found by bringing parallel rays from a slit and collimator arrangement to a focus on a screen and measuring the distance of the screen from the nearest lens. The process is repeated with a reverse arrangement of the lens system. Thus the positions of both the focal points are determined. To find the positions of the principal points (H_1, H_2) and the nodal points (N_1, N_2), we make these two pairs identical by placing the system in air so that the medium of incidence and emergence are the same. Now if a parallel pencil of rays from a collimator be brought to a focus at a point Q on a screen in the second focal plane (Fig. 149), that particular ray which is incident through the first nodal or principal point N_1 emerges out along N_2Q parallel to its direction of incidence where N_2 is the emergent nodal point. If the system now undergoes a small rotation about N_2, the particular incident ray which passes through N', the new position of N_1, will emerge out along N_2Q. Hence the image Q on the screen does not alter by small rotations about N_2. This provides a practical method of finding the nodal points of a lens system.

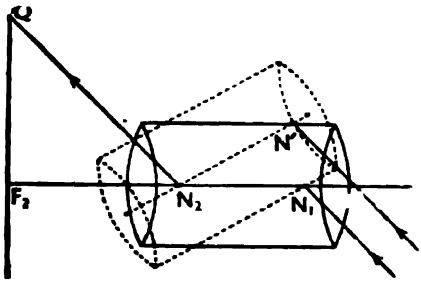


Fig. 149

For the actual determination we are to take a *nodal slide*, as shown in Fig. 150. It consists of a horizontal wooden or metal board W, capable of rotation about a vertical axis which is rigidly mounted on a heavy base B. A graduated scale S is attached to this board and the optical system whose cardinal points are to be determined is mounted on a sliding carriage C, provided with two rails R on which the mounted components of the optical system can be slided and their separation can be varied. The sliding carriage C, can, in its turn, be slided alongside the scale S. The frame C carries an index mark, I, that indicates its position relative to the scale.

In actual laboratory experiments illuminated slits are only occasionally used. Generally a vertical pin P is taken in front of the nodal slide such that its tip lies on the axis of the system and a plane mirror M is placed on the other side of the apparatus. The nodal slide, however is to be levelled initially. Now the apparatus is to be so adjusted that the axis of the lens system intersects the axis of rotation of the nodal slide. To ensure this the position of the lens system and the plane mirror are to be adjusted so that a sharp image of P is obtained as P'

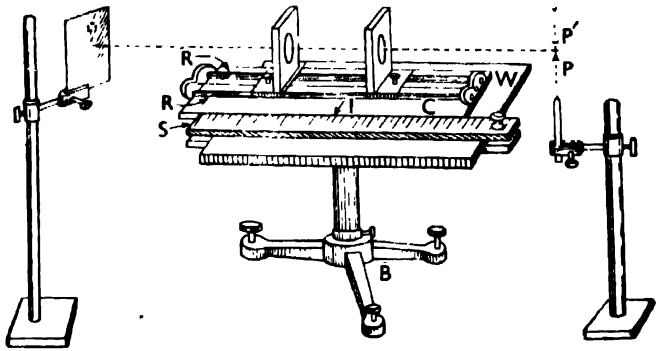


Fig. 150

in coincidence without any parallax. The system is then rotated through 180° about the rotation axis and the image of P is again made to coincide with it. This is done half by adjusting the pin position and half by moving the lens system. These operations are repeated till the coincidence is secured both in the direct setting as well as after a rotation of 180°. This ensures the intersection of the axis of the lens system with the rotation axis.

The carriage C is now moved along the scale and by adjusting the pin position, if the latter can be placed in the focal plane, the emergent rays will be parallel. These parallel rays, when reflected from the normally set mirror M, will retrace their paths and will be incident on the lens (on the distant side of P) as parallel rays and will be brought to a focus at P' in the focal plane on emergence from the front lens. If now on rotating the system about the vertical axis, we find that the coincidence of P and P' is disturbed, then it is evident that the nodal point on the emergent side (*i.e.* on the side of the front lens) have not coincided with the rotation axis. So by adjusting the position of the system alongside the scale, this coincidence of P and P' without parallax is ensured for a considerable rotation of the system about the axis. Now the second nodal point N_2 coincides with the axis of rotation and the

image finally formed by the system due to the parallel rays incident on the distant lens would be fixed in position in the focal plane as discussed above. The position of the carriage index I against the scale is noted in this position. The distance of the axis of rotation from any of the component lenses gives the position of the second nodal point from that lens and the distance between the pin and the rotation axis is the equivalent focal length F. The separation of the component lenses may also be noted as d. The system is now rotated through 180° and the position of coincidence again adjusted (keeping d constant). The distance between the axis of rotation and pin gives the equivalent focal length and that from any component lens to the axis the corresponding distance of the first nodal point. The position of the carriage index I against the scale is again noted. The mean of the two focal lengths gives the correct equivalent focal length and the difference between the carriage index readings, the nodal separation N_1N_2. For different values of separation d, of the component lenses, the equivalent focal lengths and the nodal separations are noted. Thus noting different d's, finding different F's and getting the different nodal separations N_1N_2, the relating curves like (*a*) F *vs.* d, (*b*) N_1N_2 *vs.* d, (*c*) N_1N_2 *vs.* F and (*d*) $\frac{1}{F}$ *vs.* d can be drawn.

Examples

1. An eye-piece is formed of two thin convergent lenses of focal lengths 4 cm. and 10 cm. at a distance of 7 cm. apart. Find the focal length of the eye-piece and the positions of the cardinal points of the system.

Ans. According to the convention followed in the book, we have,
$f_1 = -4$ cm., $f_2 = -10$ cm. and $d = 7$ cm.

$\therefore$ For the 1st lens, $\frac{1}{v_1} - \frac{1}{u} = -\frac{1}{4}$ ... ... (1)

and for the 2nd lens, $\frac{1}{v} - \frac{1}{v_1 + 7} = -\frac{1}{10}$ ... ... (2)

To determine the position of the first principal focus F_1, we put $v = \infty$, when $u = L_1F_1$.

$\therefore$ From (2), $v_1 + 7 = 10$; or, $v_1 = 3$.

Substituting in (1), $\frac{1}{3} - \frac{1}{u} = -\frac{1}{4}$

$\therefore$ $L_1F_1 = u = \frac{12}{7} = 1\frac{5}{7}$ cm.

To find the position of the second focus, F_2, we put $u = \infty$, when $v = L_2F$
Then from (1), $v_1 = -4$ cm.

Substituting in (2), we have, $\frac{1}{v}-\frac{1}{3}=-\frac{1}{10}$.

$\therefore\quad L_2F_2=v=\frac{30}{7}=4\frac{2}{7}$ cm.

Linear magnification, m, is given by the relation.

$$\frac{1}{m}=\frac{v_1+7}{v}\cdot\frac{u}{v_1}=\left(1-\frac{v}{10}+\frac{7}{10}\right)\left(\frac{u}{v_1}\right)\text{ [from (2)]}$$

$$=\left(\frac{3-v_1}{10}\right)\times\frac{u}{v_1}=\frac{3}{10}\frac{u}{v_1}-\frac{u}{10}$$

$$=\frac{3}{10}\left(1-\frac{u}{4}\right)-\frac{u}{10}\text{ [from (1)]}$$

$$=\frac{3}{10}-\frac{7u}{40}.$$

When $m=1$, $u=L_1H_1=-4$ cm.

The first focal length of the combination is $F_1H_1=L_1F_1-L_1H_1=5\frac{4}{7}$ cm.

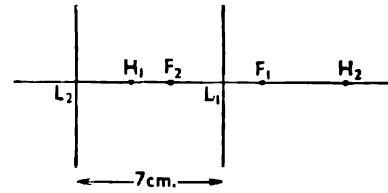


Fig. 150(a)

Hence the second focal length $=-5\frac{4}{7}$ cm., which is conventionally the focal length of the combination,

$\therefore\quad F_2H_2=-5\frac{4}{7}$ cm.

But $L_2F_2=4\frac{2}{7}$ cm.

$\therefore\quad L_2H_2=L_2F_2-F_2H_2=10$ cm.

The nodal points evidently coincide with the unit points.

2. Find the positions of the cardinal points of a thick double-convex lens in air whose surface have radii of curvature 5 cm. and 4 cm., the thickness of the lens being 3 cm. and $\mu=1{\cdot}5$.

Ans. Here since the lens is double convex, $r_1=-5$ cm. and $r_2=4$ cm. ; $\mu=1{\cdot}5$; $d=3$ cm.

This problem can be solved exactly in the same way, as have been followed in Ex. 1, above. It can, however, be more easily solved by direct application of the formulae already deduced.

Positions of unit points (or nodal points) :—

$$h_1=OH_1=\frac{r_1t}{\mu(r_2-r_1)-(\mu-1)t}=\frac{-5\times3}{1{\cdot}5(4+5)-0{\cdot}5\times3}=-1{\cdot}25\text{ cm.}$$

$$h_2=O_1H_2=\frac{r_2t}{\mu(r_2-r_1)-(\mu-1)t}=\frac{4\times3}{1{\cdot}5(4+5)-0{\cdot}5\times3}=1\text{ cm.}$$

Focal Points.—

$$OF_1=\frac{-r_1\{\mu r_2-t(\mu-1)\}}{(\mu-1)\{\mu(r_2-r_1)-(\mu-1)t\}}$$

$$=\frac{5\{1{\cdot}5\times4-3\times0{\cdot}5\}}{0{\cdot}5\{1{\cdot}5(4+5)-0{\cdot}5\times3\}}=3{\cdot}75\text{ cm.}$$

$$O_1F_2=\frac{r_2\{\mu r_1+t(\mu-1)\}}{(\mu-1)\{\mu(r_2-r_1)-(\mu-1)t\}}=\frac{4(1{\cdot}5\times-5+0{\cdot}5\times3)}{0{\cdot}5\{1{\cdot}5(4+5)-0{\cdot}5\times3\}}=-4\text{ cm.}$$

Or, Alternative method with the help of the coefficients A, B, C and D.

[Vide. Art. 87 (*ii*) (*A*) and (*B*)]

Here $\mu_0=\mu_2=1$ and $\mu_1=\mu=1{\cdot}5$

Hence, $T=\dfrac{t}{\mu_1}=\dfrac{3}{1{\cdot}5}=2$.

$$k=\frac{1{\cdot}5-1}{-5}=-0{\cdot}1 \quad \text{and} \quad k_2=\frac{1-1{\cdot}5}{4}=-0{\cdot}125$$

$\therefore$ $A=Tk_1k_2+k_1+k_2=2\times-0{\cdot}1\times-0{\cdot}125-0{\cdot}1-0{\cdot}125=-0{\cdot}2$

$B=Tk_2+1=2\times-0{\cdot}125+1=0{\cdot}75$

$C=Tk_1+1=2\times-0{\cdot}1+1=0{\cdot}8$

$D=2$.

So, we have, $OF_1=-\dfrac{B}{A}=\dfrac{0{\cdot}75}{0{\cdot}2}=3{\cdot}75$ cm.,

$$O_2F_2=\frac{C}{A}=\frac{0{\cdot}8}{-0{\cdot}2}=-4 \text{cm.},$$

$$OH_1=\frac{1-B}{A}=\frac{1-0{\cdot}75}{-0{\cdot}2}=-1{\cdot}25 \text{ cm.},$$

$$O_2H_2=\frac{C-1}{A}=\frac{0{\cdot}8-1}{-0{\cdot}2}=1 \text{ cm.}$$

Questions (Chapter V)

1. Show that in the case of a single spherical surface, the principal points coincide with the pole of the surface. (C. U. H. 1957)

2. Deduce an expression for the focal length of a thick lens in air. Show that the nodal points coincide with the principal points in this case. (C. U. H. 1959)

3. Explain what you mean by cardinal points of a thick lens.

A glass hemisphere of radius r and refractive index μ is treated as a lens, rays passing through it being limited to those nearly coinciding with the axis. Find out the principal points of this lens. Prove that its focal length is given by $f=\dfrac{r}{1-\mu}$.

(C. U. H 1948)

4. Find out the equivalent focal length of two lenses separated by a distance and find out the principal points of the system.

5. Calculate the positions of the cardinal points in the case of a thick lens in air and explain the general principle of the determination of these cardinal points.

6. Show that for a telescopic system the reciprocal of the lateral magnification is equal to the angular magnification. (C. U. H. 1957)

7. A combination is formed of two thin converging lenses of focal lengths 15 cm. and 5 cm. separated by an air space of 10 cm. Find the positions of the cardinal points.

[*Ans.* $L_1F_1=-7{\cdot}5$ cm. ; $L_2F_2=-2{\cdot}5$ cm. , $L_1H_1=-15$ cm. ; $L_2H_2=5$ cm.]

8. Deduce Helmholtz's equation showing the relation between linear and angular magnifications of two conjugate planes in an optical system.

Indicate how the equation is modified (*a*) when one of the conjugate planes is at infinity and (*b*) when the system is telescopic. (C. U. H. 1960)

92. Introduction :—In the deduction of the different relations for spherical mirrors and lenses, it has been assumed that accurate images of objects are obtained only under the condition that the rays make such small angles with the axis of the mirrors or the lenses, as the case may be, that sin θ may be replaced by θ. The rays for which the above approximation may be taken to be true are called *paraxial rays* and the region close to the axis of the mirrors and lenses are known as the paraxial region. Though under the above conditions we get images sufficiently accurate for all practical purposes, the image is, however, never the exact reproduction of the object when the latter is an extended one. This is so because the lateral magnification does not become equal to the longitudinal magnification, except when the longitudinal magnification is unity. Theoretically a point image of a point object may be easily obtained with paraxial rays. But an exact reproduction of an extended object is possible only when both the longitudinal and lateral magnifications are unity for the spherical mirrors and lenses.

In actual instruments we often require large aperture mirrors or lenses for the necessary brightness of the images and as such the incident rays are not limited to the paraxial regions only. Moreover, the pencils of rays received by the instruments are often oblique and subtend sufficiently large angles with the axis of the reflector or refractor. So the formulae deduced so far should be corrected for the large value of the aperture and the large inclination of the rays with the axis.

Seidel studied the formation of images with non-paraxial rays incident on spherical surfaces and showed that if these rays are monochromatic (of the same wavelength) in nature, five different types of defects occur in the images formed. These defects are known as the *monochromatic aberrations* or *Seidel aberrations*. They are classified as : (*a*) **Spherical aberration**, (*b*) **Coma**, (*c*) **Astigmatism**, (*d*) **Curvature**, (*e*) **Distortion**

Important monochromatic aberrations that occur in mirrors have already been discussed in Chapter II. In the following articles we shall mainly discuss the above five monochromatic aberrations with reference to a lens and also the methods for their removal.

93. Spherical Aberration :—Let L be a lens and OF_1 its principal axis. A beam of parallel rays incident in a direction parallel to the axis of the lens is refracted through it at full aperture. The paraxial rays

(1, 1) of the beam meet at a point F_1 on the principal axis after refraction (Fig. 151). F_1 evidently forms the paraxial focus of the lens. Again the marginal rays (3, 3) after refraction meet at F_2 on the principal axis and F_2 forms the marginal focus of the lens. The intermediate rays like (2, 2 *etc.*) are focussed at points intermediate between F_1 and F_2. This wandering (*i.e.* shift) of the principal focus of a lens along the principal axis due to refraction at defferent zones of a lens is known as the *spherical* aberration. The distance between the paraxial focus (F_1) and the focus for the extreme marginal rays (F_2) along the principal axis of the lens is a measure of the **axial** or **longitudinal spherical aberration.** If a screen be placed perpendicular to the axis at F_1, a circular patch of light with a bright centre and a faint rim will be visible. If this screen be shifted on to the position F_2, the patch of light will have a bright rim. In between these two positions, the patch of light obtained on the screen at some position, say, DE is found to possess the smallest section and the illumination throughout the patch nearly uniform. This is the nearest approach to a point image in case of a large aperture lens. This circular patch at DE is known as the *circle of least confusion* or the **circle of least spherical aberration.** The diameter of this circle is a measure of the **transverse or lateral spherical aberration.** Actual calculation shows that this diameter is one-fourth that of the circular patch formed at F_1 (*i.e.* qq_1).

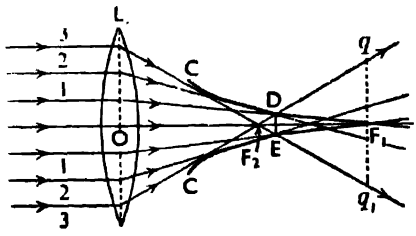


Fig. 151

Again if we refer to Fig. 151, we find that a curve (CDF_1 or CEF_1) can be so drawn as to touch all the refracted rays. This curve is known as the **Caustic Curve.** Since all the refracted rays spread out symmetrically with respect to the axis, the surface touching all these refracted rays (*i.e.* the envelope of the refracted rays) is obtained by rotating the curve CDF_1 about the axis OF_1. This enveloping surface is very bright and is generally known as the *caustic surface*, the apex F_1 of which is known as the *cusp* of the caustic surface.

It is evident from the above discussions on spherical aberration that since the focal lengths of different zones of lens is different, the image of an object formed by rays, refracted through different zones of the lens, will not be sharply defined at any one particular position. So to have a well-defined image of an object, the spherical aberration, in a lens is to be reduced, to a minimum—the complete elemination, of this aberration being impossible.

94. Methods of minimising spherical aberration :—For minimising the spherical aberration the following methods are usually employed.

(a) Since the spherical aberration in a lens depends on the aperture of the lens, *stopping down* the lens aperture with the help of a coaxial stop which confines the incident rays to a region close to the axis, the above aberration can be sufficiently reduced and a better definition of the image can be secured.

But when the marginal rays are cut off by a stop, the brightness of the image is diminished and the resolving power of the lens is also reduced. According to Rayleigh's suggestion the central region of the lens should be stopped and only the marginal rays should be utilised in formation of images. This does not reduce the resolving power of the lens. It is, however, found that if the stops are not placed in proper positions distortion of the image ensues. So in using stops the best possible position should be first ascertained.

(b) A lens of given focal length can be so designed as to reduce the spherical aberration to a minimum. Mathematical analysis shows that for a parallel beam of incident monochromatic light, the spherical aberration is minimum when the ratio of the radii of curvature of the lens $\left(i.e.\ \frac{r_1}{r_2} \right)$ is given by the relation,

$$\sigma = \frac{r_1}{r_2} = \frac{2\mu^2 - \mu - 1}{\mu(1+2\mu)} \quad \ldots \quad \ldots \quad (1)$$

where μ is the refractive index of the lens material,

This ratio, σ, is known as the **shape factor** of the lens.

If for the lens material, the refractive index is 1·5 (*i.e.* $\mu = 1{\cdot}5$) then from relation (1), we get $\sigma = \frac{r_1}{r_2} = \frac{-1}{6}$.

Thus in order to have minimum spherical aberration in a lens of refractive index 1·5 the radii of curvature of the lens must be in the ratio 1 : 6. Since r_1 corresponds to the incident surface and $\frac{r_1}{r_2} = \frac{1}{6}$, the surface of the smaller radius of curvature, *i.e.* of larger curvature must be turned towards the incident light. The negative sign of σ simply indicates that r_1 and r_2 must be oppositely directed, *i.e.*, the lens must be either a double convex or a double concave one. The minimum value of the spherical aberration (axial) for such a lens is found to be 1·07 from actual calculations.

Again if $\mu = 2$, $\frac{r_1}{r_2} = +\frac{1}{5}$ and the corresponding spherical aberration becomes only 0·44. Here since σ is +ve, the lens must be a meniscus.

But materials of such a high value of μ are seldom used for the construction of lenses of ordinary optical instruments. The lenses of $\sigma = \frac{1}{7}$ or $\frac{1}{8}$ are known as **Crossed lenses** and are one of the best types of lenses that can be designed for reducing the axial spherical aberration to a minimum value. But as the construction of such lenses involves much labour and cost, they are generally used only in high precision instruments. Ordinary instruments are fitted with lenses which are much less costly and easier to construct but wherein the spherical aberration is also considerably reduced at the same time. Plano-convex lenses with their convex side turned towards the incident rays are found to be the best type for easy construction in which the spherical aberration is also very small.

If $\mu = 1{\cdot}5$ (crown glass) and $\frac{r_1}{r_2} = 0$, (*i.e.* $r_2 = \infty$, *i.e.* the plane side is away from the incident light), the spherical aberration is found to be equal to 1·17 only which is very near the value obtained in a crossed lens. So these lenses are commonly used in ordinary optical instruments like eye-pieces etc. If, however, $\mu = 1{\cdot}686$, and $\frac{r_1}{r_2} = 0$, the spherical aberration becomes very small (nearly 0·7). But as the glass of $\mu = 1{\cdot}686$ is very costly, this type of plano-convex lenses are used only in more costly types of instruments. Again for substances of $\mu = 2$, if $\frac{r_1}{r_2} = 0$, the spherical aberration reduces to 0·5. So in all types of lens material, we find that the spherical aberration in a plano-convex lens nearly approaches the value corresponding to a crossed lens of the same material. So plano-convex lenses with their convex sides towards the light is preferred to the crossed lens as the latter involves considerable labour and cost. (*Note* : In the numerical values of the axial spherical aberration given in the above cases, the radius of the lens aperture is supposed to be 10 cm. and the focal length of the lens 100 cm.).

(c) We know that the spherical aberration of a lens increases as the aperture increases. It can be shown that as the lens aperture increases, the deviation suffered by a ray also increases. Hence in order to make the spherical aberration small, it is necessary to have the deviation of the refracted ray a minimum. As a lens may be well supposed to be comprised of a number of thin prisms the condition for minimum deviation should be the same as that for a prism, *i.e.* the incident and the emergent rays should be equally inclined to the surfaces of incidence and emergence and the total deviation should be equally distributed between these surfaces. If a number of coaxial lenses separated by a distance from a converging

system, the condition of minimum deviation and hence of minimum spherical aberration necessitates an equal distribution of deviation between the two lenses.

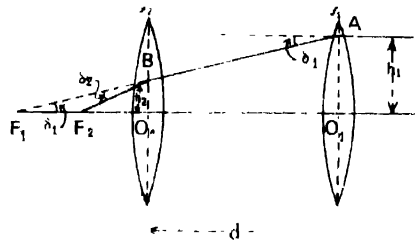


Fig. 152

Let two such converging lenses of focal lengths f_1 and f_2 be separated by a distances d (Fig. 152). If a ray strikes the first lens parallel to the axis at a distance h_1 from the optical centre O_1, the deviation of this ray is given by $\delta_1 = \frac{h_1}{f_1}$ where $f_1 = O_1F_1$.

As evident from the figure, $\angle F_1BF_2 = \delta_2$ is the deviation suffered by the ray in the second lens.

But as $\delta_1 = \delta_2$ for minimum aberration, $F_1F_2 = BF_2 = F_2O_2$ when the incident ray is paraxial.

For the second lens F_1 is the virtual object and F_2 its image. Hence applying the formula for a lens, we get, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$.

Here $u = -F_1O_2$ and $v = -F_2O_2 = -\frac{F_2O_2}{2}$ and $f = -f_2$.

$$\frac{2}{F_1O_2} - \frac{1}{F_1O_2} = \frac{1}{f_2}\ ;\ \text{or,}\ F_1O_2 = f_2.$$
$$d = O_1F_2 - F_1O_2 = f_1 - f_2.$$

Thus the condition for minimum spherical aberration for two converging lenses separated by a distance d is that this distance is to be equal to the difference of the focal lengths of the two lenses.

(d) Since the spherical aberration of a convergent lens for objects beyond its focus is positive and that for a divergent lens is negative, by suitably combining a concave, and a convex lens in contact the spherical aberration of a particular zone may be reduced to zero at the same time making the aberrations of the other zones as small as possible.

(e) In a single lens spherical aberration, in general, cannot be totally eliminated. But if a transparent sphere is used as a lens a pair of points can always be found such that when a very small object (theoretically a point object) is placed at one of these points, an image of it (also very small) will be formed at the other and there will be no spherical aberration (neither any coma) in the image so formed. This

pair of points is called the **aplanatic points** of the spherical lens. (For aplanatic surfaces and points refer to Art. 29). But as the object in this case should be practically a point object, such lenses are only suitable for microscope objectives and are not in general use.

*95. **Aplanatic Points of a transparent sphere and its application** :—Let the transparent refracting sphere QLM of radius r ($=OM$) be taken in air. Draw two concentric outer and inner dotted spheres of radii μr and r/μ respectively, μ being the refractive index of the material of the sphere relative to the outer space.

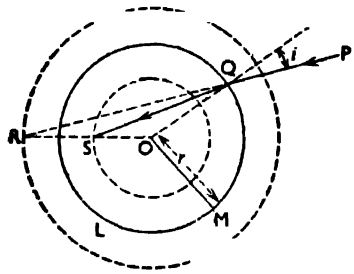


Fig. 153

Let a ray PQ be incident on the sphere at Q such that it appears to cut the outer sphere at a point R (Fig. 153). Join RO intersecting the inner sphere at S. Then QS is the refracted ray in the sphere and S is the image point corresponding to the virtual object point R. To establish this let us consider $\triangle s$ QSO and QRO wherein $\frac{OQ}{OS}=\mu=\frac{OR}{OQ}$ and the angle at O is common.

Hence the $\triangle s$ are similar.

$\therefore$ $\angle OSQ=\angle OQR=$angle of incidence i.

In $\triangle QSO$, $\frac{\sin OSQ}{\sin SQO}=\frac{OQ}{SO}=\mu$;

or, $\sin i=\mu \sin SQO=\mu \sin \phi$

$\therefore$ $\angle SQO=\phi$ is the angle of refraction. Thus the ray OS is the refracted ray.

The result is independent of the position of the point of incidence. So all the rays, irrespective of their divergence, appearing to converge to R, will after refraction through the spherical lens actually converge to the point S. So S and R are aplanatic points. The converse is also true, *i.e.*, all the rays diverging from S will after refraction appear to diverge exactly from R. It can also be shown from simple geometry of the figure that coma is also absent in this case. If θ_1 and θ_2 represent the slope angle subtended by the object of infinitesimally small length dy_1 at R and the image of length dy_2 at S respectively, then,

$$\frac{dy_2}{dy_1}=\frac{v}{u}=\frac{SO}{RO}=\frac{r/\mu}{\mu r} \quad \ldots \quad \ldots \quad (1)$$

But $\mu = \frac{\mu_2}{\mu_1}$, where μ_2 = refractive index of the sphere and μ_1 = refractive index of the surrounding space.

Again from similar $\triangle s$ SOQ and QOR,
$\angle ORQ = \angle \theta_1 = \angle \phi$, and $\angle OSQ = \angle \theta_2 = \angle i$

$$\text{From (1)}, \quad \frac{dy_1}{dy_2} = \frac{\frac{\mu_2}{\mu_1} \cdot r}{r \Big/ \frac{\mu_2}{\mu_1}} = \frac{\mu_2}{\mu_1} \frac{\mu_2}{\mu_1}$$

$$= \frac{\mu_2}{\mu_1} \cdot \frac{\sin i}{\sin \phi} = \frac{\mu_2}{\mu_1} \quad \frac{\sin \theta_2}{\sin \theta_1}$$

$$\therefore \quad \mu_1 \, dy_1 \sin \theta_1 = \mu_2 \, dy_2 \sin \theta_2.$$

Hence Abbe's sine condition is satisfied by the above aplanatic points, which are, as a consequence, free from **comatic error**

The principle has been successfully employed in the construction of microscope objectives of large magnification known as **Abbe's Oil Immersion objectives.** Fig. 154 explains the principle of such an objective L_1 repesents a hemispherical lens of glass and O is the object on the watch-glass placed in front of L_1. But as O cannot be embedded in the glass medium of the lens, it is immersed in ceda -wood oil having the same refractive index (1·51) as the glass By adjusting the position of L_1 O is placed at the inner aplanatic point of the lens L_1. Since O may now be supposed to be situated in the medium of the lens L_1, its image I_1 is formed at the outer aplanatic point of the spherical lens. A concavc-convex lens L_2 of the same glass is placed behind L_1 such that I_1, is at the centre of curvature of the lower surface. So the ray from I_1 passes through the lower surface normally and falls on the upper surface of the meniscus lens whose curvature is so chosen that I_1 is the inner aplanatic point of the sphere of the glass of radius equal to the radius of curvature of the upper surface of L_2. So the final image will now be formed at the outer aplanatic point I_2. The rays finally refracted through L_2 appear to come from I_2. Thus the original wide angle pencil from the point object O is now transformed into a pencil of smaller angle diverging from I_2. In this way be adding more meniscus lenses a large image free from spherical aberration and coma may be obtained. This principle is known as **Amici's Principle.**

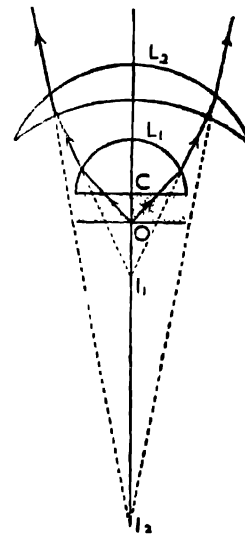


Fig. 154

In actual practice, however, only the first two lenses are constructed on this principle as otherwise chromatic errors will be predominant. The rays after passing through these lenses are subsequently corrected for chromatic effects by achromatic systems of lenses as will be fully described in the article on 'High Power Microscopes.'

96. **Coma and its removal** :—Coma is really due to non-axial spherical aberration of the oblique rays. Coma affects the rays that come from point objects off the axis or even close to the axis of the lens. This type of aberration arises due to the fact that for the non axial point objects the images are magnified to different extents by the different zones of the lens or in other words coma arises due to the different lateral magnifications of the different zones of a lens. The different magnifications of the different zones are the outcome of the asymmetrical position of the object relative to the lens.

To illustrate the mode of formation of Coma, let us consider the formation of image of a small object PQ by a pencil of rays whose

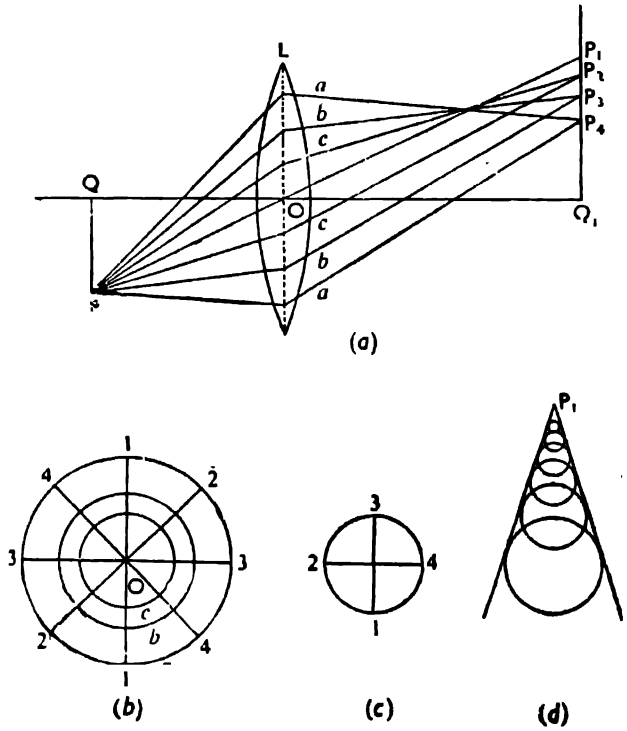


Fig. 155

principal ray POP_1 passes through the centre of the lens L [Fig.

155(*a*)]. If the lens is corrected for axial spherical aberration the paraxial rays from the image of PQ at P_1Q_1, P_1 being the image of the non-axial object point P. But the rays like *cc*, *bb* and *aa* form the images of P at P_2, P_3 and P_4 respectively. As the rays are refracted through the outer zones, the image of P gradually shift away from the paraxial images towards the axis due to the gradual decrease of the lateral magnification. We have so far considered only the rays lying in the plane containing the principal axis and the point P. If we now consider the *skew rays* not lying in this plane, the nature of the image will be more complicated: To study this complicated nature of the image, let us consider Fig. 155(*b*), which gives the view of the lens at one of its faces. Let it be divided into zones like *a*, *b*, *c* etc. each of which is divided into diametrically opposite points like, 11, 22, 33, 44. If rays from P pass through points 11 of the outer zone, they will come to a focus at a point marked 1 in the focal plane [Fig. 155 (*c*)]. Similarly ray through 22 are focussed at 2 and so on, the points 1, 2, 3, 4 where the ray from a particular zone are focussed lying on the circumference of a circle, known as the *Comatic circle*. The point 1 of this circle corresponding to image P_4. Thus the image of P formed by the outer zone is a circular patch of light, P_4 being a point on this circle. Similarly as the gradually inner zones are considered smaller circular patches will be formed in the focal plane with their centres gradually shifted towards the paraxial image P_1 as shown in Fig. 155(*d*). Due to the overlapping of these circular patches the appearance of the image will resemble the shape of a *comet* from which the name *coma* has been derived. The image so formed has the maximum illumination at its pointed end P_1. When the tip of the comma is away from the axis of the lens the comatic aberration is said to be *negative* as illustrated in the figure. If the coma tapers towards the axis, it is said to be *positive* as is found in the case of an image of a non-axial point object placed in front of a convex lens at an infinite distance. It is to be noted that the defect in an image due to coma is generally small and is often masked by the greater defects arising out of astigmatism.

Since coma is a result of the variation of lateral magnification of the different zones of lens, the method of eliminating this aberration lies in designing the lens in such a way that the lateral (*i.e.* linear) magnification $\frac{y_2}{y_1}$ is the same for all zones, where y_1 and y_2 are the extents of the object and image at right angles to the axis.

From Abbe's sine condition, which holds equally for a single refracting surface as well as for a lens, we know,

$$\mu_1 y_1 \sin \theta_1 = \mu_2 y_2 \sin \theta_2 \quad \ldots \quad \ldots \quad (1)$$

where μ_1 and μ_2 are the refractive indices of the object and image spaces and θ_1, θ_2 are the angles subtended by the zone radius at the axial points of the object and image spaces respectively.

Hence from (1), $\frac{y_2}{y_1} = \frac{\mu_1 \sin \theta_1}{\mu_2 \sin \theta_2}$ (2)

So if in an optical system $\frac{\mu_1 \sin \theta_1}{\mu_2 \sin \theta_2}$ = constant for certain values of the slope angles, θ_1 and θ_2, the system will be free from comatic aberration for the conjugate points for which the slope-angles of the typical rays satisfy the above condition. Such conjugate points are known as *Aplanatic points.*

***Direct Deduction of Sine Condition :—**

In the figure (Fig. 156) LH_1 and MH_2 represent the unit planes of

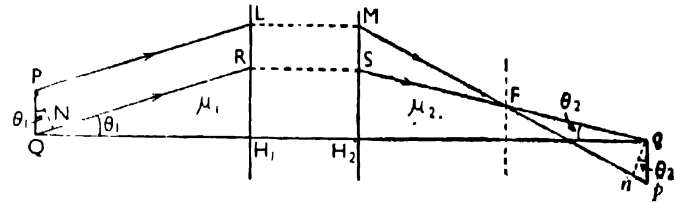


Fig. 156

an optical system whose principal axis is Qq. Let μ_1 and μ_2 be the refractive indices of the object and image spaces and Q, q be a pair of conjugate points on the principal axis *for which there is no spherical aberration.* Let PQ be a small line object at Q and pq its image at q. Let $PLMFp$ and $QRSFq$ be the optical paths of the two rays starting from P and Q. F is a point in the focal plane where these two rays meet. Let θ_1 and θ_2 denote the inclinations of the conjugate rays QR and Sp with the principal axis in the object and image spaces respectively. Draw PN perp. to QR and qn to Sq.

Then $\angle QPN = \theta_1$ and $\angle pqn = \theta_2$. As θ_2 is small (object itself being small) $\angle Fqn$ may be taken to equal to $\angle Fnq$ (=a right angle) for all practical purposes.

From Fermat's principle, the optical path, $[PLMFnp]$ = a constant $= K_1$ (say)

and the optical path $[QNRSFq]$ = another constant $= K_2$ (say).

By direct subtraction,

$[PLMF] + [Fn] + [np] - [QN] - [NRSF] - [Fq] = K_1 - K_2$

But $[PLMF] = [NRSF]$ and $[Fq] \simeq [Fn]$

$\therefore \quad [np] - [QN] = K_1 - K_2$

But $[np] = y_2 \sin \theta_2 \times \mu_2$ ($y_2 = pq$ = linear dimension of image)

and $[QN] = y_1 \sin \theta_1 \times \mu_1$ ($y_1 = PQ$ = linear dimension of object)

$\therefore \quad \mu_2 y_2 \sin \theta_2 - \mu_1 y_1 \sin \theta_1 = K_1 - K_2 \quad \ldots \quad \ldots \quad (1)$

This condition holds for all values of θ's. But for paraxial rays $\theta_1 = \theta_2 = 0$. Putting this boundary condition in (1), we have,

$$K_1 - K_2 = 0.$$

$$\therefore \quad \mu_1 y_1 \sin\theta_1 = \mu_2 y_2 \sin\theta_2 \qquad \ldots \qquad \ldots \qquad (2)$$

This relation (2) gives us the required sine condition of Abbe.

97. Astigmatism and its removal :—If a narrow pencil of rays, either parallel or diverging from a point, be such that its central ray meets the lens obliquely, the emergent pencil will not come to a point focus. Such an oblique pencil is called an astigmatic pencil of rays. Instead of a single point focus the pencil is found to pass through two *focal lines* at right angles to each other. As discussed in the case of spherical mirrors (vide Art. 28), in lenses also the formation of these focal lines can be explained with reference to Fig. 157. Let us consider a parallel pencil of rays whose central ray makes a large angle with the axis of the lens to be incident on a lens LL_1 whose meridional plane passes through LL_1 (Fig. 157). The rays like P_1, P_2 and Q_1, Q_2 which lie in planes parallel to that passing through LL_1 are brought to foci

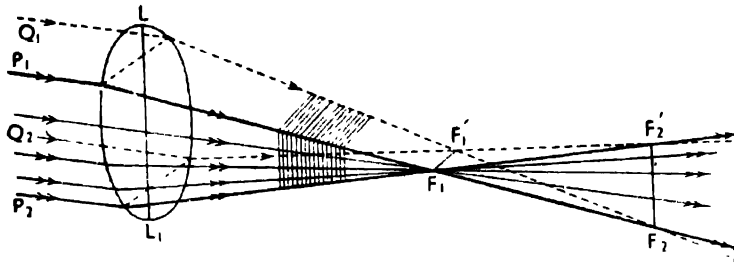


Fig. 157

respectively at F_1 and F_1' which lie along a line nearly perpendicular to LL_1. This line (F_1F_1') is known as the **first (or meridional) focal line.** Again the rays like P_1, Q_1 and P_2, Q_2 that lie in planes perpendicular to LL_1 are focussed at points F_2 and F_2' lying in a line nearly perpendicular to the first focal line. This line (F_2F_2') is known as the **sagittal or the second focal line.** The focussing of the rays in two different focal lines is due to the fact that the oblique rays after refraction through a lens lie on wave-fronts of unequal curvatures in different planes. The above phenomenon exhibited by an oblique pencil of rays wherein it cannot be focussed to a single point but is actually focussed along two focal lines is known as **astigmatism.** If the aperture through which the light falls on the lens be circular, the sections of the emergent pencil in the neighbourhood of the focal lines will be elongated ellipses. Somewhere in between F_1 and F_2 the axes of the ellipse will be equal and the section is circular. **This section is the circle of least confusion** and the best image of an object is formed in this plane. *If* θ be the angle of inclination of the oblique rays with the axis of the lens, the **astigmatic difference** (F_1F_2) is found to be proportional to $\sin^2\theta$, or, to θ^2 if θ is small.

The astigmatic defect can be removed in many ways. (*i*) Since the astigmatic difference is in opposite directions for a concave and a convex lens, by suitably placing two such lenses at a distance apart to form a compound lens, the astigmatic errors can be corrected to a large extent. The lenses combined must satisfy Petzval's condition, $\sum \frac{1}{\mu f}=0$, where μ is the refractive index and f the focal length of the lens. (*ii*) It can be removed by using two plano-cylindrical lenses separated by a suitable distance. (*iii*) It is eliminated in a *toric lens* where the lens has surfaces of different horizontal and vertical curvatures. (*iv*) Astigmatism may also be reduced by cutting off the oblique rays by means of apertures placed coaxially with the lens. Such apertures are known as stops. The stops should be placed judiciously at the best possible position—which is generally very close to the surface of the lens in the case of a single lens and somewhere in between the two lenses when they are used as a combination.

98. **Curvature and its removal** :—Even if a lens be free from spherical aberration, coma and astigmatism, the image of an extended object formed by the lens becomes curved due to the fact that the off-axial points of the object are farther away from the lens than the axial point. The off-axial points cast their images at points closer to the lens than the images of the axial points and the image as a whole though well-defined will be curved. In the case of a real image formed by a convex lens this is illustrated in Fig. 158(*a*), where in the image space Q_1 represents the image of the axial point Q of the object and P_1 that of the off-axial point P. The distance of P from the centre of the lens, *i.e.*, OP being greater than OQ, OP_1 will be less than OQ_1 because in case of real image formation, as u increases v diminishes. Hence P_1 is shifted towards the lens and the image P_1Q_1 of the object PQ becomes curved. In the case of a virtual image cast by a convex lens the curvature will be in the opposite sense as shown in Fig. 158 (*b*). This is so because if the object be placed within the focal length, the image is virtual. Here u and v are both $+ve$ and focal length f is $-ve$. Let $OQ=u$ and $OQ_1=v$ and $OP=u+x$ where x is the increase of the object distance due to the off-axial position.

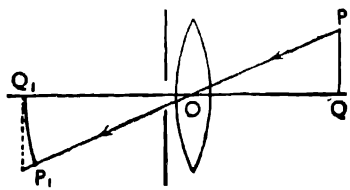


Fig. 158 (*a*)

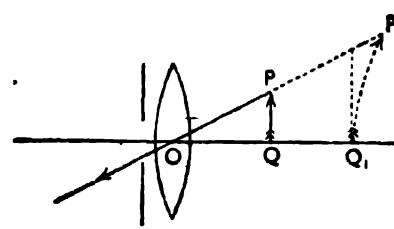


Fig 158 (*b*)

For the axial image, we have,

$$\frac{1}{v}-\frac{1}{u}=-\frac{1}{f}; \quad \text{or,} \quad m=\frac{v}{u}=\frac{f}{f-u} \text{ (on simplification).}$$

For off-axial image if OP_1 be v_1, we have,

$$\frac{1}{v_1}-\frac{1}{u+x}=-\frac{1}{f}, \quad \text{or,} \quad m_1=\frac{v_1}{u+x}=\frac{f}{f-u-x}.$$

$\therefore$ As $m_1>m$, $v_1>v$; or, $OP_1>OQ_1$.

So the image curves away from the lens. In the case of a concave lens wherein always a virtual image it formed, it can be shown by algebraic calculations that the image will be curved as shown in Fig. 159. Here since u, v and f are all $+ve$, for the axial image Q_1,

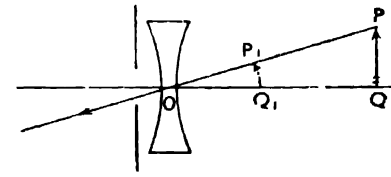


Fig 159

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}, \quad \text{or,} \quad m=\frac{v}{u}=\frac{f}{f+u},$$

and for off-axial image P_1, taking $OP_1=v_1$ and $OP=u+x$,

$$\frac{1}{v_1}-\frac{1}{u+x}=\frac{1}{f}; \quad \text{or,} \quad m_1=\frac{v_1}{u+x}=\frac{f}{f+u+x}.$$

$\therefore$ As $m_1<m$, $v_1<v$; or, $OP_1<OQ_1$.

So the image is curved towards the lens as shown in the figure.

Thus we find that the curvature of the image produced by a convex lens is always positive whereas that produced by a concave lens is always negative.

In order to remove this defect the field curvature should be flattened by judiciously placing a stop at the suitable position as has been indicated in the case of astigmatism.

Since the field is to be flattened, the lens system should be such as to conform to Petzval's, condition *i e.* $\sum\frac{1}{\mu f}=0$. Actually as referred to in astigmatism, by combining one converging lens with a diverging one at a distance apart such that the condition $\frac{1}{\mu_1 f_1}+\frac{1}{\mu_2 f_2}=0$ is satisfied, both astigmatic errors as well as the field curvature can be simultaneously reduced to a negligible amount.

99. Distortion and its removal —When the stop used for eliminating the curvature, astigmatism etc. in a lens, is not placed at its best position, the defect known as *distortion* creeps in. Though the image is still sharp, it gets distorted and curved. If we consider the formation of a real image by a converging lens and if the stop S be placed on the side of the object PQ at a distance from the lens, the object distance u measured along the rays will be greater than OP, *i.e.* the object distance if the rays from P passed through the central region of the lens (Fig. 160). Since the magnification in the case of a real image formed by a convex lens is $m=\frac{f}{u-f}$ and as u is greater than that in the case of a central stop, magnification of the edge of the image will be less than that with a central stop. So the image of the edge will appear to be pulled towards the axis and the image of a rectangular object becomes *barrel-shaped*. This is why the distortion of real images formed by a converging lens with a stop on the object side is known as *barrel-shaped distortion*.

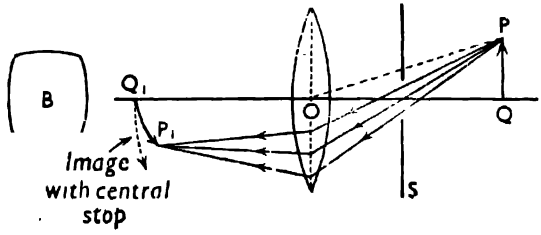


Fig. 160

Again if the stop is placed on the image side of the lens at a certain distance, the image of PQ will be formed at P_1Q_1 (Fig. 161). Here if we consider the rays (from P) passing through the stop, the object distance measured along the path of rays, *i.e.* u is less than

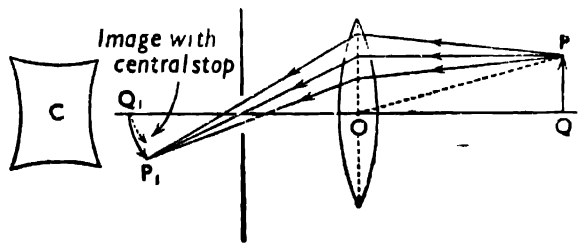


Fig. 161

that of the rays passing through a central stop *i.e.*, less than OP. Hence magnification of the image P_1Q_1 will be greater than that of the image formed with the rays through the central stop. So the edge of the image will be pulled away from the axis

So if a rectangular object is taken its image will appear *cushion-shaped.* Because of this the distortion in a real image formed by a convex lens with a stop on the image side is known as *pin-cushion* or simply *cushion-shaped* distortion. In the cases of virtual images formed by a convex lens, the nature of distortion in the two cases considered above will be reversed.

To eliminate this defect due to distortion a stop is to be placed between the two identical components of a combination of lenses. By this arrangement the cushion-shaped distortion of one component will cancel the barrel-shaped distortion of the other. This type of arrangement with an intermediate stop is generally made in a good photographic camera and the combination of lenses fitted with such a stop is known as *a rapid rectilinear lens system.*

Example

A concave mirror has a radius of curvature of 20 cm. and a diameter of 4 cm. Find the longitudinal spherical aberration for a ray from an axial point at infinity meeting at the edge of the mirror. (*C. U. H. 1965*)

Ans. In Fig. 162, i denotes the angles of incidence and reflection.

From the figure, $\sin i = \frac{LD}{LC} = \frac{2}{20} = \frac{1}{10}$

$\therefore \cos i = \sqrt{1-\sin^2 i} = \sqrt{\cdot 99} = \cdot 995.$

Since u and $\frac{1}{v}+\frac{1}{u}=\frac{2}{r}$ we have

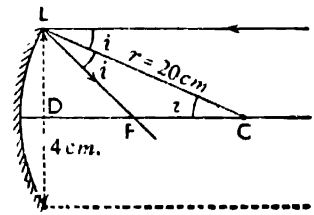


Fig. 162

$\frac{1}{v}=\frac{2}{r}$, or, $f_p = v = \frac{r}{2}$ for paraxial rays.

Again from Art. 19(3), for marginal rays, $\frac{1}{v_1}+\frac{1}{u}=\frac{2\cos i}{r}$. $\therefore \frac{1}{v_1}=\frac{2\cos i}{r}$

when $u=\infty$, or, $f_m = v_1 = \frac{r}{2\cos i}$.

$\therefore$ Longitudinal spherical aberration $= f_m - f_p = \frac{r}{2}\left(\frac{1}{\cos i}-1\right) = 10\left(\frac{1}{\cdot 995}-1\right)$

$= 0\cdot 05$ cm.

Questions (Chapter VI)

1. Explain how the effects due to spherical aberration are minimised in the case of a Huygens' eye-piece.

2. State clearly and fully what is meant by spherical aberration. Discuss the various ways by which it is reduced to a minimum, bringing out the physical principles involved.

3. State and explain the defects in the image of an object formed by a lens. How may these defects be reduced to a minimum in the case of a large aperture convex lens ? (*cf.* Utk. U. '70s , C. U. H. 1950)

4. What do you mean by the term aplanatic foci of a lens system ? Find the positions of the aplanatic foci of a spherical refracting surface. How is the principle utilised in the construction of high power microscope objectives ?

5. Explain what you mean by 'Coma in image formation by an optical system and deduce the condition for its removal (C. U. H. 1950)

6. In what way does the spherical aberration spoil the image? Can it be ever completely removed? If so, describe fully how it can be done, showing that a perfect point image is formed of a point object.

7. Discuss how spherical aberration can be minimised in the case of a thin lens, by adjusting the radii of curvature of its surfaces. (C. U. H. 1959)

8. Describe the various Seidel aberrations in a lens. What are the means to correct them?

9. What are the various monochromatic aberrations in a lens? Fully describe comatic aberration and its removal.

CHAPTER VII — DISPERSION: CHROMATIC ABERRATION: RAINBOW

100. Dispersion :—The refractive index of a material depends on the nature of the material itself and it varies with the wavelength of light. As the wavelength increases, the refractive index gradually diminishes. So when a pencil of composite light enters a prism or a lens, on emergence it is found to be dispersed into rays of different wavelengths or colours. It was Sir Isaac Newton who first demonstrated the decomposition of sunlight into its seven coloured components when it was made to pass through a transparent prism. This phenomenon of resolution of a composite light into its constituent rays of different wavelengths is known as *dispersion* and the dispersed band of component colours is known as the *spectrum*. In general the rays of larger wavelength suffer less deviation due to dispersion because the refractive index of the material of the prism is less for these wavelengths. Again the rays of shorter wavelength suffer larger deviation because the refractive index is greater for these rays. Thus the spectrum of a pencil of composite light has the wavelengths of the component rays arranged in order, the ray having the largest wavelength being least deviated from the incident beam and that having the smallest wavelength being most deviated. The type of dispersion, as stated above, wherein the rays are arranged in order of their wavelengths is known as **normal dispersion.** If there be any change in this wavelength order the dispersion is no more normal and may be termed as **anomalous dispersion.**

101. Angular Dispersion and Dispersive Power :—When white light from a narrow slit enters a prism, on emergence it is divided into seven different colours. The red component being the least refrangible

will be the least deviated while the violet component being the most refrangible will suffer the maximum deviation as has been stated before.

If we consider a ray PQ of white light incident on the prism ABC. on emergence O_1R and O_2V will respectively be the directions of the red and the violet components of the ray (Fig. 163). The angular separation between these two extreme components is known as the *angle of dispersion* or the *angular dispersion* of the prism. In actual practice since the red and violet components spread over a definite range of wavelengths, two definite spectral lines of known λ are sometimes chosen as the extreme rays for defining the angular dispersion.

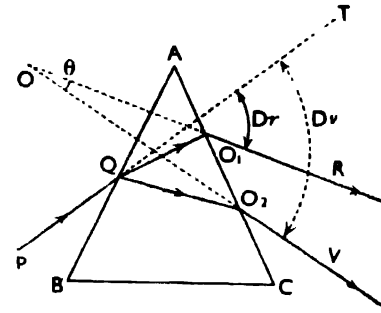


Fig. 163

The choice of these wavelengths depends on the particular occasion for which the knowledge of dispersion is required. Generally the red C-line ($\lambda=6563A$) and either the blue F-line ($\lambda=4881A$) or the violet H-line ($\lambda=3968A$) are taken as the extreme lines for the measurement of angular dispersion. In our calculations, however, we shall use the subscripts r and v for the two extreme rays.

In defining angular dispersion as also the dispersive power of a material, it is always done with reference to a thin prism of the material. This is so chosen because the results obtained in the case of a thin prism can directly be applied to the case of lenses which may be supposed to be composed of a number of thin prisms.

As is evident from the geometry of Fig. 163 the angular dispersion between the red and the violet components is $\angle ROV=\theta=D_v-D_r$, where D_v and D_r represent the deviations of the violet and the red components.

But in a thin prism the deviation is

$D=A(\mu-1)$...(1), where A is the angle of the prism.

$\therefore \quad \theta=D_v-D_r=A(\mu_v-\mu_r)$.

In calculus notation, dispersion (D_v-D_r) is written as dD.

$\therefore$ From (1), on differentiation, we get,

$$dD=A.\,d\mu.$$

The **dispersive power** of a material is defined as the angular dispersion produced per unit mean deviation in a thin prism of the material. Now since the mean deviation corresponds to the deviation of the mean ray (yellow) of the spectrum, we can very well write it as

$$D=A(\mu-1).$$

Hence the dispersive power, generally designated by the letter ω, is given by the expression,

$$\omega=\frac{D_v-D_r}{D}=\frac{A(\mu_v-\mu_r)}{A(\mu-1)}=\frac{\mu_v-\mu_r}{\mu-1}$$

In calculus notation, $\omega = \frac{dD}{D} = \frac{A\,d\mu}{A(\mu-1)} = \frac{d\mu}{\mu-1}$.

Since the calculus method is easier to tackle we shall use calculus in most of our calculations and give the ordinary algebraic method as an alternative treatment.

Example

White light falls on a 15° prism. What is the angle of dispersion between the red and the blue components if the refractive indices for these two colours are 1·64 and 1·66 respectively ?

Ans. Here dispersion angle $\theta = D_v - D_r = A(\mu_v - \mu_r)$, since the prism is fairly thin.

$\therefore \quad \theta = 15^c(1\cdot66 - 1\cdot64) = 15^c \times 0\cdot02 = \frac{15 \times 60 \times 2}{100} = 18$ minutes.

102. Achromatic combination of prisms. (Deviation without dispersion) :—A prism ordinarily produces deviation on an incident ray. This deviation is also attended with dispersion when the ray is composite in nature. But it is desirable in certain experiments to avoid this dispersion effect, so that only a known amount of deviation is obtained. In a single prism this is never possible because μ_v and μ_r must be different for the material. So a number of prisms may be conveniently arranged such that the dispersion produced by some of them is cancelled by that produced by the others. For the present discussion we shall consider only two thin prism C and F placed such that the total dispersion produced by them is nil (Fig. 164).

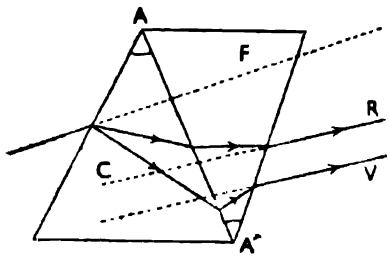


Fig. 164

If A and A' be the angles of these thin prisms, and if μ, μ' and ω, ω' refer to their refractive indices and dispersive powers respectively, then, the dispersion produced by the prism C is

$A(\mu_v - \mu_r)$ and that by the prism F is $A'(\mu'_v - \mu'_r)$.

So the total dispersion is $A(\mu_v - \mu_r) + A'(\mu'_v - \mu'_r)$. In order that the combination is achromatic, this total dispersion must vanish.

$$\therefore \quad A(\mu_v - \mu_r) + A'(\mu'_v - \mu'_r) = 0,$$

$$\text{or,} \quad \frac{\mu_v - \mu_r}{\mu'_v - \mu'_r} = -\frac{A'}{A} \qquad \ldots \qquad \ldots \quad (1)$$

Relation (1) can be written also in terms of the dispersive powers of the prism materials because,

$$\mu_v - \mu_r = (\mu - 1)\,\omega \text{ and } \mu'_v - \mu'_r = (\mu' - 1)\omega'.$$

$$\therefore \quad \text{From (1),} \quad \frac{(\mu-1)\omega}{(\mu'-1)\omega'} = -\frac{A'}{A} \qquad \ldots \qquad \ldots \quad (2)$$

So in order that the combination of the two prisms may be achromatic, the above relation (1) or (2) must hold good. The negative sign

indicates that A and A' must be oppositely directed as shown in Fig. 164, because neither ω nor μ of a material is negative. If the two prisms be of the same material, $\omega=\omega'$ and $\mu=\mu'$, and hence $A=A'$ (numerically). So the two prisms must be identical in size and as such they would combine to form a parallel faced slab of glass producing no dispersion and neither any deviation. In general, however, the combination consists of a crown glass prism (C) and a flint glass prism (F) of different dispersive powers and different indices of refraction. In that case $\omega'>\omega$ and $\mu'>\mu$ and hence it follows from relation (2) that $A>A'$. So in an achromatic combination of two prisms—one of crown glass and the other of flint glass—the crown glass prism must be of greater refracting angle. Actually if a white ray enters such a combination, the deviated component rays will come out parallel to each other and they will overlap on emergence. If a slit of definite width is used as a source, the coloured components will overlap only in the central portion of the emergent beam while its edges will still be coloured.

The total deviation produced by the above achromatic combination of two prisms can also be easily calculated. The deviation produced by the prism C is given by

$$D=A(\mu-1)$$

and that produced by the prism F is $D'=A'(\mu'-1)$

$\therefore$ Total deviation is $D+D'=A(\mu-1)+A'(\mu'-1)$

$$=A(\mu-1)\left[1+\frac{A'}{A}\frac{(\mu'-1)}{(\mu-1)}\right].$$

Substituting from (2), total deviation $=A(\mu-1)\left[1-\frac{\omega}{\omega'}\right]$.

As has been stated above, if the two prisms are of the same material, $\omega=\omega'$ and the total deviation is nil.

103. Dispersion without Deviation (in Prisms) :—The combination of thin prisms wherein their is no deviation of the mean ray but in which dispersion is produced as usual, is used in the construction of *direct vision spectroscopes*. Here also the prisms used must be of different types of glass. If a combination of two thin prisms is wanted one of which is of crown glass (C) and the other is of flint glass (F), the angle of the flint glass prism should be made greater than that chosen for the achromatic combination. This would make the total deviation of the mean ray zero while due to the greater

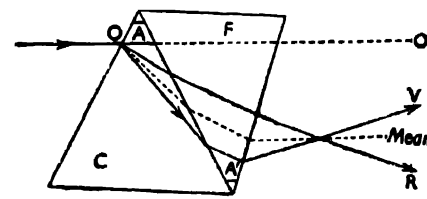


Fig. 165

dispersion of the flint glass there will be a resultant dispersion of the combination.

If A and A' be the angles of the two thin prisms C and F' (Fig. 165) respectively and if μ, μ' and ω, ω' refer respectively to their refractive indices and the dispersive powers, then the total deviation of the *mean ray* will be given by

$$D+D'=A(\mu-1)+A'(\mu'-1).$$

In order that this deviation reduces to zero, we must have,

$$A(\mu-1)+A'(\mu'-1)=0\ ;$$

$$\text{or,}\quad \frac{A}{A'}=-\frac{(\mu'-1)}{(\mu-1)} \qquad \ldots \qquad \ldots \qquad \ldots \quad (1)$$

As $(\mu'-1)$ and $(\mu-1)$ are positive, the negative sign indicates that A and A' must be oppositely directed. Again as $(\mu'-1)$ for the flint glass is greater than $(\mu-1)$ for the crown glass, A is greater than A'. It can be easily shown that A' in the case is greater than that in the case of the achromatic combination. The relation (1) gives the condition that is to be satisfied in order that the combination of the two prisms may produce no deviation of the mean ray.

The total dispersion in this combination satisfying the above relation is given by

$$\begin{aligned}
&A(\mu_v-\mu_r)+A'(\mu_v'-\mu_r')\\
&=A\left\{(\mu_v-\mu_r)+\frac{A'}{A}(\mu_v'-\mu_r')\right\}\\
&=A\left\{(\mu_v-\mu_r)-\frac{\mu-1}{\mu'-1}(\mu_v'-\mu_r')\right\} \qquad [\text{substituting from (1)}]\\
&=A(\mu-1)\left\{\frac{\mu_v-\mu_r}{\mu-1}-\frac{\mu_v'-\mu_r'}{\mu'-1}\right\}\\
&=A(\mu-1)(\omega-\omega') \qquad \ldots \qquad \ldots \qquad \ldots \quad (2)
\end{aligned}$$

Since $\omega \neq \omega'$, the total dispersion will have a definite value. Again as $\omega'>\omega$, the dispersion is negative, *i.e.* the violet ray will be deviated in the direction of the refracting edge of the first prism (Fig. 165). This is the principle of the direct-vision spectroscope wherein instead of two prisms a number of prisms are used as described below.

104. The Direct-vision Spectroscope : This instrument is

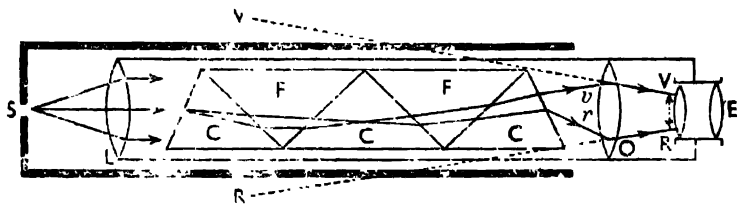


Fig. 166—Direct-vision Spectroscope

designed to obtain a spectrum in the direction of the incident light. Thus there is only dispersion and no deviation and hence all the different parts of the spectroscope, *viz.* the collimator, the prism system and the telescope are all mounted in one straight tube. Often it is small enough to be carried about in the pocket and so it is also called a *pocket spectroscope* which serves for the quick qualitative examination of spectra.

Description.—It consists of the following parts (Fig. 166).—

(1) An outer tube having an adjustable slit S at one end through which the light is admitted.

(2) An inner tube sliding coaxially within the outer one and it carries the following elements :

(*a*) the collimator lens L which is an achromatic convex lens ;

(*b*) the combination of prisms, ordinarily three of crown (C) glass, and two of dense flint (F) glass, placed alternately, with their refracting edges turned in opposite directions ;

(*c*) a telescope having the objective, O, and the eye-piece, E.

Adjustments.—The slit, after its width is properly adjusted, is made parallel to the refracting edges of the prisms. It is then turned towards the source of light. The inner tube is slided so that the slit S is placed at the focal plane of the collimator L from which then a beam of parallel rays falls on the combination of prisms. The spectrum produced by the prisms is brought to focus by the objective, O, and when looked through the eye-piece, a virtual magnified image of the spectrum extended on either side of the mean colour is seen.

105. Chromatic Aberration in Lenses :—A lens may be looked upon as the combination of a number of thin prisms. So when a beam of white light is incident on any zone of a lens in a direction parallel to the principal axis of the lens it is not brought to a point focus after refraction because each of the rays will be dispersed by the lens into its constituent colours. The red components being least refrangible will be focussed at F_r on the principal axis. This is the principal focus for the red rays and the violet components will be more deviated and the corresponding principal focus for the violet rays will be F_v, a point which is much closer to the lens surface than F_r (Fig. 167). This *wandering of the principal focus* along the axis of the lens due to the composite nature of the incident beam

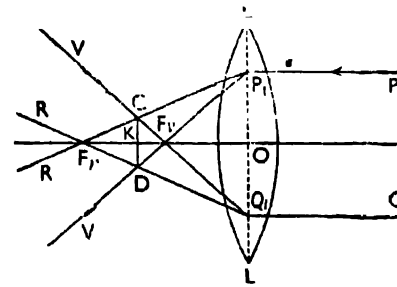


Fig. 167

Is known as the *axial or longitudinal chromatic aberration* of the lens. If a screen be placed at F_v, the centre of the coloured patch will be intensely violet with the outer edge coloured red and if the screen be shifted on to the position F_r, the centre of the coloured path will be intensely red with its outer edge coloured violet. If the screen be gradually moved along the axis to a certain position CD the patch will be more or less uniformly illuminated and colour effect will be minimum there. This patch is known as the *circle of least confusion or, of least chromatic aberration* and the diameter of this circular patch at CD is a measure of the *lateral or transverse chromatic aberration.* The lateral chromatic aberration is actually the difference in magnification of the two extreme coloured images formed due to the chromatic error in a lens. The longitudinal chromatic aberration is measured by the difference in the focal lengths of the two extreme coloured components and is vitally associated with the positions of the images formed by the lens while the lateral chromatic aberration is associated with the question of magnifications of the images. It is due to chromatic aberration that a real image produced by an uncorrected single lens is found to be fringed with the colours of the spectrum.

106. Calculation of Chromatic Aberration :

(a) Longitudinal Chromatic Aberration—As has been stated above, the longitudinal chromatic aberration is measured by the difference in the focal lengths of the two extreme coloured components of the incident composite beam, *i.e.*, the longitudinal chromatic aberration $= f_r - f_v$.

We know, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

$\therefore \quad \frac{1}{f_v} = (\mu_v - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

and $\quad \frac{1}{f_r} = (\mu_r - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

$\therefore \quad \frac{1}{f_v} - \frac{1}{f_r} = (\mu_v - \mu_r)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

$= \left(\frac{\mu_v - \mu_r}{\mu - 1}\right) \times (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

$= \omega \cdot \frac{1}{f}$ (where f = mean focal length)

Hence $\quad \frac{f_r - f_v}{f_r f_v} = \omega \cdot \frac{1}{f}$.

But $f_r f_v = f^2$ for all practical purposes.

$$\therefore \quad \frac{f_r - f_v}{f^2} = \omega \cdot \frac{1}{f};$$

$$\text{or,} \quad \omega = \frac{f_r - f_v}{f} \qquad \ldots \qquad \ldots \qquad \ldots \quad (1)$$

This is the expression for *the dispersive power of a lens.*

From (1), we get, $f_r - f_v$ = longitudinal chromatic aberration $= f\omega$.

*(b) **Lateral or Transverse Chromatic Aberration**—The lateral chromatic aberration is measured by the diameter of the circle of least chromatic aberration, *i.e.*, by CD in Fig. 167. To deduce an expression for CD in terms of the constants of the lens, let us refer to Fig. 167, where in the similar $\triangle s$ CDF_v and $P_1Q_1F_v$,

$$\frac{KF_v}{CD} = \frac{OF_v}{P_1Q_1} = \frac{f_v}{P_1Q_1} \text{ (here } OF_v = f_v) \qquad \ldots (\alpha)$$

Again in the similar $\triangle s$ CDF_r and $P_1Q_1F_r$,

$$\frac{KF_r}{CD} = \frac{OF_r}{P_1Q_1} = \frac{f_r}{P_1Q_1} \quad (\because \quad OF_r = f_r) \qquad \ldots \qquad \ldots (\beta)$$

From (α) and (β),

$$\frac{KF_v + KF_r}{CD} = \frac{f_v + f_r}{P_1Q_1};$$

$$\text{or,} \quad \frac{F_rF_v}{CD} = \frac{f_v + f_r}{P_1Q_1}; \quad \text{or,} \quad \frac{f_r - f_v}{CD} = \frac{f_v + f_r}{P_1Q_1}.$$

Here $f_r - f_v = f\omega$, $f_r + f_v = 2f$ (approx.) and $P_1Q_1 = d$, the effective aperture of the lens.

$$\therefore \quad \frac{f\omega}{CD} = \frac{2f}{d}$$

$$\text{or,} \quad CD = \frac{\omega d}{2}.$$

This is the expression for the lateral chromatic aberration of a lens.

107. Achromatism in lenses :—

(a) Single lens.—In a single lens of mean focal length f, the chromatic aberration is given by $f_r - f_v$. So in order to make the lens achromatic, the principal focus for the red rays must be made coincident with the principal focal for the violet rays. So for achromatism, $f_r - f_v = 0$.

Again since $\frac{1}{f_v}-\frac{1}{f_r}=\frac{f_r-f_v}{f_v f_r}$, the condition of achromatism demands that $\left(\frac{1}{f_v}-\frac{1}{f_r}\right)$ should also be zero.

According to the convention of calculus, we can write the condition of achromatism as $d\left(\frac{1}{f}\right)=0$, the differentiation evidently being with respect to the wavelength.

We know, $\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$.

$$\therefore\ d\left(\frac{1}{f}\right)=d\mu\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=\frac{d\mu}{\mu-1}.(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

$$=\omega.\frac{1}{f},\quad\left(\text{where } \omega=\text{dispersive power}=\frac{d\mu}{(\mu-1)}\right)$$

$\therefore$ For achromatism of a single lens,

$$d\left(\frac{1}{f}\right)=\frac{\omega}{f}=0.$$

But $\omega\neq 0$. $\therefore$ The condition demands that $f=\infty$. So no single lens of finite focal length can be made achromatic.

(b) Two lenses in contact :—

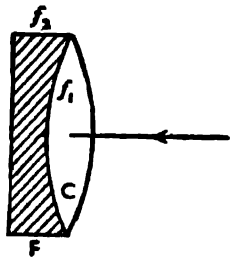


Fig. 168

When two lenses are in contact, the equivalent focal length is given by F, where,

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}\qquad\ldots\qquad(1)$$

Here f_1 and f_2 are the focal lengths of the component lenses.

Due to chromatic aberration, the principal foci for the red and the violet components will be different and for achromatism of the system the difference between the focal lengths of the red and the violet components must vanish.

$\therefore$ The condition of achromatism is that $F_r-F_v=0$.

Again as $\frac{1}{F_v}-\frac{1}{F_r}=\frac{F_r-F_v}{F_v F_r}$, the condition of achromatism demands that

$\frac{1}{F_v}-\frac{1}{F_r}=0$; or, $d\left(\frac{1}{F}\right)=0$. But $d\left(\frac{1}{F}\right)=d\left(\frac{1}{f_1}\right)+d\left(\frac{1}{f_2}\right)$.

We know, $\frac{1}{f_1}=(\mu_1-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$;

or, $d\left(\frac{1}{f_1}\right)=d\mu_1\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=\frac{d\mu_1}{\mu_1-1}(\mu_1-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=\frac{\omega_1}{f_1}$,

where ω_1 is the dispersive power of the lens C (Fig. 168) of focal length f_1. Similarly $d\left(\frac{1}{f_2}\right)=\frac{\omega_2}{f_2}$, where ω_2 is the dispersive power of the lens of focal length f_2.

$\therefore\ d\left(\frac{1}{F}\right)=\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}$.

$\therefore$ For achromatism of the combination, $d\left(\frac{1}{F}\right)=0$;

or, $\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}=0$. ... ... (1)

When relation (1) is satisfied for a combination of two lenses in contact, it becomes achromatic for the two extreme colours with reference to which ω_1 and ω_2 have been defined.

From (1), we have $\frac{\omega_1}{\omega_2}=-\frac{f_1}{f_2}$.

Here ω_1 and ω_2 are positive. Hence f_1 and f_2 must be of opposite signs. If f_1 refers to a convex lens, f_2 would refer a concave lens. If both lenses are of same material, $f_1=f_2$ numerically ($\because\ \omega_1=\omega_2$). This would make $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}=0$. So the purpose of the lens combination is not served here. So $\omega_1\neq\omega_2$. If ω_1 refers to crown glass and ω_2 refers to flint glass, ω_2 being greater than ω_1, f_2 must be greater than f_1. Thus in order to make a converging achromatic combination of two lenses in contact, a convex lens of crown glass of smaller focal length is to be placed in contact with a concave lens of flint glass of greater focal length so that the above relation (1) is satisfied. In achromatic doublets generally the flint glass lens is made plano-concave (Fig. 168) the concave surface of which is placed in contact with a suitable convex lens of crown glass. If the convex surface faces the incident ray, this achromatic doublet also sufficiently reduces the spherical aberration.

If a number of lenses are put in contact to form an achromatic combination, the condition is given by $\Sigma\frac{\omega}{f}=0$, where the summation (Σ) extends over all the component lenses.

[**Alternative treatment** :—For achromatism, $F_r - F_v = 0$.

Now, $\frac{1}{F_v} - \frac{1}{F_r} = \frac{F_r - F_v}{F_v F_r}$ $\therefore$ For achromatism, $\frac{1}{F_v} - \frac{1}{F_r} = 0$.

We know, $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$.

$\therefore$ $\frac{1}{F_v} = \frac{1}{f_{1v}} + \frac{1}{f_{2v}}$ and $\frac{1}{F_r} = \frac{1}{f_{1r}} + \frac{1}{f_{2r}}$.

$\therefore$ $\frac{1}{F_v} - \frac{1}{F_r} = \left(\frac{1}{f_{1v}} - \frac{1}{f_{1r}}\right) + \left(\frac{1}{f_{2v}} - \frac{1}{f_{2r}}\right)$ ... ... (A)

Now, $\frac{1}{f_{1v}} = (\mu_{1v} - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$ and $\frac{1}{f_{1r}} = (\mu_{1r} - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$.

$\therefore$ $\left(\frac{1}{f_{1v}} - \frac{1}{f_{1r}}\right) = (\mu_{1v} - \mu_{1r})\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

$= \left(\frac{\mu_{1v} - \mu_{1r}}{\mu_1 - 1}\right)(\mu_1 - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

$= \omega_1 / f_1$,

where ω_1 is the dispersive power of the lens of mean focal length f_1.

Similarly, $\left(\frac{1}{f_{2v}} - \frac{f}{f_{2r}}\right) = \omega_2 / f_2$,

where ω_2 is the dispersive power of the lens of mean focal length f_2.

$\therefore$ For achromatism, $\frac{1}{F_v} - \frac{1}{F_r} = \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$;

or, $\frac{\omega_1}{\omega_2} = -\frac{f_1}{f_2}$.]

(c) Two lenses separated by a distance :—The equivalent focal length F in this case is given by $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{a}{f_1 f_2}$, where a is the separation between the two component lenses.

For achromatism, $d\left(\frac{1}{F}\right) = 0$, the differentiation being with respect to wavelength.

But $d\left(\frac{1}{F}\right) = d\left(\frac{1}{f_1}\right) + d\left(\frac{1}{f_2}\right) + \frac{a}{f_1} d\left(\frac{1}{f_2}\right) + \frac{a}{f_2} d\left(\frac{1}{f_1}\right)$,

and $d\left(\frac{1}{f_1}\right) = \frac{\omega_1}{f_1}$ and $d\left(\frac{1}{f_2}\right) = \frac{\omega_2}{f_2}$.

$\therefore$ Substituting in the expression for $d\left(\frac{1}{F}\right)$, we have,

$$d\left(\frac{1}{F}\right)=\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}+\frac{a\omega_2}{f_1f_2}+\frac{a\omega_1}{f_1f_2}.$$

$\therefore$ For achromatism,

$$\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}+\frac{a}{f_1f_2}(\omega_1+\omega_2)=0 \qquad (\alpha)$$

If the lenses are of the same material, $\omega_1=\omega_2=\omega$.

$\therefore$ $\frac{\omega}{f_1}+\frac{\omega}{f_2}+\frac{2a\omega}{f_1f_2}=0$, (for achromatism)

[If the component lenses are convex this condition would be

$$\frac{1}{f_1}+\frac{1}{f_2}=\frac{2a}{f_1f_2}\ ]$$

or, $\frac{f_1+f_2}{f_1f_2}=-\frac{2a}{f_1f_2}$,

or, $a=-\frac{f_1+f_2}{2}$ ... ... (β)

Thus relation (α) gives the condition of achromatism for two lenses separated by a distance when the lens materials are not necessarily the same. The relation (β) gives the condition of achromatism for the two lenses of the same material separated by a distance. This principle is utilised in the construction of the Huygens' eye-piece.

In relation (β) ω being absent, this condition of achromatism is valid for all colours. But it must be remembered that the equivalent focal length F is used here only in a restricted sense. So the achromatisation of focal length here indicates that the achromatisation of magnification is only ensured and not necessarily that of position. It can be easily shown that for complete achromatisation of the system both for the magnification as well as for the position, each of the component lenses must itself be achromatic.

In order to make the system achromatic for two colours for any position of the object, it would not only be necessary to give F the same value but also to give the equivalent planes the same positions for both the colours. This is not possible ; neither it is possible to make the system achromatic for one position of the object. This is evident from the following treatment. Let pq be the final image of PQ formed by the lenses L_1 and L_2, p_1q_1 being the first image formed by L_1. Let $PQ=h_1$, $p_1q_1=h_2$ and $p_1=h_3$. Let $L_1Q=u$, $q_1L_1=v_1$, $L_2q_1=u_1$ and $L_2q=v$.

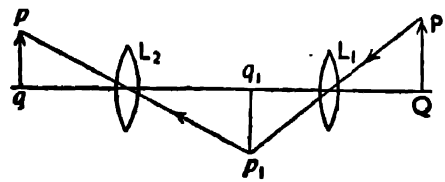


Fig. 168 (a)

Then $\frac{h_1}{h_2}=\frac{u}{v_1}$ and $\frac{h_2}{h_3}=\frac{u_1}{v}$

$$\therefore \quad \frac{h_3}{h_1}=v_1v/u_1u \qquad \ldots \qquad \ldots \qquad \text{(I)}$$

If the images formed by two colours coincide in position and size, u, v and h_3/h_1 must be the same for both colours. Hence v_1/u_1 should also be the same for these colours. But L_1L_2 is a fixed distance. Hence in order to have v_1/u_1 = constant, q_1 must be a fixed position for both colours, which automatically demands that both L_1 and L_2 must be achromatic themselves.

[**Alternative treatment.—**

We know $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{a}{f_1f_2}$.

$$\therefore \quad \frac{1}{F_v}=\frac{1}{f_{1v}}+\frac{1}{f_{2v}}+\frac{a}{f_{1v}f_{2v}} \qquad \ldots \qquad \ldots \qquad \text{(I)}$$

$$\text{and,} \quad \frac{1}{F_r}=\frac{1}{f_{1r}}+\frac{1}{f_{2r}}+\frac{a}{f_{1r}f_{2r}} \qquad \ldots \qquad \ldots \qquad \text{(II)}$$

For achromatism, $\frac{1}{F_v}-\frac{1}{F_r}=0$.

We know, $\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$. So if we suppose the material of the two component lenses to be the same, μ and ω are the same for both the lenses and hence we can write,

$$\frac{1}{f_{1v}}=\frac{\mu_v-1}{\mu-1}\cdot(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=\frac{\mu_v-1}{\mu-1}\cdot\frac{1}{f_1},$$

$$\frac{1}{f_{2v}}=\frac{\mu_r-1}{\mu-1}\cdot\frac{1}{f_2},\ \frac{1}{f_{1r}}=\frac{\mu_r-1}{\mu-1}\cdot\frac{1}{f_1} \text{ and } \frac{1}{f_{2r}}=\frac{\mu_r-1}{\mu-1}\cdot\frac{1}{f_2}.$$

Substituting in (I) and (II), we have,

$$\frac{1}{F_v}=\frac{\mu_v-1}{\mu-1}\left(\frac{1}{f_1}+\frac{1}{f_2}\right)+\frac{a}{f_1f_2}\left(\frac{\mu_v-1}{\mu-1}\right)^2$$

$$\text{and} \quad \frac{1}{F_r}=\frac{\mu_r-1}{\mu-1}\left(\frac{1}{f_1}+\frac{1}{f_2}\right)+\frac{a}{f_1f_2}\left(\frac{\mu_r-1}{\mu-1}\right)^2$$

$\therefore$ For achromatism,

$$\frac{\mu_v-\mu_r}{\mu-1}\left(\frac{1}{f_1}+\frac{1}{f_2}\right)+\frac{a}{f_1f_2}\times\frac{(\mu_v-1)^2-(\mu_r-1)^2}{(\mu-1)^2}=0;$$

$$\text{or,} \quad \omega\left(\frac{1}{f_1}+\frac{1}{f_2}\right)+\frac{a}{f_1f_2}\cdot\frac{(\mu_v-\mu_r)}{(\mu-1)}\cdot\frac{(\mu_v+\mu_r-2)}{(\mu-1)}=0.$$

But $\mu_v+\mu_r=2\mu$ approximately.

$\therefore$ The condition of achromatism becomes,

$$\omega\left(\frac{1}{f_1}+\frac{1}{f_2}\right)+\frac{2a}{f\,f_2}\cdot\omega=0;$$

or, $\frac{1}{f_1}+\frac{1}{f_2}+\frac{2a}{f_1 f_2}=0$; or, $a=-\frac{f_1+f_2}{2}$.]

108. Rainbow :—The rainbow is a beautiful solar spectrum formed when the sun shines on rain drops or fountain spray. The formation of the rainbow is due to reflection, refraction and dispersion of the rays by water-drops and an observer standing with his back to the sun sees a circular arc of spectral colours in the sky, the red being at the top or the outer edge and the violet at the bottom or the inner edge. This is called the *Primary Rainbow.* The primary rainbow subtends at the eye of the observer an angle of nearly 41° with the direction of the sunrays. Sometimes another arc is seen outside the first with the order of the spectral colours reversed *i.e.* the violet at the top and the red at the bottom. This is known as the *Secondary Rainbow.* It subtends an angle of about 53° at the eye of the observer. The region of the sky immediately above the secondary bow and immediately below the primary bow appear to be somewhat brighter than the rest of the sky whereas the space between the two rainbow seems to be rather darker than the rest of the sky. The formation of primary and secondary rainbows can be explained from the standpoint of the simple geometrical theory.

109. Theory of the formation of Rainbow :—The geometrical theory of the formation of rainbow requires a knowledge of the manner in which a pencil of parallel rays is reflected and refracted in a spherical droplet of water.

Suppose *SABCS* be the path of a ray of sunlight which comes out of a spherical droplet of water after suffering one internal reflection at *B* [Fig. 169(*a*)].

Let i be the angle of incidence at *A* and r be the angle of refraction at *A*. r also represents the angles of incidence at *B* and *C* and the angle of reflection at *B*. Since the angle of incidence inside the droplet at *C* is r, the angle of emergence at *C* will be i.

The deviation of the ray due to refraction at *A* is $(i-r)$, that due to

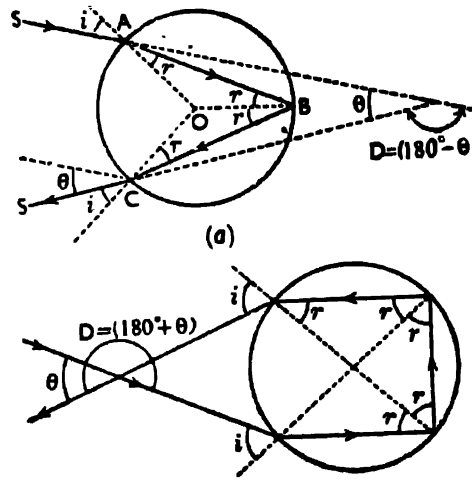


Fig. 169

reflection at B is $(180^\circ - 2r)$ and on emergence at C the deviation is again $(i-r)$.

If instead of one internal reflection there are two internal reflections as in Fig. 169(b), the deviation due to the reflections will be $2(180^\circ - 2r)$.

Similarly for n internal reflections the deviation is $n\ (180^\circ - 2r)$. As there are only two refractions at the point of incidence and emergence, the total deviation D suffered by the emergent ray after n internal reflections will be $D = 2(i-r) + n(180^\circ - 2r)$

$$= n \times 180^\circ + 2i - 2r(n+1) \quad \ldots \quad \ldots \quad (1)$$

For the minimum value of D, we have $\frac{dD}{di} = 0$. That $\frac{dD}{di} = 0$ represents the condition of minimum deviation is clear from the fact that if $\frac{d^2D}{di^2}$ is calculated, it is found to be positive.

So for minimum deviation, we have,

$$\frac{dD}{di} = 2 - 2(n+1)\frac{dr}{di} = 0\ ;$$

$$\text{or,}\quad \frac{dr}{di} = \frac{1}{n+1} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

If μ be the refractive index of the material forming the droplet (*i.e.*, water), then from Snell's law, $\sin i = \mu \sin r$ ... ... (3)

Differentiating with respect to i, $\cos i = \mu \cos r \frac{dr}{di}$.

$$\therefore \quad \cos i = \frac{\mu}{n+1}\sqrt{1 - \sin^2 r} \quad \ldots \quad \ldots \quad [\text{from (2)}]$$

$$= \frac{\mu}{n+1}\sqrt{1 - \frac{\sin^2 i}{\mu^2}} = \frac{\sqrt{\mu^2 - \sin^2 i}}{n+1}.$$

Squaring we get,

$$(n+1)^2 \cos^2 i = \mu^2 - \sin^2 i = \mu^2 - 1 + \cos^2 i.$$

Re-arranging, $(n^2+2n)\cos^2 i = \mu^2 - 1$

$$\text{or,}\quad \cos i = \sqrt{\frac{\mu^2 - 1}{n^2 + 2n}} \quad \ldots \quad \ldots \quad (4)$$

For one internal reflection $n = 1$, $\cos i = \sqrt{\frac{\mu^2 - 1}{3}}$ ... (5)

and for two internal reflections $n=2$, and so

$$\cos i = \sqrt{\frac{\mu^2 - 1}{3}} \quad \ldots \quad (6)$$

So if μ of water for a particular coloured ray is known and if the number of internal reflections suffered by the ray in the droplet be known, i can be found out from (4) or (5) or (6). Knowing i, r can be computed from (3). Now as these values of i and r refer to minimum deviation condition, the value of minimum deviation can also be calculated by substituting these values in relation (1).

For water droplets μ for red is 1·329 and that for violet is 1·349. For a single internal reflection, calculations show that,

D_m for red rays $=(180^\circ - 43^\circ)$ nearly and that for violet rays $=(180^\circ - 41^\circ)$ nearly. For two internal reflections it is found that for the red rays, D_m $180^\circ + 51^\circ$ and for the violet rays D_m $180^\circ + 54^\circ$ nearly.

When a beam of sun rays falls on the water droplets, each of these droplets disperses these white rays into the colour components. The above calculations show that due to a single internal reflection, the red components suffer a deviation, whose minimum value is $(180^\circ - 43^\circ)$. So if we draw a cone of semi-vertical angle 43° with the droplet as its apex (Fig. 170) and the direction of sun rays as its axis, the red rays suffering minimum deviation will all lie on the surface of this cone and the rays undergoing a greater deviation will lie within this cone. It is actually found that the rate of change of deviation with the angle of incidence is very little in the neighbourhood of minimum deviation. So the emergent red rays will be more concentrated near the surface of this cone thereby making the intensity of the red rays maximum in the direction of the minimum deviation. Again since the minimum deviation for the violet components is $(180^\circ - 41^\circ)$, the violet rays will be intensely visible in the direction of the surface of a cone of semi-vertical angle 41°. In the annular conical space between 41° and 43°, other intermediate spectral colours will concentrate. So above 43° there will be practically no ray reaching after a single internal reflection but below 41° there may be the presence of rays which suffer deviations greater than the minimum.

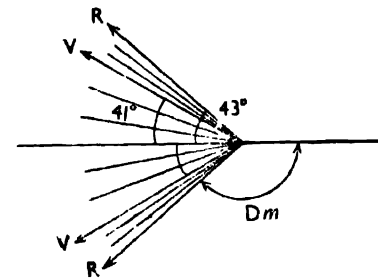


Fig. 170

Again from the calculations given above, we know that due to two internal reflections, the red rays suffer a deviation whose

minimum value is (180°+51°). So if with the droplet as the vertex and the direction of sun rays as the axis we draw a cone of semi-vertical angle 51°, the red rays suffering minimum deviation will lie on its surface and there will be a concentration of red rays near this surface. The red rays suffering greater deviation will be outside this cone as shown in Fig. 171. For violet rays suffering two internal reflections the cone of minimum deviation will have a semi-vertical angle 54° as is evident from the above calculations. So the violet rays will be copiously visible in a direction 54° with the sun rays. The intermediate colours lie within the annular conical space between 51° and 54°. On the outside of the cone of semi-vertical angle 54°, some rays suffering deviations greater than the minimum will always be present but there can be no ray within the cone of 51° semi-angle.

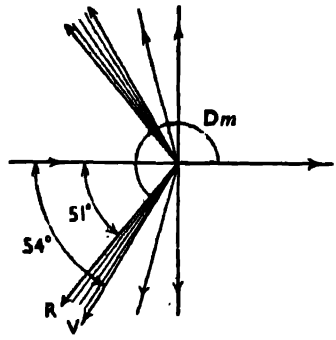


Fig. 171

110. Explanation of the formation of rainbow :—In order to explain the formation of rainbow let us consider the water-droplets

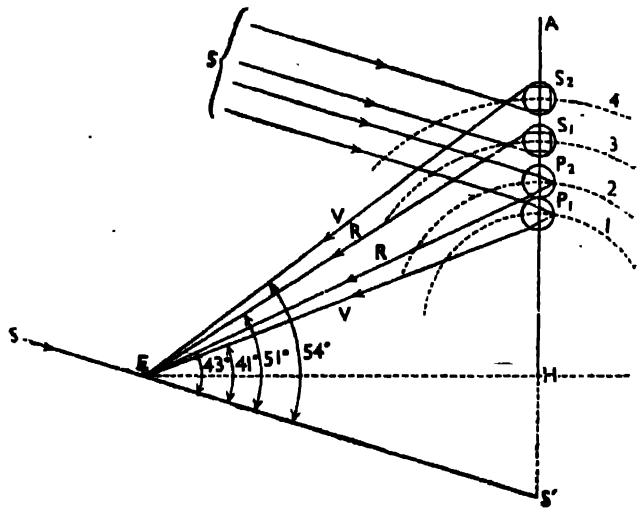


Fig. 172

P_1, P_2, S_1, S_2 (shown magnified in the figure) to be suspended in air in a vertical line AH (Fig. 172). Let E be the eye of the

observer with his back to the sun and EH the horizontal line through E. The sun rays are supposed to be incident on the droplets in a direction parallel to SES', where S' is the point of intersection of the line SES' with AH produced and is commonly known as the *counter-sun*. If the angle $S'EP_1$ be 41°, the rays undergoing single internal reflection within P_1 will allow their violet components to come out copiously along P_1E, as the deviation in this direction corresponds to minimum deviation for the violet rays (*i.e.*, 180° − 41°). If P_1E is now rotated about ES', P_1 will describe an arch (1) of angular radius 41°. All the water-droplets lying on this arch (1) will send to E only the violet rays prominently and the arch would appear violet. Again if $\angle P_2ES'$ be 43° and if the line P_2E be rotated about the axis ES', P_2 will describe another arch (2) of angular radius 43°. All the water-droplets lying on this arch (2) will send to the observer only the red rays which have suffered minimum deviation after one internal reflection within the droplet. So the arch (2) appears red. The other coloured arches appear in between these two extreme arches, (1) and (2). Thus the formation of the **primary rainbow** is interpreted from the simple geometrical theory. The angular width of this primary rainbow is only about 2° As has been discussed in the last article, the droplets below P_1 which subtend angles less than 41° to the eye will send some rays of all colours after one internal reflection and as a result, the region just below the primary rainbow will appear to be somewhat brighter than the remaining portions of the sky. But no droplet above P_2 (*i.e.*, subtending angles greater than 43°) can send rays to E after one internal reflection.

Now suppose the angle S_1ES' is 51°. The rays undergoing two internal reflections within S_1 will allow their red components to come out copiously along S_1E, as the deviation is minimum for the red rays in this direction (*i.e.*, at 180°+51°). If S_1E is now rotated about ES', S_1 describes an arch (3) of angular radius 51°. All the water-droplets lying on this arch (3) will send to E only the red rays and the arch appears strongly red in colour. Again if the angle S_2ES' be 54° and if the line S_2E be rotated about the axis ES'. S_2 describes the arch (4) of angular radius 54° All the water-droplets lying on this arch (4) will send to the eye of the observer only the violet rays which have undergone minimum deviation after two internal reflections within the droplet. So the arch (4) appears violet. The intermediate colours occur in between the arches (3) and (4) and the **secondary rainbow** is seen with its order of colours reversed with respect to the primary bow. The angular width of this secondary bow is nearly 3°. From the discussions of the last article, it is evident that water-droplets lying above S_2 (*i.e.*, subtending angles greater than 54° to the eye) send some rays of all colour to the eye after two internal reflections. So the region just above the secondary rainbow is somewhat brighter. Again the water-droplets lying below S_1 (*i.e.*,

subtending angles less than 51° to the eye) can send no ray after two internal reflections. So the water-droplets lying in the region between the two rainbows—primary and secondary can send no rays to E after one or two internal reflections. This is why this region appears to be darker than the remaining parts of the sky.

Rainbows formed due to more number of internal reflections lack in intensity compared to the bright sun rays and are as such rarely observed.

Discussions :—(*i*) It is evident from the above explanation of the formation of rainbows that if the observer ascends higher up in balloon or in an aeroplane, the position of the counter-sun will be raised. As the counter-sun represents the centre of the coloured arches of the rainbow, the latter will appear to be more and more raised and the coloured arcs gradually approach the shape of complete circles. (*ii*) Again if two observers simultaneously see the rainbow, as the positions of the observers are different, the positions of the plane of formation of the rainbow will also be different. (*iii*) If an observer approaches a rainbow, he can still see the rainbow formed in a shifted plane provided there are sufficient numbers of water-droplets in that plane. (*iv*) If there is a sheet of water or a horizontal plane mirror in front of an observer, he would see a second rainbow due to reflection in the above horizontal reflecting surface over and above the one seen directly in the sky. Now the question to be discussed here is whether the rainbow seen by reflection is an image of that seen directly by the observer. Let E be the eye of the observer and MM_1 the horizontal reflecting surface. P is the position of the rainbow directly seen by the eye. Let E' be the image of E (Fig. 173) as formed by the reflecting surface due to reflection. Now if we suppose the eye to be at E' and MM_1 to be removed, a rainbow can only be seen by the eye when it is formed at P_1 where $E'P_1$ is parallel to EP. In the presence of MM_1 the rays from P_1 will be actually reaching E after reflection at MM_1 and the rainbow will be seen in the direction EOQ_1. Q_1 being the apparent position of the reflected bow. Q_1 is evidently the image of the bow formed at P_1 and not of that at P.

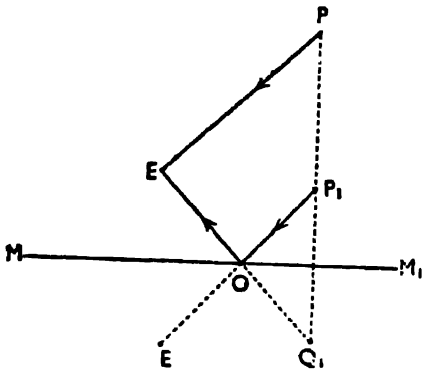


Fig. 173

111. Experimental Demonstration of Rainbow :—In the laboratory, rainbows can be produced by taking a large round bottomed flask in which a small quantity of water is poured. A piece of cork is now fitted to the neck of the flask so as to make the arrangement air tight. A glass tube is inserted inside the flask through a previously bored hole in the cork. The flask is now inverted and clamped at a convenient height. The glass tube is then connected to an air-pump through an air-chamber and a stop-cock. First the stop-cock is closed and the air-pump is worked for sometime. The flask is now illuminated by strong arc light or sun light and the stop-cock suddenly opened. As the stop-cock is opened a partial vacuum is created in the flask where upon small droplets of water are formed inside it due to adiabatic cooling. As these droplets are illuminated by white light rainbows will be seen within the flask by an eye suitably placed with respect to the flask, provided the back of the observer is in the direction of the source of light.

***112 Supernumerary bow** :—On a closer examination of a rainbow it is found that there is a number of narrow coloured rings (or bands) in which the succession of colours is apparently irregular. These rings are known as *supernumerary bows*. They occur frequently near the inner edge of the primary rainbow and less frequently at the outer edge of the secondary rainbow. Though the succession of colours are irregular in supernumerary bows, when looked through a coloured glass the rings of the bows appear as alternate dark and bright bands which closely resemble the pattern of diffraction rings formed round the shadow of an opaque circular disc. The formation of these rings has been partly explained from the interference principle by Young while some of the scientists have suggested a theory based on the diffraction phenomenon. Details of the discussions of these theories are beyond the scope of this book.

Examples

1. The mean refractive indices of two specimens of glass are 1·52 and 1·66 respectively. The differences in the refractive indices for the same two lines in the spectrum are ·013 for the first and ·022 for the second. Find the focal length of the lens of the second glass which when combined with a convex lens of 50 cms focal length of the first will make an object glass achromatic. (*C. U. 1946*)

Ans. Here, $\omega_1 = \frac{\mu_v - \mu_r}{\mu - 1} = \frac{\cdot 013}{1\cdot 52 - 1} = \frac{\cdot 013}{\cdot 52} = \frac{1}{40}$

and $\omega_2 = \frac{\cdot 022}{1\cdot 66 - 1} = \frac{\cdot 022}{\cdot 66} = \frac{1}{30}$.

For achromatism, $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$

and from the given data, $f_1 = -50$ cm. $\therefore \quad -\frac{1}{40 \times 50} + \frac{1}{30 \times f_2} = 0.$

$\therefore \quad \frac{1}{40 \times 50} = \frac{1}{30 \times f_2}$, or, $f_2 = \frac{200}{3} = 66\cdot 6$ cms.

2. *The refractive indices of crown glass for red and violet lights are 1·515 and 1·523 respectively and the corresponding values for the dense flint glass are 1·614 and 1·632. If an achromatic plano-convex doublet of 30 cm. focal length is to be designed, what will be the radii of curvature of the lenses ?*

Ans. For crown glass, $\omega_1 = \dfrac{\mu_v - \mu_r}{\mu - 1} = \dfrac{1{\cdot}523 - 1{\cdot}515}{\dfrac{1{\cdot}523 + 1{\cdot}515}{2} - 1} = \dfrac{{\cdot}008}{1{\cdot}519 - 1}$

$$= \frac{{\cdot}008}{{\cdot}519} = {\cdot}0154.$$

For flint glass, $\omega_2 = \dfrac{1{\cdot}632 - 1{\cdot}614}{\dfrac{1{\cdot}632 + 1{\cdot}614}{2} - 1} = \dfrac{{\cdot}018}{{\cdot}623} = {\cdot}0289.$

For achromatism, $\dfrac{\omega_1}{f_1} + \dfrac{\omega_2}{f_2} = 0$;

or, $\dfrac{{\cdot}0154}{f_1} = -\dfrac{{\cdot}0289}{f_2}$;

or, $\dfrac{1}{f_1} = -\dfrac{289}{154 f_2}$ ... ... (A)

Again we know, $\dfrac{1}{F} = \dfrac{1}{f_1} + \dfrac{1}{f_2}$ and $F = -30$ cm.

$\therefore \quad -\dfrac{1}{30} = -\dfrac{289}{154 f_2} + \dfrac{1}{f_2}$. Hence $f_2 = 26{\cdot}30$ cm.

Substituting in (A), $f_1 = -14{\cdot}01$ cm.

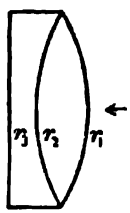


Fig. 174

So for the achromatism of the plano-convex doublet, a convex lens of crown glass of focal length $-14{\cdot}01$ cm. and of mean $\mu = 1{\cdot}519$ is to be placed in contact with a plano-concave lens of flint glass of focal length 26·30 cm. and of mean $\mu = 1{\cdot}623$.

Supposing light to be incident on the convex surface, r_1 is $-$ve, r_2 is $+$ve and $r_3 = \infty$

Considering the plano-concave lens,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_2} - \frac{1}{r_3}\right).$$

$\therefore \quad \dfrac{1}{26{\cdot}30} = (1{\cdot}623 - 1)\left(\dfrac{1}{r_2} - \dfrac{1}{\infty}\right)$.

or, $r_2 = {\cdot}623 \times 26{\cdot}30 = 16{\cdot}38$ cm.

Since the second convex surface of the crown glass lens usually exactly fits in the first concave surface of the flint glass lens, r_2 is also the radius of curvature of the second surface of the convex lens.

Again for the convex lens, $r_2 = +16{\cdot}38$ cm.

and $\mu = 1{\cdot}519$ and $f = -14{\cdot}01$ cm.

$$\therefore \quad \frac{1}{-14{\cdot}01} = (1{\cdot}519 - 1)\left(\frac{1}{r_1} - \frac{1}{16{\cdot}38}\right)$$

or, $-\dfrac{1}{14{\cdot}01 \times 519} = \dfrac{1}{r_1} - \dfrac{1}{16{\cdot}38}$;

or, $\dfrac{1}{r_1} = \dfrac{1}{16{\cdot}38} - \dfrac{1}{14{\cdot}01 \times {\cdot}519} = \dfrac{1}{16{\cdot}38} - \dfrac{1}{7{\cdot}271}$. Hence $r_1 = -13{\cdot}08$ cm.

3. *An achromatic converging combination of focal length 60 cms. is formed with a convex lens of crown glass and a concave lens of flint glass placed in*

contact with each other. Calculate their focal lengths if the dispersive power of the crown glass is ·03 and that of the flint glass is ·05.

Ans. For achromatism, $\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}=0$;

or, $\frac{\cdot 03}{f_1}=-\frac{\cdot 05}{f_2}$; or, $\frac{1}{f_1}=-\frac{5}{3f_2}$

Since the equivalent focal length $F=-60$ cm. from the relation $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$, we have,

$$-\frac{1}{60}=-\frac{5}{3f_2}+\frac{1}{f_2}=-\frac{2}{3f_2}.$$

Hence $f_2=\frac{60\times 2}{3}=40$ cm. and $f_1=-\frac{3f_2}{5}=-24$ cm.

4. Calculate the angle of the flint glass prism which when combined with a crown glass prism of 5° produces (a) dispersion but no deviation and (b) deviation but no dispersion. Find also the value of deviation in (a) and dispersion in (b). [The refractive indices of the crown glass for C, D and F lines are 1·514, 1·517, 1·523, respectively and those for the flint glass are 1·644, 1·650, 1·664.]

Ans. (a) For no deviation,

$$(\mu_D-1)A=(\mu'_D-1)A' \text{ (numerically) ;}$$

or, $\cdot 517\times 5^\circ=650\times A'$.

$$\therefore\quad A'=\frac{\cdot 517}{\cdot 130}=3^\circ 58'\cdot 6.$$

Dispersion by the crown prism $=(\mu_F-\mu_C)A=(1\cdot 523-1\cdot 514)5^\circ=\cdot 009\times 5^\circ=(0\cdot 045)^\circ$

Dispersion by the flint prism $=(\mu'_F-\mu'_C)A'=\cdot 02+\frac{517}{130}=(0\cdot 0795)^\circ$.

$\therefore$ The resultant dispersion $=0\cdot 0795-0\cdot 045=\cdot 0345^\circ=2'\cdot 07$.

(b) For no dispersion,

$(\mu_F-\mu_C)A=(\mu'_F-\mu'_C)A'$ (numerically).

Hence, $\cdot 009\times 5^\circ=\cdot 02\times A'$

or, $A'=\frac{\cdot 045}{\cdot 02}=2^\circ\cdot 25=2^\circ 15'$

Deviation by the crown prism $=A(\mu_D-1)=5^\circ\times\cdot 517=\cdot 2^\circ\cdot 585$
Deviation by the flint prism $=A'(\mu'_D-1)=2^\circ\cdot 25\times\cdot 650=1^\circ\cdot 4625$
Hence the total deviation $=2\cdot 585-1\cdot 4625=1^\circ\cdot 1225$.

5. A convex lens of focal length 35 cms. achromatic for C and F lines is to be made from two glasses, data for which are as follows :—

Hard crown of $\mu_d=1\cdot 5176$, $\mu_f-\mu_d=\cdot 00856$
Dense flint of $\mu_d=1\cdot 6214$, $\mu_f-\mu_d=\cdot 01722$.
Find the focal lengths of the components. (*I.A.S. Exam., 1951*)

Ans. ω_1 (of crown glass) $=\frac{\cdot 00856}{1\cdot 5175-1}=\frac{\cdot 00856}{\cdot 5175}=\cdot 01654$;

ω_2 (of flint glass) $=\frac{\cdot 01722}{1\cdot 6214-1}=\frac{\cdot 01722}{\cdot 6214}=\cdot 02901$.

For achromatism,

$$\frac{\cdot 01654}{f_1}=-\frac{\cdot 02901}{f_2}.$$

$$\therefore\quad f_2=-\frac{\cdot 02901}{\cdot 01654}f_1=-\frac{2901}{1654}f_1.$$

Again, $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$.

or, $-\frac{1}{35}=\frac{1}{f_1}-\frac{1654}{2901f_1}=\frac{1}{f_1}\left[\frac{2901-1654}{2901}\right]=\frac{1247}{2901f_1}$.

$\therefore\quad f_1=-\frac{1247\times 35}{2901}=-15{\cdot}03$ cm.

Hence $f_2=15{\cdot}03\times\frac{2901}{1654}=26{\cdot}37$ cm.

6. The refractive indices of the hard crown glass for red and violet light are 1·553 and 1·567 respectively. If a beam of white light falls on a convex lens of the same glass in a direction parallel to the axis and if the radii of curvature of the two surfaces of the lens be 30 cm. and 20 cm., calculate the axial chromatic aberration of the lens for the above two colours.

Ans. Here the dispersive power of the glass is,

$$\omega=\frac{1{\cdot}567-1{\cdot}553}{\frac{1{\cdot}567+1{\cdot}553}{2}-1}=\frac{{\cdot}014}{1{\cdot}56-1}=\frac{1}{40}.$$

The focal length of the lens is given by the relation,

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\text{(numerically)}$$

$$=(1{\cdot}56-1)\left(\frac{1}{30}+\frac{1}{20}\right)={\cdot}56\times\frac{1}{12}=\frac{{\cdot}14}{3}\quad\therefore\ f=\frac{300}{14}\text{ cm.}$$

The dispersive power of a lens is $\omega=\frac{f_r-f_v}{f}$ where f_r-f_v is the required axial chromatic aberration.

$$\therefore\quad f_r-f_v=\omega f=\frac{1}{40}\times\frac{300}{14}=0{\cdot}5357\text{ cm.}$$

7. Show that for a single refraction at a plane surface the dispersion is proportional to the tangent of the angle of refraction.

Ans. Deviation due to a single refraction is $D=i-r$ ... ... (1)

Since the angle of incidence is constant, differentiating (1), we get, $dD=-dr$

Here dD is evidently the dispersion.

Again for the refraction at a plane surface, $\sin i=\mu\sin r$.

$\therefore$ Differentiating we get, $0=d\mu.\sin r+\mu\cos r\,dr$,

or, $-dr.\,\mu\cos r=\sin r.d\mu$,

or, $-dr=\frac{\tan r}{\mu}d\mu$

Hence $dD=-dr=\frac{d\mu}{\mu}\tan r$

Thus the dispersion is proportional to the tangent of the angle of refraction at a plane surface.

Questions (Chapter VII)

1. Explain the terms 'dispersion' and 'dispersive power.' (Utk. U. 1960)
How would you form an achromatic combination of prisms ? (C. U. 1953)

2. What are chromatic errors ? How are they remedied ?
Deduce the condition of achromatism of two lenses in contact with each other.
(*cf.* Utk. U. 1966 , C U. 1946)

3. Explain what is meant by the dispersive power of a substance. Give the theory and construction of a direct vision spectroscope. (C. U. 1954)

4. Explain the dispersion produced by a single lens and show how the defect may be corrected.
Why is such correction unnecessary in the case of a simple convex lens used as a magnifying glass held close to the eye ? (C. U. 1947)

5. What is meant by chromatic aberration ? Show how it is eliminated in combination of two lenses. (*cf.* Del. U. '67 ; Luck. U. 1949, '53)

6. Explain what is meant by achromatism. Obtain the condition that must be satisfied in order that a combination of two lenses of focal lengths f_1 and f_2 and a distance d apart be achromatic. (Punj. U. 1951)

7. What do you mean by chromatic aberration ? Explain how chromatic aberration is removed by a combination of two lenses of different materials placed in contact. (C. U. 1955, '67)

8. Explain what you mean by the dispersion of light and the dispersive power of a lens.

Show that an achromatic combination of two lenses with dispersive powers ω_1 and ω_2 and of focal lengths f_1 and f_2 respectively, is possible when, $\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}=0$.

(C. U. 1949)

9. Describe the condition of achromatism of two thin lenses separated by a distance. (Del. U. 1960 ; Agra U. 1946 ; Ban. U. 1948)

10. What do you mean by achromatic combination of two lenses ? Establish the condition of achromatism of two lenses in contact. (C. U. 1952)

11. What is chromatic aberration ? Find the condition of achromatism of two lenses in contact with each other.

The dispersive powers of crown and flint glasses are 0·036 and 0·064 respectively. Find the focal length of two thin lenses which will form a convergent achromatic lens of focal length 50 cm. when put in contact with each other. (C. U. 1957)

Ans. −21·88 and 38·89 cm.

12. Discuss the condition for achromatism of two thin lenses placed in contact. Given the dispersive powers of crown and flint glass as 0·2 and 0·4 respectively, find the focal lengths of the two components of an achromatic combination of focal length 20 cm. (Punj. U. 1953)

Ans. −10 cm. (convex) and 20 cm. (concave).

13. What is an achromatic combination of lenses ? Describe its construction and principle of action.

Calculate the focal length of a lens of flint glass of dispersive power 0·45 which will render achromatic a converging lens of crown glass of focal length 75 cms and of dispersive power 0·21. (Pat. U. 1950)

Ans. 160·7 cm. (concave).

14. Show how it is possible to construct an achromatic combination from two lenses of the same material. (Gau. U. 1957)

15. **Explain the terms 'Dispersive power' and 'Dispersion'.**

Deduce the condition of achromatism of two prisms.

The dispersive powers of crown and flint glass are 0·03 and 0·05 respectively. If the difference in the refractive indices of red and violet colours is ·014 for crown glass and ·023 for flint glass, calculate the angles of two prisms which when combined will be achromatic but will produce a deviation of 10°. (C. U. 1956)

Ans. 53°·6 (crown) and 32°·6 (flint) approx.

16. **Two thin convex lenses of the same glass but of different focal lengths are mounted coaxially at a distance apart. Work out the condition of achromatism of the combination. Discuss the importance of the result in obtaining achromatism in Huygens' eye-piece.** (C. U. H. 1958)

17. **What do you mean by spherical and chromatic aberrations of a lens ? Explain how they are caused.**

How do you correct for chromatic aberration in the case of a lens-system in contact ? (Agra U. 1947 ; Utk. U. 1960)

18. **What are cardinal points ? Derive the formula for the equivalent focal length of two thin lenses separated by a distance x in air. State how such a combination is used to remove chromatic aberration.** (Del. U. 1968)

19. **Write a short note on Rainbow.**
(C. U. 1945 ; Luck. U. 1947, '51 , Utk. U. '70s)

20. **Give the theory of the formation of the primary rainbow.**

In what order the colours are arranged ? (C. U. 1948, '52)

How would the appearance of the bow change to a man ascending in a balloon ?
(C. U. 1949)

21. **A ray of light is incident on the surface of a transparent sphere of refractive index 1·32. After refraction it is internally reflected (not total) and then refracted out of the sphere. Find the deviation suffered by the ray and the angle of incidence to the nearest degree, so that the deviation may be minimum. Hence explain the formation of rainbow.** (C. U. 1949)

Ans. $D = 136°$ and $i = 60°$.

22. **Give an elementary theory of the formation of primary and secondary rainbows. Will two observers see the same rainbow ? Explain your answers.**
(Punj. U. 1960)

23. **Discuss the theory of the formation of primary and secondary rainbows. Can you give an explanation of the supernumerary bows ? Describe a method for the demonstration of the rainbows to a large audience.**
(C. U. 1967 ; C. U. H. 1939, '48, cf. 1962)

24. **Give the theory of the formation of a primary rainbow. In what order are the colours arranged and why ? In what part of the sky is a rainbow seen in the morning ?** (Ban. U. 1947 ; Gau. U. 1959)

25. **Explain the formation of the primary rainbow. What is the semi-vertical angle of the cone of light reaching the eye of the observer and in what order are the colours arranged ? Will there be any change in the angle if the observer advances towards the rainbow by some appreciable distance ?** (C. U. 1946 ; cf. 1937)

113. The Simple Microscope :—If an object be placed between a convex lens and its principal focus, an erect, magnified and virtual image will be visible to an eye placed behind the lens. The convex lens, thus used, forms a simple microscope. This is popularly called a **Magnifying Glass** or a **Reading Glass** (Fig. 175).

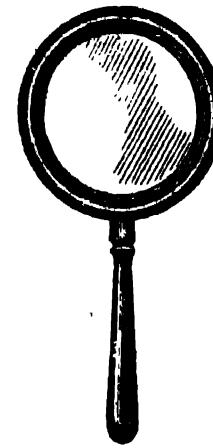


Fig. 175—Reading Glass.

An object becomes visible when it forms an image on the retina of the eye. The apparent size of the object is proportional to the size of the image formed on the retina, and the size of the image depends upon the angle subtended by the object at the eye. It should be clearly remembered that *for the greatest advantage in vision, the angle subtended at the eye by the object should be as large as possible.* This angle increases as the object is brought nearer to the eye, and so the object appears to be larger. But as we cannot see objects clearly when too near the eye, there is obviously a limit to the distance up to which an object can be brought near the eye, if clear vision is to be obtained. This distance is known as the **least distance of distinct vision** (*D*). For the normal eye it is about 25 cms.

In Fig. 176, OQ' be the least distance of distinct vision. So an eye without the aid of a lens cannot clearly see an object placed at Q. For clear vision the maximum angle is subtended at the eye by an object when it is placed at Q'. If the object be brought nearer, the subtended angle would be larger, but the image would be indistinct, because for clear vision OQ' is the least distance. But it will be clear from Fig. 176, that by the aid of the lens L, the image of PQ is formed at $P'Q'$, and that both the object and the image subtend the same angle at the centre of the lens, i.e. at the eye placed close to the lens. So the image appears both distinct and magnified. Thus the effect of a simple microscope, or a simple magnifying glass is to enable an object to be brought very close to the eye when the image formed at the least distance of distinct vision is magnified and yet distinctly visible.

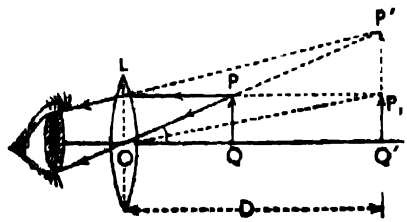


Fig. 176—The Simple Microscope.

The simple microscopes or reading glasses may consist of two plano-convex lenses with their curved surfaces facing each other and mounted on a thick rim.

Magnifying Power.—*It is defined as the ratio of the angle subtended at the eye by the image to that subtended by the object when placed at the least distance of distinct vision.*

For distinct vision the angle subtended at the eye by the image $= \angle P'OQ' = \tan \angle P'OQ'$ (the angle being small is taken as proportional to the tangent of the angle) $= \frac{P'Q'}{OQ'} = \frac{\text{size of image}}{D}$ (Fig. 176). Similarly the angle subtended by the object when placed at the least distance of distinct vision $= \angle P_1OQ' = \tan \angle P_1OQ'$

$$= \frac{P_1Q'}{OQ'} = \frac{PQ}{OQ'} = \frac{\text{size of object}}{D}.$$

Therefore, the magnifying power, according to the definition, is $m = \frac{P'Q'}{OQ'} \div \frac{PQ}{OQ'} = \frac{P'Q'}{PQ} = \frac{\text{size of image}}{\text{size of object}} = \frac{v}{u}$. But for a lens, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$.

Multiplying every term by v and transposing, $\frac{v}{u} = 1 - \frac{v}{f}$. Since the lens is convex and the image is formed at the least distance of distinct vision, we have, $v = D$, and f is negative.

$$\therefore \quad m = \frac{v}{u} = 1 + \frac{D}{f}.$$

The magnification in the case of a simple microscope is limited, because a powerful lens must be thick and very thick lenses cannot produce sharp images and the images are also curved and distorted.

Best position where the eye is to be placed.—If the distance of the eye from the lens is a, the magnifying power will be given by

$$m = \frac{v}{u} = 1 + \frac{v}{f} = 1 + \frac{D - a}{f}.$$

From this it is clear that the best position of the observing eye will be where m is maximum, *i.e.* $a = 0$. In other words, the eye should be placed as close to the lens as possible.

Achromatism of a Magnifying glass.—With a magnifying glass different coloured images should be produced due to dispersion. But if the eye is placed very close to the magnifying glass, no such chromatic error is produced. Since corresponding points of the image and the object and the centre of the lens (*e.g.* P', P and O in Fig. 176) lie in the same straight line, the visual angle subtended at the eye by the red image is the same as that subtended by the blue image. The same is the case for every other colour. Thus the images of different colours subtend the same visual angle at the eye and are of the same size. Therefore they superpose on each other on the retina and the image seen is achromatic.

Coddington lens is very often used as a pocket-magnifier. It consists of a transparent sphere of glass or any other refracting material wherein a deep groove has been cut off all the way round leaving only a small central aperture. The groove is filled with a suitable opaque substance. This arrangement prevents distortion.

114 Some Important Terms used in connection with Optical Instruments :—

(a) Field of view :—The *field of view* literally means the extent of space actually visible to an eye. In the case of an optical instrument this region of space is limited by the stops of the component lenses of the instrument. So the region of the object space which is imaged by an instrument is known as its field of view.

In order to elucide the concept of field of view of an optical instrument, let us consider the object plane PQ, the stop ST used in front of the objective so as to limit the rays in the object space (i.e., the entrance pupil) and the image stop st of the instrument (Fig. 177).

From the figure it is evident that the whole beam of rays entering ST

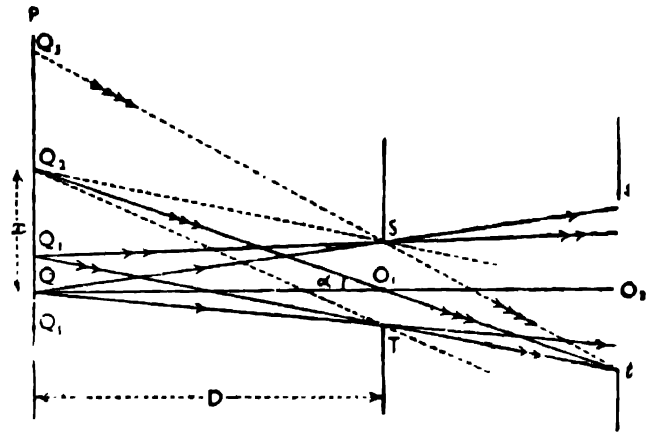


Fig 177

from Q and Q_1 can pass through the image stop st. If we consider a point like Q_1 only half of beam entering ST can pass through st. For intermediate points between Q_1 and Q_2 more than half of the beam can enter st. Again for the point Q_2, none of the rays emitted from it will pass through st. So the point Q_2 limits the extent of field of view. So the maximum area that can be seen by the instrument extends from Q to Q_2 all around the point Q. This area gives the *complete field of view*. The region round Q extending upto the distance QQ_1 is known as

the *field of full illumination*. The area round Q extending upto Q_2 consists of the central zone (upto Q_1) of full illumination and an annular zone (Q_1 Q_2) of gradually diminishing illumination the outer edge of which has only half the illumination of the central zone. This circular zone QQ_2 is generally known as the *field of view* of the instrument. The angle (α) subtended by QQ_2 at the centre (O_1) of the entrance pupil is called the *angular radius of the field of view* or simply the *angular field of view* of the instrument. It is given by the relation $\tan \alpha = \frac{QQ_2}{QO_1} = \frac{x}{D}$. Though the field of view is conventionally defined as the zone round Q of radius QQ_2, it really extends upto the region of the complete field of view, *i.e.*, of radius QQ_3. But the annular space of thickness Q_2Q_3, surrounding the field of view is a *ragged* or *vignetted region*. In a good optical instrument this outer ragged region is to be cut off by a stop placed suitably. When a sharply defined image is required, this stop should be so adjusted as to limit only the field of full illumination.

(b) Apertures and Stops in optical systems :—

Any circular opening of a suitable size in a small opaque screen and placed coaxially with the optical system is called a *stop*. When the rays of light pass through an optical system, they will be limited by the apertures of the various lenses or other components of the system. The mounts holding these components are, as a consequence, considered as stops. Over and above, additional stops may also be introduced in the system in order to cut out the unwanted rays.

Consider an optical system containing two lens apertures or stops at S and T. S_1, T_1 are the images of these stops formed in the object space (by the rays passing through the system from right to left). If light from an axial point O is to pass through the aperture at T, it must pass through the corresponding image T_1 before entering the system and will be limited to cone of rays OAB. The

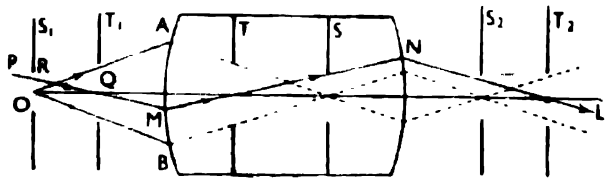


Fig. 177 (a)

subsequent course of these rays are shown by the dotted lines. So the amount of light passing through the system is limited by the aperture at T, which is known as the *aperture stop* of the system.

Its image, T_1 in the object space is called the *entrance pupil* and its image, T_2, in the image space is called the *exit pupil*.

Consider the ray, represented by the continuous line *PQMNL*, coming from an extra-axial point *P* and passing through the centre of the entrance pupil, This will be considered as the *principal ray* so far as the system of rays coming from *P* and passing through the optical system is concerned. The field of view in the plane of S_1 is limited by diameter of the aperture in S_1. So no light from points further than *R* from the axis will pass through the system. The actual stop *S* which limits the field of view in this way is called the *field stop*. Its images S_1, in the object space is called the *entrance window* or *entrance port*, and the image, S_2, in the image space is known as the *exit window* or *exit port*. So the *field of view* is measured by the angle subtended by the entrance port at the centre of the entrance pupil. (For *stop number* See, Art. 134)

(c) **Depth of Focus** :—As the human eye has a finite resolving power, the existence of indistinctness in any image, whether due to incorrect focussing, aberration or diffraction, cannot always be recognised. Consider an object and its image. The former may be moved over a range beyond or within the conjugate position while the fixed image does not show detectable blurring. This range is known as object space depth of focus (*depth of field*). Again keeping the object fixed, the range over which the image screen may be displaced without blurring is known as the image space depth of focus (or simply *depth of focus*). In camera lenses depth of focus is of special interest. (Vide, Art. 134).

115. The Eye-pieces or Oculars :—In optical instruments like telescopes and microscopes, the objective or the lens system facing the object casts an image of that object on the other side of the lens system and an eye-lens, placed very close to the eye, finally magnifies this first image and serves the purpose of a simple magnifier. But with a single eye-lens it is not possible to receive all the rays from the first image because the eye-lenses are of small apertures so that all rays passing through them may not enter the pupil of the eye. If the diameter of the eye lens be made larger, then rays passing through the outer part of eye-lens would not enter the pupil of the eye. This diminishes the field of view of the instrument. So in order to increase the field of view and hence to make all the rays from the first image to pass through the eye-lens an additional lens of comparatively larger aperature known as the field-lens is to be placed in between the objective and the eye-lens. This field-lens together with the eye-lens forms an eye-piece or Ocular. Three different types of arrangements of lenses are generally made in eye-pieces.

(A) Kellner's Eye-piece.—It consists of an achromatic eye-lens and a plano- or bi-convex field lens. The focal lengths of both the eye-lens and the field-lens are equal. The separation between the two lenses is

also equal to this focal length. Since $f_1 = f_2 = f$ and $d = \frac{f_1 + f_2}{2} = f$, the lateral chromatism is avoided in this eye-piece. Distortion effects are also eliminated here. But the main disadvantage of this type of eye-piece arises due to the fact that a single speck of dust or a scratch mark on the field lens would be magnified highly and this would make the final image rather ill-defined—this lack of definition resulting partly from the light reflected successively from the inner and the outer surfaces of the lens.

(B) Huygens' Eye-piece.—It consists of two plano-convex lenses of the same kind of glass with their curved faces turned towards the object. Both the chromatic and the spherical aberrations have been ingeniously reduced to nearly a minimum value by the proper choice of the focal lengths and the separating distance of the component lenses. The focal length of the field-lens is thrice that of the eye-lens. In Fig. 178, F and E respectively indicate the field-lens and the eye-lens

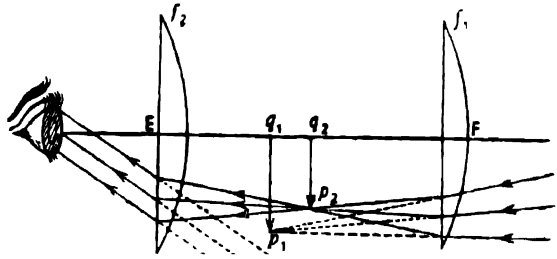


Fig. 178

of the ocular. Let f_1 and f_2 be their respective focal lengths. In order to avoid lateral chromatic aberration, the separation, d, between these two lenses are made equal to $d = \frac{f_1 + f_2}{2}$. Again for minimum spherical aberration, $d = f_1 - f_2$.

Hence, $f_1 - f_2 = \frac{f_1 + f_2}{2}$;

or, $f_1 = 3f_2$.

Thus the separating distance is to be made equal to $\frac{f_1 + f_2}{2} = 2f_2$ in order that both the aberrations are minimised. The equivalent

focal length of the ocular is given by the relation, $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{d}{f_1 f_2}$.

As both f_1 and f_2 refer to converging lenses they are negative.

$$\therefore \quad \frac{1}{F}=-\frac{1}{3f_2}-\frac{1}{f_2}+\frac{2f_2}{3f_2.f_2}=-\frac{2}{3f_2}.$$

The negative sign shows that the system acts also as a converging one The equivalent focal length being numerically equal to $F=\frac{3}{2}f_2$.

When the instrument, to which the ocular is attached, is focussed for parallel rays, the rays emerging out of the eye-lens must be parallel. So the position of the first image formed by the objective should be such that the second image formed by the field-lens is at a distance f_2 from the eye-lens (*i. e.*, in its focal plane) Thus Eq_2 must be equal to f_2. But as $EF=d=2f_2$, Fq_2 is also equal to f_2 numerically. Let $v(=Fq_1)$ be the distance of the first image formed by the objective from the field-lens. Now since $\frac{1}{f_1}=\frac{1}{v}-\frac{1}{u}$ and $f_1=-3f_2$ ($-ve$, because converging), $v=Fq_1=-f_2$, we have,

$$-\frac{1}{3f_2}=-\frac{1}{f_2}-\frac{1}{u}\text{ ; or, }\quad u=-\tfrac{3}{2}f_2=-\frac{f_1}{2}.$$

Thus the first image should be formed at p_1q_1 at a distance of $\frac{f_1}{2}$ behind the field-lens which actually intercepts the rays from the objective and finally casts its image at p_2q_2 in the focal plane of the eye-lens.

The main *advantages* of the Huygens' eye-piece are that here field of view is increased and that both the lateral chromatic aberration and the spherical aberration are reduced to a minimum. The longitudinal chromatic aberration may also be avoided by using achromatic doublets for both the eye-lens and the field-lens.

The main disadvantage of this ocular is that here the first image p_1q_1 has no real existence and the first real image that we get is formed at p_2q_2. So if at all a cross-wire is to be used for any kind of measurement it is to be placed at p_2q_2 and will, therefore, be magnified only by the eye-lens. So spherical aberration will creep in. As the cross-wire would not be magnified to the same extent by the eye-piece as the first image p_1q_1 is magnified, true measurements are impossible So generally Huygens' eye-pieces are not provided with cross-wires excepting when the magnification is sufficiently low. In this eye-piece as the first image is formed on the negative side of the field-lens it is generally known as a ***negative eye-piece***. It is used only to examine the image formed and not for any kind of work involving and measurements of angles or distances.

***The Cardinal Points of a Huygens' Eye-piece :-**

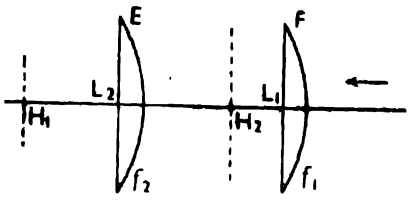


Fig. 178(a)

Let L_1 and L_2 represent the optical centres of the field-lens F and the eye-lens E respectively of focal lengths f_1 and f_2. Let H_1 and H_2 denote the first and second principal points of the eye-piece and F_1 and F_2 the corresponding principal foci. Then from Art. 69(*A*), we have,

$$L_1H_1 = \frac{1-B}{A} \text{ and } L_2H_2 = \frac{C-1}{A},$$

$$F_1H_1 = -\frac{1}{A} \text{ and } F_2H_2 = +\frac{1}{A}$$

where $A = dk_1k_2 + k_1 + k_2$, $B = dk_2 + 1$, $C = dk_1 + 1$

and $k_1 = \frac{1}{f_1}$, $k_2 = \frac{1}{f_2}$, d = lens separation.

For a Huygens' eye-piece, $f_1 = 3f_2$, (both lenses being of $-$ve focal length).

$$d = (f_1 + f_2)/2 = 2f_2 \quad \text{and} \quad A = dk_1k_2 + k_1 + k_2$$

$$= 2f_2 . \left(-\frac{1}{3f_2}\right)\left(-\frac{1}{f_2}\right) - \frac{1}{3f_2} - \frac{1}{f_2} = -\frac{2}{3f_2}.$$

$$\therefore \quad L_1H_1 = \frac{-dk_2}{A} = \frac{-2f_2 . \left(-\frac{1}{f_2}\right)}{-\frac{2}{3f_2}} = -3f_2 ;$$

$$L_2H_2 = \frac{dk_1}{A} = 2f_2 . \left(-\frac{1}{3f_2} \Big/ -\frac{2}{3f_2}\right) = f_2 ;$$

$$F_1H_1 = -\frac{1}{A} = \frac{3}{2}f_2 \text{ and } F_2H_2 = \frac{1}{A} = -\frac{3}{2}f_2.$$

As the system is placed in air, the nodal points coincide with principal points.

(c) Ramsden's Eye-piece :—This eye-piece consists of two plano-convex lenses of the same focal length with their convex surfaces turned towards each other. The separation between the lenses is equal to two-thirds of the focal length of each of these component

In Fig. 179, F and E respectively represent the field-lens and the eye-lens of a Ramsden eye-piece. In this eye-piece neither the spherical nor the chromatic aberration is reduced to a minimum possible value. For lateral achromatism the separation should be $\frac{f+f}{2}=f$, where f is the focal length of each of the two lenses. But as this arrangement makes the final image blurred due to presence of scratch-marks or dust particles present in the field lens (*cf.* Kellner eye-piece) which become highly magnified when the eye-piece is focussed

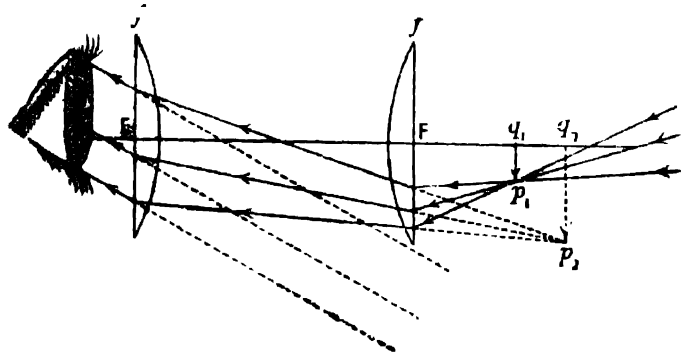


Fig. 179

for parallel rays, the separation has intentionally been reduced to $\frac{2}{3}f$. Thus chromatic aberration is not completely eliminated in this type of oculars. Again the condition of minimum spherical aberration, namely $d=f_1-f_2$, is also not satisfied in this eye-piece. But inspite of these defects Ramsden eye-pieces are extensively used in optical instruments whenever actual measurements of distances or angles are involved. This is because in this type of eye-piece, a cross-wire or a reference scale can be used for the actual measurement of the image formed by the objective.

When focussed for parallel rays, the second image cast by the field-lens must lie in the focal plane of the eye-lens. Let p_2q_2 be the position of this second image.

Then $Eq_2=f$. But $EF=\frac{2}{3}f$. $\therefore$ $Fq_2=f-\frac{2}{3}f=\frac{1}{3}f$. But p_2q_2 is the image of the first image p_1q_1 and it is formed by the field-lens. If the position of the first image be at a distance $u\,(=Fq_1)$ from the field-lens, then $-\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=\frac{3}{f}-\frac{1}{u}$ or, $u=\frac{f}{4}$. Thus the first image formed by the objective should be in front of the field-lens at a distance 1/4 from the lens. As this first image p_1q_1 is real and lies in front of the eye-piece a *cross-wire* or a *scale etched on a glass plate* can be conveniently placed there. This cross-wire or scale will be magnified by

both the lenses of the eye-piece exactly to the same extent as p_1q_1. Thus an accurate measurement is possible with such an eye-piece. As the first image is on the positive side of the field-lens, this eye-piece is called a *positive eye-piece*. The equivalent focal length of this eye-piece will be given by the relation,

$$\frac{1}{F} = -\frac{1}{f} - \frac{1}{f} + \frac{\frac{3}{4}f}{f.f} = -\frac{2}{f} + \frac{2}{3f} = -\frac{4}{3f}$$

$$\therefore \quad F = -\frac{3}{4}f.$$

The main advantages of this ocular are (*i*) that here the field of view is increased and the illumination of the image is made uniform by putting a ring-stop (fitted with cross-wire) at the focal plane of the objective, which cuts off the ragged edge of the field of view; and (*ii*) that as a cross-wire or a scale can be fitted at the position of the image formed by the objective, accurate measurements are possible. The disadvantages of this eye-piece lie in the facts that neither the spherical aberration is reduced to a minimum nor the condition of achromatism is satisfied here.

*Cardinal points of Ramsden's eye-piece :—

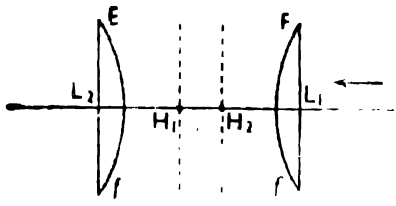


Fig 179 (*a*)

Let L_1 and L_2 represent the optical centres of the field-lens F and the eye-lens E. Let H_1, H_2 and F_1, F_2 respectively denote the first and second principal points and the first and second principal foci. From Art. 89(*A*), we have,

$$L_1H_1 = \frac{1-B}{A}; \quad L_2H_2 = \frac{C-1}{A}$$

$$F_1H_1 = -\frac{1}{A} \text{ and } F_2H_2 = \frac{1}{A}$$

where A, B, C have their usual notations as in Art. 89(*A*).

For a Ramsden's eye-piece, $f_1 = f_2 = f$ ($-$ ve, because convergent),

$$d = \frac{2}{3}f; \text{ and } A = \frac{2}{3}f.\left(-\frac{1}{f}\right)\left(-\frac{1}{f}\right) - \frac{1}{f} - \frac{1}{f} = -\frac{4}{3f}$$

$$\therefore \quad L_1H_1 = -\frac{dk_2}{A} = -\frac{2}{3}f.\left(-\frac{1}{f}\right)\Big/\left(-\frac{4}{3f}\right) = -\frac{f}{2},$$

$$L_2H_2 = \frac{dk_1}{A} = \frac{\frac{2}{3}f.\left(-\frac{1}{f}\right)}{-4/3f} = \frac{f}{2}.$$

$$F_1H_1 = -\frac{1}{A} = 3f/4 \text{ and } F_2H_2 = \frac{1}{A} = -\frac{3f}{4}$$

Here also the nodal and principal points are coincident.

116. The Principle of a Compound Microscope :—This instrument is used for producing a much greater magnification than what is possible with a simple microscope.

It consists of two converging lens-systems O and O' (Fig. 180) placed coaxially in a tube at a distance from each other. The lens-system O is turned towards the object and is called the **objective**, while the system

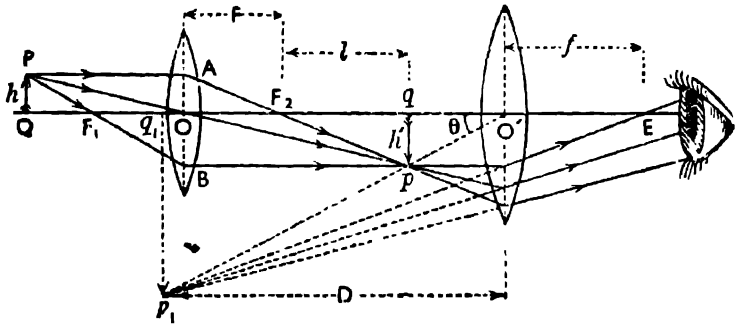


Fig. 180—The Compound Microscope.

O' behind which the eye of the observer is placed is called the **eye-piece**. The objective is of short focal length and short aperture. The eye-piece is also of short focal length (greater than that of the eye-piece) but of wider aperture. It can be slided within the main tube whereby its distance from the objective can be altered. The microscope tube is fitted on a slider by which it can be moved as a whole towards or away from an object (PQ).

In an experiment, the microscope tube is so adjusted that the object PQ is placed just beyond the principal focus (F_1) of the objective which thus produces a real, inverted and magnified image pq of the object. The eye-piece O' is then so adjusted that the image pq formed by the objective falls within the focal length of the eye-piece and the final image p_1q_1, virtual, magnified and inverted (erect with respect to pq but inverted with respect to the object PQ) is formed at the least distance of distinct vision (D) for an eye placed just behind the eye-piece.

117. A High Power Microscope :—As has been referred to in the above simple principle of a compound microscope, both the objective and the eye-piece comprise a number of lenses and as such the above simplified diagram of a compound microscope is to be modified. In Fig. 181 the formation of the image of an object in *a high power microscope* is shown.

The **objective** of a microscope of high magnifying power and high resolving power should be of a very short focal length. As in a microscope the object is placed just outside the principal focus of the objective,

the object distance is also very small. So the rays incident on the objective from the object will have large inclinations and this will make the spherical aberration very prominent. This is, however, avoided in an Abbe's immersion objective wherein the properties of the aplanatic foci of a transparent sphere are taken help of.

The objective consists of a hemispherical lens (H), a concavo-convex lens (L_1) and an achromatic lens system (L_2). In some forms of objective

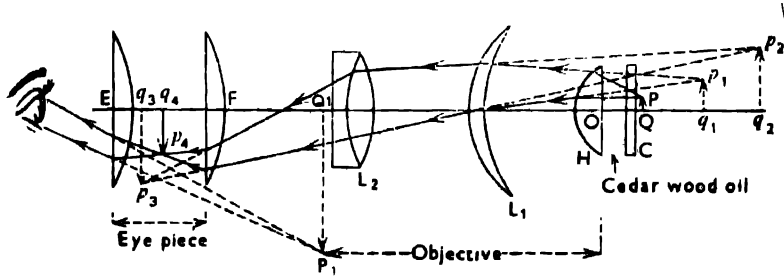


Fig. 181

seven to ten lenses are used in order to reduce the various types of aberrations. The object PQ is kept between two cover glasses one of which is shown as C in the figure and is then placed on the stage of the microscope. The space between the cover glass and the plane face of the hemispherical lens is filled with cedar-wood oil whose refractive index is equal to that the glass of the hemispherical lens. The objective described here is known as an *oil-immersion objective* and the system of lenses used in order to eliminate the chromatic errors is known as the apochromat.

The eye-piece of a high power microscope is of the Huygens' type. The field-lens F and the eye-lens E are both plano-convex with their convex sides turned towards the objective. The lens F has three times the focal length of the lens E and their separation is twice this focal length. So the conditions of minimum spherical aberration and of minimum lateral chromatic aberration are satisfied in this eye-piece. But no cross-wire can be used in this eye-piece. So whenever any measurements are to be made, as with ordinary laboratory travelling microscopes, a Ramsden's eye-piece is to be used because here cross-wires can be fitted.

Besides the objective and the eye-piece, a good microscope must be fitted with a *condenser* below its stage. It consists of a number of convergent lenses which concentrate the light from a strong source on to the object through a hole in the stage. This increases the divergence of the rays from the object falling on the objective. Thus the maximum advantage of the large numerical aperture (see Physical Optics) is secured by the condensing arrangements.

In Fig. 181, PQ is the object put between two cover glasses and placed on the stage of the microscope. From below the stage light from a strong source is concentrated on PQ with the help of the condenser. As in between C and H there is cedar-wood oil whose refractive index is the same as the material of H, Q may be supposed to be a point within a transparent sphere of radius equal to the radius of curvature of the curved surface of H. Now adjusting the position of H, Q is placed at a distance $\frac{r}{\mu}$ from the centre O of the hemisphere H. So its image will be formed at the other aplanatic point q_1 at a distance μr from O. Thus a virtual image is formed at p_1q_1 due to refraction at H. Again q_1 is at the centre of curvature of the front surface of L_1 and hence rays enter this lens normally and fall on the back surface of L_1. The lens L_1 is so chosen that q_1 is at the first aplanatic point of the glass sphere formed by this surface. Its image q_2 will be formed at the second aplanatic point and there will be no spherical aberration and coma in the image so formed. Now as PQ is apparently shifted to a sufficient distance at p_2q_2, the inclinations of the rays are considerably reduced. The achromatic lens system L_2 (also so designed as to have a minimum spherical aberration) then tends to cast an image of p_2q_2 at p_3q_3. But before this image is actually formed, the field-lens F of the Huygens' eye-piece intercepts the rays and forms a real and inverted image at p_4q_4. This image p_4q_4 is then finally magnified by the eye-lens E and the final magnified and virtual image is formed at P_1Q_1 which is inverted with respect to PQ.

In an actual high power biological microscope (Fig. 182) the eye-piece E and the objective O are fitted to draw-tubes sliding in a coaxial hollow metal tube. So the distance between the objective and the eye-piece can be adjusted. Usually two or three objectives of different magnifications are supplied in a good microscope, any one of which can be used according to necessity. In order to make the highly magnified image more intense there is an arrangement to illuminate the object strongly by light reflected from a mirror R and concentrated by the condenser C both of which are placed below the stage S on which the object is placed.

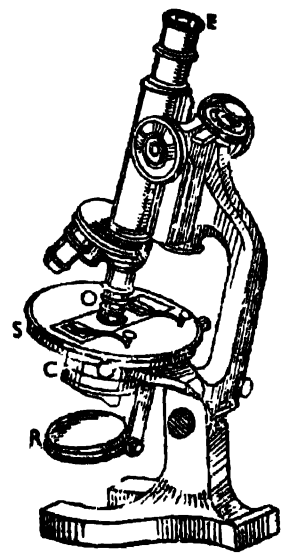


Fig. 182—A Compound Microscope.

118. Magnifying Power:—Magnification by the object-glass (Fig. 180): $m_1 = \frac{rq}{PQ} = \frac{v}{u}$ (without considering signs). The second magnification is due to the eye-piece and is obtained as in the case of a simple microscope by the following way.

$\frac{p_1q_1}{pq} = m_2 = 1 + \frac{D}{f}$ (where f is the focal length of the eye-piece).

$\therefore$ The total magnification, $m = \frac{p_1q_1}{PQ} = \frac{p_1q_1}{pq} \times \frac{pq}{PQ} = m_1 \times m_2 = \left(1 + \frac{D}{f}\right)\frac{v}{u}$.

Conditions for Large Magnifying Power:—The equation for magnification shows that m is large when,

(i) u is small, which means that the focal length of the objective should be small; (ii) f is small, which means that the focal length of the eye-piece should also be small; (iii) v is large, which means that the eye-piece and the objective should be separated as far as possible, *i.e.* the tube length should be large.

Note:—Alternative method for magnifying power (or normal adjustment). In Fig. 180, the ray PA is incident on the objective in a direction parallel to the principal axis and hence after refraction passes through the second principal focus F_2 and the ray PB passing through the first focus F_1, after refraction runs parallel to the axis and after second refraction through the eye-piece passes through its focus E. Let h be the height of the object and h' that of the first image. Let $l = F_2q$ and F and f be the focal lengths of the objective and the eye-piece.

The visual angle subtended by the image at infinity (*i.e.* for normal adjustment) $= \theta = \frac{h'}{f}$.

From similar $\triangle s$, AOF_2 and pqF_2, $\frac{h}{F} = \frac{h'}{l}$ or, $h' = \frac{lh}{F}$

$$\therefore \quad \theta = \frac{lh}{Ff}.$$

Again the visual angle subtended by the object at the least distance of distinct vision (D) is $\theta' = \frac{h}{D}$.

$$M = \frac{\theta}{\theta'} = \frac{lh}{Ff} \Big/ \frac{h}{D} = \frac{Dl}{Ff}.$$

Since F and f are small in a microscope, l is very nearly equal to the tube length and is known as the optical tube length. In case when the image is formed at the least distance of distinct vision, f in the formula should be replaced by x, the distance of the first image from the eye-piece and l should be the distance between the focus of the objective and the first image.

Examples

1. The focal length of the object-glass of a microscope is $\frac{1}{2}$ an inch, that of the eye-piece is 1 inch. Taking the least distance of distinct vision to be 12 inches, find the distance between the object-glass and the eye-piece when the object viewed is $\frac{3}{4}$ of an inch from the object-glass.

Ans. Objective . $u' = +\frac{3}{4}$, $f = -\frac{1}{2}$, $v' = ?$. Then, $\frac{1}{v'} - \frac{4}{3} = -2$, or, $v' = -\frac{3}{2}$ in.

Eye-piece : $v = +12$, $f = -1$; $u = ?$, Then, $\frac{1}{12} - \frac{1}{u} = -1$; or, $u = \frac{12}{13}$ in.

$\therefore$ The distance between the objective and eye-piece $= v' + u = \frac{3}{2} + \frac{12}{13} = 2\frac{11}{26}$ in.

2. The objective of a compound microscope has a focal length of half an inch and is placed at a distance of $2\frac{1}{2}$ inches from the eye-piece. What must be the focal length of the eye-piece if the final image of an object, placed $\frac{3}{4}$ inch from the object-glass is formed at the least distance of distinct vision, which is 10 inches?

Ans. Objective : $u = +\frac{3}{4}$, $v = ?$, $f = -\frac{1}{2}$. Then, $\frac{1}{v} - \frac{4}{3} = -2$; whence $v = -\frac{3}{2}$ in.

i.e. the image is at a distance of $1\frac{1}{2}''$ behind the objective.

$\therefore$ The object distance u' of the eye-piece $= \left(2\frac{1}{2} - 1\frac{1}{2}\right) = 1''$

Eye-piece : $u' = 1''$, $v = 10''$; $f = ?$, Then $\frac{1}{10} - 1 = \frac{1}{f}$, whence $f = -\frac{10}{9} = -1\frac{1}{9}''$.

3. A microscope comprises an objective of focal length 0·5 in. and an eye-piece of focal length 2 in. The separation of the two lenses is 6 in. Calculate the magnifying powers (i) when the final image is at infinity and (ii) when it is formed at the least distance of distinct vision (10 in.).

Ans. (*i*) In this case the optical length of the tube,

l = separation of the foci of the objective and the eye-piece

$= 6 - (2 + 0{\cdot}5) = 3{\cdot}5''$.

$\therefore$ $M = \frac{D\,l}{F\,f} = \frac{10 \times 3{\cdot}5}{0{\cdot}5 \times 2} = 35$.

(*ii*) Here since the final image is formed at the least distance of distinct vision and it is on the same side of the eye-piece as the first image cast by the objective lies, $v = +10$ in. If u be the distance of the first image from the eye-piece, we have,

$-\frac{1}{2} = \frac{1}{10} - \frac{1}{u}$; or, $\frac{1}{u} = \frac{6}{10} = \frac{3}{5}$. $\therefore$ $u = \frac{5''}{3}$.

The optical length of the tube in this case is l = distance between the focus of the objective and the first image

$$= 6 - \left(\frac{1}{2} + \frac{5}{3}\right) = 6 - \frac{13}{6} = \frac{23}{6}.$$

Here f should be replaced by $u = \frac{5''}{3}$ $\therefore$ $M = \frac{D\,l}{F u} = \frac{10 \times \frac{23}{6}}{\frac{1}{2} \times \frac{5}{3}} = 46.$

119. Other Types of Microscope :—

(a) **Reflecting Microscope.**—Fig. 183 shows the arrangements of the condenser and the objective of a reflecting microscope. The specimen S is placed on a cover glass and it is illuminated by strong light with the help of the condenser C. The rays diverging at large angles from S are reflected from the large concave aspherical reflector R on the small aspherical convex reflector r. From this small reflector (r) the rays pass through a circular hole in S and are ultimately received by the eye-piece placed behind R.

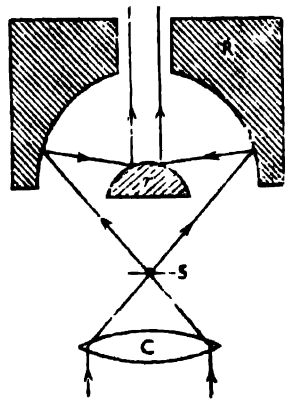


Fig. 183

Because of the large divergence of the pencil starting from S, the numerical aperture and hence the resolving power of the instrument is sufficiently increased. Moreover the aspherical nature of the reflectors indicates that the spherical aberration is almost eliminated in the system. The chromatic aberration does not creep in because no refracting systems are used in this type of objective. As dispersion and absorption does not practically occur in R and r, ultraviolet rays can very well be used thereby increasing the resolving power of the instrument. But in that case the eye-piece is to be replaced by a photographic arrangement.

(b) **Ultra-Microscope.**—In high-power microscopes the light from the condenser directly enters the microscope objective and thereby makes the field of view sufficiently bright. So very small objects are not very well defined due to lack of contrast. In order to improve this contrast Siedentopf and Zsigmondy used a bright beam at right angles to the axis of the microscope and the space below the sample was made dark by using a stop. By this method very minute particles like those exhibiting Brownian motion were detected. This arrangement is known as an *ultra-microscope arrange-*

men' with a *dark ground* illumination. In this arrangement no light directly can enter the microscope and only the scattered light from the specimen is received by its objective. Now-a-days instead of an original transverse illumination device, various types of condensers have been devised whereby the specimen is strongly illuminated but the illuminating beam gets totally reflected from the cover glass and is thus debarred from entering the objective. Fig. 184 shows such a type of condenser. It is known as *Zeiss paraboloidal condenser*. This condenser *C* consists of the glass block the upper and lower surfaces of which are plane. The curved surface is generated by the revolution of a parabola. The sample *S* is placed at the focus of the parabola on a glass platform and covered by a cover-glass. The space between *S* and *C* is filled with convenient oil of the same refractive index as glass. *D* is a stop placed below *C* to provide for the dark ground illumination. The rays from a strong source incident on the curved surface of the condenser are concentrated on *S* and are then totally reflected back to the medium below *S*. The condenser is free from chromatic and spherical aberrations. The objective which is placed just beyond *S* thus receives only the light scattered from *S* and none from the condenser. So the contrast of the minute objects is sharp and they can be easily detected and studied by this arrangement.

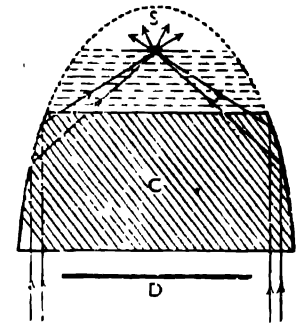


Fig 184

120 Telescopes :—Telescopes are instruments used for viewing distant objects. Because of the distance from the observer, the apparent size of the object becomes small and the function of a telescope is to increase the visual angle of the final image without altering the accommodation of the eye.

Telescopes may be of two classes : (1) Refracting and (2) Reflecting : The distinctive feature between them is that in a refracting telescope or refractor, as it is commonly called, the objective consists of a converging system of lenses of large focal length and large aperture, while in a reflecting telescope, the objective is generally a paraboloidal mirror.

Refracting telescopes are mainly of two kinds : (1) Astronomical and (2) Galilean.

121. The principle of an Astronomical Telescope :—The principle of the refracting type of an astronomical telescope is illustrated in Fig. 185. It consists of two systems of converging lenses mounted

co-axially in a tube. The objective O has a large aperture and a large focal length and the eye-piece E has a short focal length. The objective has to collect sufficient light from the distant object and so its aperture must be sufficiently large The eye-piece should also be so large as to receive all the rays coming from the objective but should never be very large, for in that case the rays passing through E may not be fully

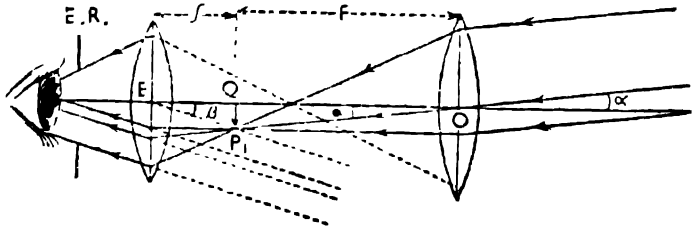


Fig 185

received by the eye placed just behind E. As telescopes are meant for seeing distant objects, the rays incident on the objective may be considered to be parallel. So the first image P_1Q_1 is formed at the focal plane of O and is real, inverted with respect to the original object and highly diminished in size. This first image is finally magnified by the eye-piece. When the telescope is to be *focussed for infinity*, the position of the eye-piece E is adjusted such that P_1Q_1 is at its focal plane. So the rays emerging from E will be all parallel and the eye placed behind E sees the final image at infinity, which is magnified, virtual and erect with respect to the first image. The instrument is now said to have been adjusted for *normal vision*. In this case $OQ_1 = F$, the focal length of the objective and $EQ_1 = f$, the focal length of the eye-piece. So for the normal adjustment the tube length of the astronomical telescope should be $(F+f)$.

If instead of focussing the instrument for infinity, a distinct image of the object is to be seen at the least distance of distinct vision, the eye-piece is slightly pushed towards the objective so that P_1Q_1 is within its focal length. A virtual, magnified image is formed at the least distance of distinct vision which is inverted with respect to the object.

When the object is situated at a finite distance, the first image is formed beyond the focal plane of O and hence for a clear view of the final image, the eye-piece is to be pulled slightly away from the objective.

As has been referred to above the eye-piece should be of such an aperture that it can receive all the rays from the objective and

han in its turn concentrate all these rays on to the eye. Actually the eye-piece E casts an image of the rim-aperture of the objective at a point slightly behind E. The aperture of E should be properly chosen so that this image of the rim-aperture is as large as the pupil of the eye. This make the image brightest. This image of the rim-aperture of the objective as formed by the eye-piece is known as the *exit pupil* of the instrument. At the position of this image a stop with a hole at the centre is to be placed. This is called the *eye-ring* (ER) whose diameter is the same as that of the image of the rim-aperture. The actual aperture of the rim in which the objective is fitted is known as the *entrance pupil*

Magnifying power —Here the magnifying power is defined as the ratio of the angle subtended by the image at the eye to the angle subtended by the object at the eye. As the object is at infinity the angle subtended by it at the objective is practically the same as that subtended at the eye. So as is shown in Fig. 185, α and β respectively are the angles subtended by the object and the image at the eye.

$$\therefore \text{ Magnifying Power, } m = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} = \frac{P_1Q_1/EQ_1}{P_1Q_1/OQ_1} = \frac{OQ_1}{EQ_1} \quad \ldots \quad (2)$$

But $OQ_1 = F$, the focal length of the objective and when the instrument is focussed for infinity $EQ_1 = f$, the focal length of the eye-piece.

$\therefore$ For the normal adjustment, the magnifying power or the angular magnification, $m = \frac{F}{f}$. ... ... (2)

If, however, the instrument is adjusted so as to form the final image at the least distance of distinct vision D, then,

$$-\frac{1}{f} = \frac{1}{D} - \frac{1}{EQ_1} \text{ ; or, } \frac{1}{EQ_1} = \frac{1}{D} + \frac{1}{f} = \left(\frac{D+f}{Df}\right).$$

$$\text{From (1), } m = F\frac{(D+f)}{Df} = \frac{F}{f}\left(1 + \frac{f}{D}\right).$$

122. (a) **The construction of an actual astronomical telescope fitted with a Huygens' eye-piece** :—In an actual astronomical telescope, the objective as well as the eye-piece consists of a number of lenses conveniently arranged so as to form a converging system having minimum spherical and chromatic aberrations. The objective O consists of a converging combination of two lenses in contact, one being a double convex lens of crown glass and the other a plano-concave lens of flint glass. The convex surface of the combination is turned towards the incident beam (*i.e.* towards the object) and this arrangement considerably reduces the spherical aberration. But as parallel rays are incident on the objective, this aberration is neither very predominant. It is the chromatic aberration that is to be more carefully avoided in a telescope

objective. Hence the above combination is so constructed that it becomes an achromatic doublet satisfying the condition, $\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}=0$.

Since the limit of resolution of the system forming the objective, is given by $\frac{1{\cdot}22\lambda}{D}$, where λ is the wavelength used and D is the diameter of the lens aperture, the greater the aperture of the objective the smaller will be the distance between two points which can be distinctly identified by the instrument.

The eye-piece is a Huygens' eye-piece consisting of the field-lens F and the eye-lens E of the same glass. Both these lenses are plano-convex with their convex surface turned towards the object. The focal length of the eye-lens E is one-third the focal length of the field-lens F and their separation is equal to twice the focal length of E. Thus the conditions of both minimum spherical and chromatic aberrations are satisfied in this eye-piece (See Huygens' eye-piece).

Fig. 186, shows the construction of an astronomical telescope and the formation of the image when a Huygens' eye-piece is fitted to it. Parallel rays from a distant object falls on O and the first real, inverted and diminished image pq would have been formed at the focal plane of O. The field-lens F of the eye-piece, however, intercepts the rays and forms a real image at p_1q_1 also inverted with respect to the object such that Eq_1 is equal to the focal length of the eye-lens. So the rays emerging out of E will be all parallel and the final image is formed at infinity

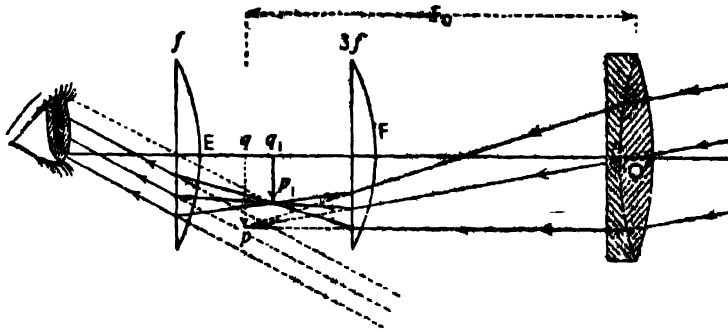


Fig. 186

This is the normal adjustment of the telescope. If the final image is to be formed at the least distance of distinct vision the position of the eye-piece is adjusted in such a way that the image p_1q_1 is slightly within the focal length of the eye-lens and a virtual magnified image inverted with respect to the original object is formed at the least distance of

distinct vision. Since in the Huygens' eye-piece no cross-wire can be fitted, it is not suitable for the purpose of measurement or comparison. In that case a Ramsden's eye-piece is to be used in the instrument. This eye-piece may be fitted with a cross-wire or a reference scale for actual measurements. The diagram tracing the rays in an astronomical telescope fitted with a Ramsden's eye-piece will be almost similar to above. Here the first image q, will be actually formed outside the eye-piece (as shown in Fig. 179 of Ramsden's eye-piece). The remaining portion of the drawing can also be done by the student exactly in the same way as given in Fig. 186. Since in a telescope fitted with a Ramsden's eye-piece a cross-wire or a scale can be placed in the focal plane of the objective, the *angular distance between two stars* can be easily measured with such a telescope. If the images of the two stars cast in the focal plane of the objective be A and B, the angle AOB subtended by them at the objective is the same as the angular distance between the stars. So measuring the distance AB with the help of the scale fitted in the eye-piece and knowing the focal length F of the objective we get, $\angle AOB = AB/F$ (approx.).

*(b) **Telescope objectives** :—In the design of an objective of a refracting telescope, the first consideration is to be paid to its focal length. The next important consideration is the achromatism of the objective. If two lenses separated by a distance be used for the objective, the longitudinal as well as lateral chromatism cannot be simultaneously removed. So the two lenses forming the objective must be placed as close as possible and their focal lengths and dispersive powers are to be properly chosen. Again as the field of view of a telescope in the image space as seen by the observer is rarely greater than 35° or 40° the corresponding field in the object space is less in proportion to its magnifying power. For an objective with $m = 35$ or 40, the angular field of view in the object space is only about 1°, *i.e.* only 0·5° on either side of the axis. With higher values of m, this is still less. For this reason, astigmatism, curvature, and distortion are of little consequence in telescope objectives, but they cannot be eliminated. In terrestrial telescopes they may be somewhat significant but in astronomical types their effects may very well be neglected. So the aberrations that count here are :—(*a*) Spherical aberration, (*b*) Coma and (*c*) Chromatic aberrations—both lateral and axial.

Of all these aberrations, the axial chromatic one is the most difficult to correct satisfactorily. In general, the corrections for the above errors can be secured in small telescopes with a doublet consisting of a double convex crown lens and a plano-concave flint lens. The combination may be made achromatic for two wavelengths λ_1 and λ_2 (C and F lines generally) according to relation $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$, (1) where ω_1 and ω_2 are the partial dispersive powers of the lens materials for these two wavelengths. The values of the two focal

lengths f_1 and f_2 are computed from the relations, (1) above and (2) given below as, $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$, ...(2), where F is the equivalent focal length of the doublet.

The combination so constructed is achromatic for λ_1 and λ_2 but the condition of achromatism slowly changes with λ. So a slight colour effect that continues is of little consequence. The sizes of the images for λ_1 and λ_2 are now the same but their positions on the axis are not so, because the principal points of the system are different for different colours. This axial chromatism increases with the thickness of the combination and may be reduced by means of the eye-piece which has to be under-corrected for axial chromatism by changing the distance between the field and eye-lenses. The objective may be achromatised for three wavelengths by using three lenses of different materials so as to satisfy the relations :

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}\ ;\ \frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}+\frac{\omega_3}{f_3}=0,$$

and $$\frac{\omega_1'}{f_1}+\frac{\omega_2'}{f_2}+\frac{\omega_3'}{f_3}=0.$$

Here $\omega_1, \omega_2, \omega_3$ refer to partial dispersive powers for λ_1 and λ_2 and $\omega_1', \omega_2', \omega_3'$ to those for λ_2 and λ_3.

For a doublet for which F has been fixed, the spherical aberration may be removed by properly choosing the four variables r_1, r_2, r_3 and r_4 (*i.e.* radii of curvature) of the lenses. When two lenses are in contact r_2 and r_3 are made identical to avoid total reflection at the interface and we have only three variables at our disposal and they can be evaluated from the relations :

$$\sigma \text{ (shape factor)}=\frac{r_1}{r_4}=-\frac{1}{6}\ ;\ \text{(for minimum spherical aberration)}$$

$$\frac{1}{f_1}=(\mu_1-1)\left(\frac{1}{r_1}-\frac{1}{r}\right) \text{ and } \frac{1}{f_2}=(\mu_2-1)\left(\frac{1}{r}-\frac{1}{r_4}\right),$$

where $r_2=r_3=r$ and mean μ for lens material is 1·5. As plano-convex lenses have spherical aberration very near the minimum value, plano-convex combinations are generally used for convenience of construction. This makes the spherical aberration very near the minimum and since one of the component lens is convex and the other concave (plano-concave) in nature, the chromatic error can also be minimised. In small aperture objectives the errors are thus minimised to the possible extent. But in the case of an objective of large diameter, the lenses cannot be cemented together as the cementing would crack due to unequal expansion of the lens material. Hence here r_2 cannot be made equal to r_3 and we shall have to utilise all the four variables r_1, r_2, r_3, r_4 for correcting spherical aberrations and coma from suitable relations.

In ordinary telescopes of small size, the common type of objective

consists of plano-concave flint lens cemented to a double convex crown lens. Fraunhoffer's objectives are also used in some telescopes. It consists of a double convex crown lens and a flint concave meniscus lens separated by a small distance. Such an objective is free from primary chromatic aberration. It is practically free from coma but the spherical aberration is corrected only for one zone.

In reflecting telescopes chromatic aberration is absent. Here by using paraboloidal mirrors, spherical aberration is reduced practically to zero at least for the parallel rays.

123. Field of view of an Astronomical Telescope :—Let O and E respectively represent the objective and the eye-piece of the telescope (Fig. 187). Let PQ be the object plane where Q_1Q_1 limits the field of view of uniform illumination. The rays from Q_1Q_1 that enter the objective through the marginal region of the rim-aperture of the objective are just allowed to reach the eye-piece. Thus the eye-piece gets the full advantage of the field of view. The ragged edge of the

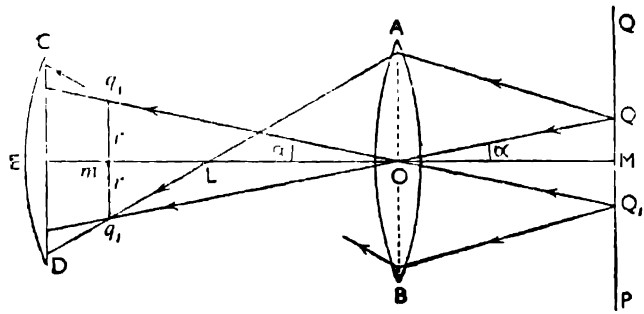


Fig 187

field is cut off by another stop-aperture at q_1q_1 where the image formed by the objective is cast. The size of this stop-aperture is also q_1q_1.

Evidently the angular *field of view* is determined by the angle $Q_1CQ_1 = 2\alpha$, where α is known as the angular radius of the field of uniform illumination. If F and f be the focal lengths of O and E respectively and if $q_1m = r$, then $\alpha = \frac{r}{F}$ ... (1)

Now to find r, let us take Y_o and Y_e as the semi-aperture of the objective and the eye-piece and consider the similar $\triangle$s mLq_1 and OLA.

Here, $\frac{r}{mL} = \frac{Y_o}{LO} = \frac{Y_o + r}{mL + LO} = \frac{Y_o + r}{mO} = \frac{Y_o + r}{F}$.

Similarly from $\triangle$s mLq_1 and ELD, $\frac{r}{mL} = \frac{Y_e}{EL} = \frac{Y_e - r}{f}$.

$\therefore \quad \frac{Y_o + r}{F} = \frac{Y_e - r}{f}$. Hence we get, $r = \frac{Y_eF - Y_of}{F + f}$

As in actual telescopes $F>>f$, the term $Y_e f << Y_e F$.

$$\therefore \quad r = \frac{Y_e F}{F+f}.$$

So the angular radius of the field of view of uniform illumination is

$$\frac{r}{F} = \frac{Y_e}{F+f} = \frac{\delta}{2(F+f)}$$ where δ is the linear aperture of the eye-piece.

124. Determination of the magnifying power of a telescope in terms of diameters of entrance and exit pupils :—

The magnifying power of a telescope can be conveniently measured in the laboratory by directly measuring the entrance pupil and the exit pupil of telescope. In Fig. 188, the diameter AB of the objective O evidently represents the entrance pupil. The image ab of AB as is cast by the eye-piece E represents the exit pupil. When the instrument is focussed for infinity the distance $OE = F+f$, where F and f respectively represent focal lengths of the objective and the eye-piece.

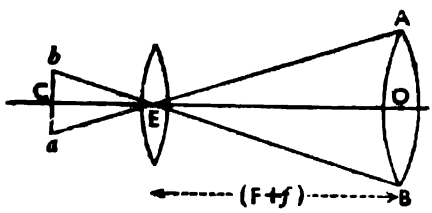


Fig. 188

Now, $AB/ab = OE/CE$...(1). Again as ab is the image of AB formed by E, we have, $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ (converging lens) ;

or, $\frac{1}{f} = \frac{1}{CE} + \frac{1}{OE} = \frac{1}{CE} + \frac{1}{F+f}$; or, $CE = \frac{f(F+f)}{F}$.

$$\therefore \quad AB/ab = OE/CE = (F+f) \Big/ \frac{f(F+f)}{F} = \frac{F}{f} = \text{magnifying power.}$$

Thus the ratio of the diameter of the entrance pupil and that of the exit pupil is the magnifying power of the telescope.

To determine the magnifying power, the telescope is first focussed for infinity. The objective is then replaced by a rectangular slit whose length is the same as that of the lens aperture. The length of the image of this slit as cast by the eye-piece is accurately measured by a travelling microscope. The length of the slit is then directly determined with the help of a divider and a diagonal scale. The ratio of the length of the slit and that of its image gives the magnifying power of the telescope.

Examples

1. In a simple form of astronomical telescope the focal length of the object glass is 30 inches and that of the eye-piece is 2 inches. Calculate the magnifying when the final image of a distant object is seen (i) a long way off, (ii) at a d of 12 inches. Find the distance between the two lenses in each case.

Ans. (i) Since the final image is seen a long way off the image due to the objective is formed at the principal focus of the eye-piece. Hence, in this case

magnifying power $= \frac{F}{f} = \frac{30}{2} = 15.$

The distance between the lenses $= 30 + 2 = 32$ inches.

(ii) In this case the distance of the final image due to the eye-piece is 12 inches We have, in the case of a lens,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{Here } v = +12, f = -2\ ;\ u = ?$$

$$\text{or,} \quad \frac{1}{12} - \frac{1}{u} = -\frac{1}{2}\ ; \text{ whence, } u = \frac{12''}{7}.$$

Hence, the magnifying power $= \frac{F}{u} = 30/\frac{12}{7} = 17{\cdot}5.$

Since, 12/7" is the distance of the image from the eye-piece, the distance between the two lenses $= 30 + 12/7 = 30 + 1{\cdot}71 = 31{\cdot}71$ inches.

2. A telescope is formed of two convex lenses of 10 cms. and 1 cm. focal length respectively. If the telescope is focussed on a scale one metre from the objective and the final image formed 25 cms. from the eye, calculate the magnification produced.

Ans. In order to form a final (virtual) image at a distance of 25 cms. the object (*i.e.* the real image formed by the objective) must be at a distance, say, u from the eye-piece, when we have,

$$\frac{1}{25} - \frac{1}{u} = -\frac{1}{1}\ ; \text{ whence, } u = \frac{25}{26} \text{ cms.}$$

Again the distance v is given by,

$$\frac{1}{v} - \frac{1}{100} = \frac{1}{10}, \text{ or, } \frac{1}{v} = -\frac{9}{100}\ ; \text{ or, } v = -\frac{100}{9} \text{ cms.}$$

$\therefore$ Total magnification, $\frac{v}{u} = \frac{100}{9} / \frac{25}{26} = 11\frac{5}{9}.$

125. The Terrestrial Telescope :—An inverted image produced by an astronomical telescope is of little use for observing terrestrial objects. For example, an erect image is essential if the instrument is to be used by navigators, surveyors, etc.

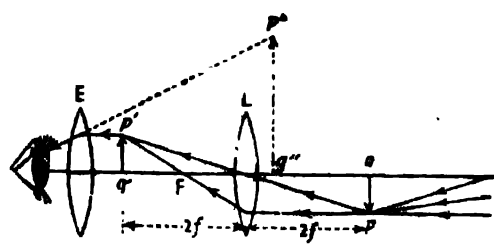


Fig. 189

This difficulty of the astronomical telescope is removed in the terrestrial telescope by the addition of a convex lens to the eye-piece so as to convert the inverted image into an erect image, though the length of the instrument is thereby inconveniently increased. Some light is

reflected and lost at each glass surface, and so the image becomes fainter by the use of the extra lens.

Due to the objective (which is not shown in the diagram), a real inverted image pq is formed (Fig. 189). A convex lens L, which is the erecting lens, is interposed between the objective and the eye-piece in such a way that the distance of the image pq from the auxiliary lens L is twice the focal length $(2f)$ of the lens L. Thus a real, inverted image $p'q'$, equal in size to pq, is formed at the same distance $2f$ on the other side of the lens L. This image being inverted with respect to pq is erect with respect to the object. The eye-piece E is so adjusted that the image $p'q'$ is formed just within its focal length and a virtual magnified image $p''q''$ is seen at the least distance of distinct vision.

Instead of one lens L, a combination of two lenses A and B (Fig. 190), called an erecting eye-piece, is also sometimes used. The first lens A of the combination is placed at a distance of its focal length from the image pq and the rays emerge from it as parallel beams and fall on B which brings them to a real focus at $p'q'$. Thus a real, erect image is obtained just within the focal length of the lens E, and a virtual, magnified image $p''q''$ is finally obtained. The lenses A and B are placed apart by a distance equal to twice their focal length.

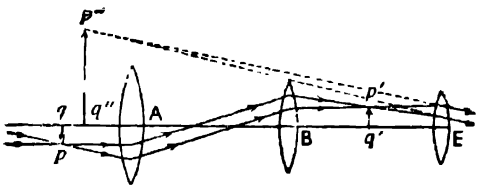


Fig. 190

126. The Galilean Telescope:—The disadvantage of the great length of a terrestrial telescope is avoided to some extent in the Galileo's telescope, which is the earliest form of a telescope. It consists of a **convergent achromatic objective** O **of large focal length** and an achromatic **concave lens** O' as the eye piece (Fig. 191). A real, inverted image $p'q'$ of the object should be formed at the focal plane of the objective, but the concave lens intercepts the rays before they reach the focus, and so the rays emerge from it as a practically parallel pencil and the eye sees a *virtual, erect* and *magnified image* $p''q''$ at infinity provided the distance $O'q'$ is equal to the focal length of the lens O'. The eye should be placed as close to the eye-piece as possible, for the rays being rendered divergent by the eye-piece the field of view diminishes as the eye recedes from it.

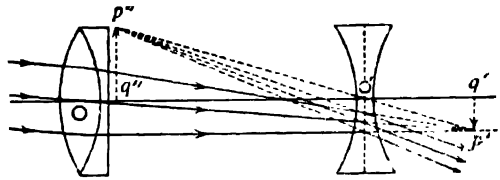


Fig. 191—The Galilean Telescope.

As the rays emerge as a parallel pencil due to the interception of the eye-piece at O'. $O'q'$ is the focal length of the eye-piece. And Oq' is the focal length of the objective O.

Magnifying Power :—As in the case of the astronomical telescope the magnification is given by,

$$m = \frac{\angle p''O'q''}{\angle p'Oq'} = \frac{\angle p'O'q'}{\angle p'Oq'} = \frac{\tan p'O'q'}{\tan p'Oq'}, \text{ (the angles being small).}$$

$$= \frac{p'q'}{O'q'} \Big/ \frac{p'q'}{Oq'} = \frac{Oq'}{O'q'}, \text{ focal length of objective / focal length of eye-piece}$$

$$= \frac{F}{f} \text{ (when the eye-piece is focussed for infinity).}$$

When the final image is formed at the least distance of distinct vision D, the magnifying power can be calculated exactly in the way followed in the case of an astronomical telescope. Here it is given by $m = \frac{F}{f}\left(1 - \frac{f}{D}\right)$.

Field of View.—In a Galilean telescope the field of view is much smaller than that of an astronomical telescope. Here as the eye-glass is a divergent lens, the exit pupil of the telescope which is the image of the objective as formed by the eye-piece is virtual and lies between the eye-glass and the objective and is as such not accessible to the eye. Hence the eye should be placed very close to the eye-glass to have the advantage of a greater field of view. Only that cone of rays, refracted by the objective and eye-glass, which fills the aperture of the pupil of the eye, limits *the field of view of uniform illumination*. The marginal rays from the eye-glass form either the ragged edge or are lost. So the effective aperture of the eye-glass should be identical with the aperture of the pupil of the eye.

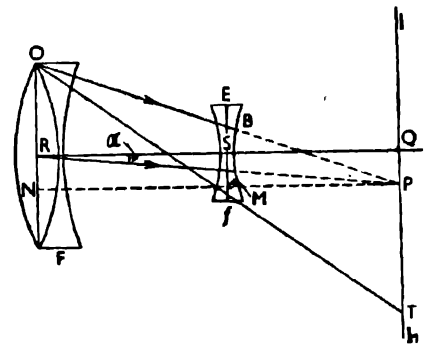


Fig. 191 (*a*)

Let O and E represent the objective and the eye-glass and II_1 the image plane of the objective in absence of the eye-glass. SB represents the effective semi-aperture of the eye-glass (or the semi-aperture of the pupil of the eye) because the marginal ray from the objective proceeds along the path OBP to give a full view of the virtual image of PQ, which is the image formed in absence of the eye-glass. Here PQ limits the field view of unifrom illumination and the angle $\alpha = \angle PRQ$ indicates the angular radius of the field of view of uniform illumination. The total field of

view, however, extends upto T in the image plane and is limited by the extreme marginal rays. (*e.g.* OT). The region PT only forms the ragged edge.

Let $SB = Y_e$ = linear semi aperture of the pupil of the eye = δ (say),

and $RO = Y_o$ = semi-aperature of the objective.

Draw PMN parallel to the telescope axis as shown in Fig. 191 (*a*),

From similar Δs PNO and PMB,

$\frac{ON}{BM} = \frac{F}{f}$ where F and f are the focal lengths of the objective and the eye-glass.

$$\therefore \quad \frac{Y_o + PQ}{Y_e + PQ} = \frac{F}{f}; \text{ or, } PQ = \frac{Y_e F - Y_o f}{F - f} = \frac{Y_e F}{F - f},$$

as $F >> f$.

Hence the angular radius of the field of view of uniform illumination is $\alpha = \frac{PQ}{F} = \frac{Y_e}{F-f} = \frac{\delta}{F-f}$.

Astronomical and Galilean Telescopes Compared.—

(*a*) Since the **focus** of the eye-piece coincides with the focus of the objective, the ***distance*** between the two lenses in a Galilean telescope is equal to the ***difference*** between their focal lengths, *i.e.* $(F-f)$; in an astronomical telescope this distance is equal to the *sum* of their focal lengths, *i.e.* $(F+f)$. A Galilean telescope is thus shorter in lengths which is a distinct advantage over the other types as it causes **less** loss of light due to absorption.

(*b*) In a Galilean telescope the image formed by the eye-piece is erect and so it is best adapted for viewing terrestrial objects. In an astronomical telescope the final image is inverted, so this type can be used only for astronomical work.

(*c*) The field of view and the magnification of an astronomical telescope are much greater than those in the Galilean type. In the Galilean telescope, the eye-piece being a divergent lens, the image is practically formed by those rays only which pass near about the centre of the lens, the marginal rays being mostly lost, and for this reason, the final image is faint, *i.e.* the field of view is limited.

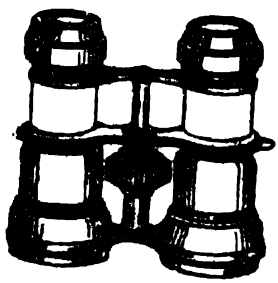


Fig. 192—Opera Glasses.

(*d*) Cross-wires cannot be fitted in the Galilean telescope, for the eye-piece being a divergent lens there is no position where the cross-wires may be placed in order that the final image formed by the eye-piece and the image of the cross-wires may be coincident in position. The focal plane of the objective is outside the instrument and no cross-wires can be placed there.

127. Opera Glasses :—The ordinary *Opera Glasses* (Fig. 192) usually consists of a pair of Galilean telescope mounted side by side with their axes parallel. The magnification in this case is small owing to the shortness of the telescope tubes. The distance between the two telescopes may be altered by adjustment to suit the two eyes of the observer.

Example

In an opera glass the focal length of the objective is 4 inches and that of the eye-piece $1\frac{1}{2}$ *in. What will be the magnifying power and also the distance between the objective and the eye-piece when focussing a distant object ?*

Ans. The magnifying power is $\frac{F}{f}$, where F is the focal length of the objective and that of the eye-piece. $\therefore$ The magnifying power $= 4 \div \frac{3}{2} = \frac{8}{3}$.

In this case, the distance between the two lenses $= F - f = 4 - \frac{3}{2} = \frac{5}{2} = 2\frac{1}{2}$ in.

128. Reflecting Telescopes :—Reflecting telescopes are principally used for studying celestial objects and for such purposes they have some definite advantages over the refracting telescopes. The main types of reflecting telescopes are described below :

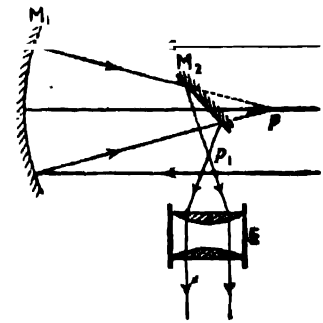


Fig. 193

(i) Newtonian Telescope :—In Newton's reflecting telescope (Fig. 193), a parallel beam of light from a distant object falls on a concave mirror M_1 made of speculum metal and after reflection a real, inverted and diminished image would have been formed at its focal plane, but before reaching that plane the rays are intercepted by a small plane mirror M_2 inclined at 45° to the axis of the instrument by which the image is shifted to a side tube where it is viewed by means of the eye-piece E. The eye-piece is so placed that the image formed by the mirror is in its focal plane and thus a virtual and magnified image is seen at infinity. The magnifying power of this telescope when focussed for infinity is also given by, $m = \frac{F}{f}$.

(ii) Cassegranian Telescope.—Here a beam of light coming from the object is received on a paraboloidal objective mirror (M_1) made of

speculum metal as in the Newtonian Telescope, but before converging at p in the focal plane of the objective, it is intercepted by a convex hyperboloidal mirror (M_2), also of the same material, whereby the image is formed at p_1, as shown in the figure (Fig. 194), and is viewed, as in the Newtonian Telescope, through an eye-piece which is fitted through a hole in the objective mirror M_1.

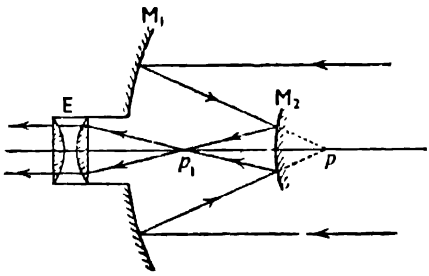


Fig. 194

The magnifying power of a Cassegranian telescope is slightly greater than F/f for normal adjustment of the instrument.

(iii) **Herschel's Telescope** :—In this type of telescope the axis of the refractor M is slightly inclined to the direction of the parallel incident rays. The image p is thrown to one side of the axis. This image is then further magnified by means of the eye-piece E (Fig. 195). When the telescope is focussed for infinity its magnification is also found to be equal to F/f.

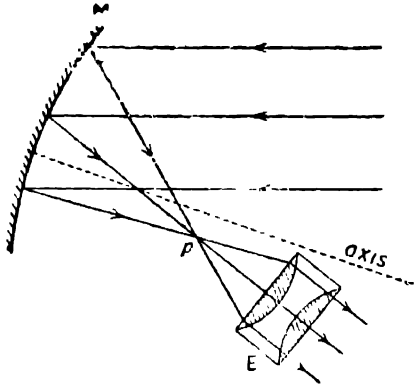


Fig. 195

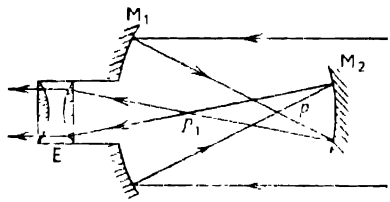


Fig. 196

(iv) **Gregory's Telescope**.—In this type of telescope, the concave reflector M_1 is pierced with a small circular aperture at the centre. The first image p due to reflection at M_1 is formed in front of a concave mirror M_2 on the axis at a distance between the centre of the curvature and the principal focus of M_2. A second image p_1 is then formed and this is finally magnified by the eye-piece E (Fig. 196).

Advantages of a Reflecting Telescope—

(*a*) In a refracting telescope, objectives of large aperture are used in order to have a large amount of light, but due to the large

aperture an appreciable portion of the light is absorbed by the lens. To avoid this, concave mirrors are used where the loss of light by reflection is much less and so the image obtained is much brighter.

(*b*) There is no chromatic aberration due to refraction, and so the image is quite sharp and free from colour defects.

(*c*) When the aperture of an objective is large, greater details of the object can be seen. It is more difficult to make large lenses used in refracting telescopes than to make reflecting mirrors of large diameters.

(*d*) By using a paraboloidal mirror spherical aberration may be prevented.

It is interesting to know about the largest telescopes, both refracting and reflecting, in the world today. The Yerkes Telescope at Lake Geneva, Wisconsin, U.S.A., is the largest refracting telescope in a tube 63 ft. long. At the Mt. Wilson Observatory, Pasadena, California, there is a 100″ reflector of focal length 42·3 ft. made of glass silvered at the front face. The largest reflector of this type is the Hale telescope set up at Mount Palomar in California where a 200″ Pyrex paraboloidal mirror coated with aluminium has been used.

129. Prism Binoculars (or Field Glasses) :—These are essentially terrestrial telescopes made in a compact form containing *two totally reflecting prisms*. It will be seen from the diagram (Fig. 197) that rays from the objective first enter a prism and are internally reflected by it. They are again internally reflected by a second prism after retraversing the length of the instrument and finally pass through an erecting eye-piece. Thus the rays from the objective travel thrice the length of the tube by means of the prisms before reaching the erecting eye-piece. That is, the distance between the objective and the eye-piece is increased without increasing the length of the instrument. So an objective of larger focal length can be used in this instrument. In the figure, the image formed by the first prism, which is placed with its refracting edge vertical, is inverted *laterally*, but not *vertically* and by the prism, which is placed with its refracting edge horizontal, *i.e.* at right angles to that of the first prism, the image is inverted *vertically*, but not *laterally*. So the final image is upright and the correct way round. Thus the purpose of the terrestrial telescope is served in a short length. This is the best form of a binocular. This instrument is termed a binocular (Lat. *bini*, two together+*oculus*, eye) as it is constructed with two telescopes one for each eye.

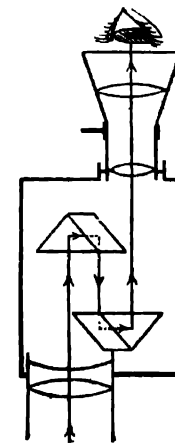


Fig. 197—The Prism Binocular

Advantages—Here by using an objective of greater focal length a greater magnification is made possible than in the case of ordinary opera glasses, and by the arrangement of prisms the objectives are placed farther apart than in opera glasses by which a wider field and an increased stereoscopic effect are obtained. Fig. 198 illustrates a modern form of prism binocular.



Fig. 198—A Prism Binocular

130. The Stereoscope:— A photograph of an object always gives a *flat* appearance because it is taken from the same angle. But, if two pictures of an object are taken simultaneously by two cameras whose lenses are separated in the same way as our eyes *i.e.* from slightly different angles, and then if the two pictures are mounted side by side and looked through lenses or tubes such that one eye looks at one picture only, they will form in the brain a **stereoscopic view**, *i.e.* the impression of one picture with *depth* (as the object is actually seen). This is the action of the stereoscope.

The action will be understood from Fig. 199. Here AB and $A'B'$ are two stereoscopic pictures of the same object, *i.e.* photographs of the same object taken by a double camera having two objectives placed side by side at the same angular distance as the eye. The two pictures are correctly mounted on a card-board which is introduced into the stereoscope. This is a wooden box having two compartments in which the two pictures are to be housed. On the same side of the box, there are two holes one for each compartment, in which two converging half lenses L_1, L_2 are fixed, behind which the eyes of the observer are placed. The positions of the lenses are so arranged that rays coming from corresponding points P_1 and P_2 of the two stereoscopic pictures, which enter the eyes after refraction through the lenses, form images which appear to combine into single image at P.

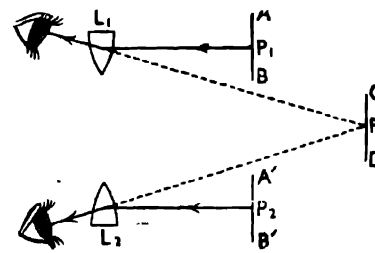


Fig. 199—The Stereoscope.

The two photographs of the same object being taken side by side at the same angular distance at the two eyes, the two images seen by

the eyes appear to combine and give the impression of only one object in relief, *i.e.* an object as actually seen.

131. The Magic Lantern (Optical Lantern) :—By means of this apparatus a real magnified image of a lantern slide, a film, or a drawing on glass, is projected on a screen so that it can be displayed to a large gathering.

The essential parts of this instrument are as follows (Fig. 200) —

(i) **A Powerful Source of Light (A)**—If it is a carbon-arc lamp, the positive carbon, on which the crater is formed, is taken to be

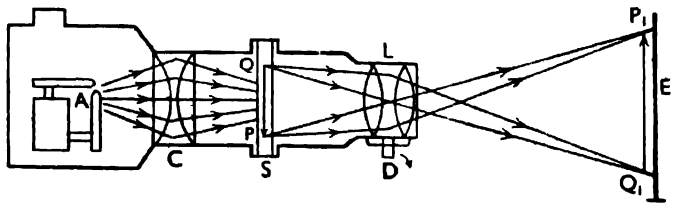


Fig. 200—The Magic Lantern.

horizontal and the other carbon-rod vertical in order that the full light produced in the crater may pass through the apparatus. Lime light, acetylene burner, or any other suitable source, may be used if electric light is not available. A suitable reflector (not shown in the figure) may be placed at the back in order to turn the back-rays to the forward direction.

(ii) **The Condenser (C)**—It consists of two plano-convex lenses placed with their curved surfaces facing each other. The convergent lens-system turns the divergent rays from the source into a convergent beam directed towards the object PQ. It concentrates the rays to illuminate the object properly and is therefore called a condenser.

(iii) **The Slide.**—It is a photograph on a glass plate of suitable size, or a film, or a drawing on glass. It is put in a wooden frame which is inserted in a groove S in the lantern. It can be changed by drawing out the wooden carrier. This constitutes the object.

(iv) **The Focussing Lens (L).**—It is a convergent combination of lenses corrected for chromatic and spherical defects. Its position with respect to the slide can be adjusted by means of a rack and pinion arrangement D. The action of the lens can be stopped by means of a cap which can be fitted on it.

(v) **A Vertical Screen (E).**—It should be preferably white.

Action.—By the action of the condenser C, the diverging rays from the source A are turned into a convergent beam directed towards the object PQ, which is thereby uniformly and strongly illuminated. The slide is put in an inverted fashion. The position of the slide is beyond the principal-focus of the focussing lens-system L and so an inverted, magnified and real image P_1Q_1 of the inverted object PQ (*i.e.* an erect

image) is produced on the distant screen E. The focussing lens L is then slightly adjusted (*i.e.* moved forward or backward) by means of the rack and pinion arrangement D until the image formed is made very distinct and of desired size.

Lantern slides are usually about 3″ square and the size of the image on the screen may be as large as 8 feet square. Thus the light passing through slide is spread over an area 1000 times as great. Hence it is obvious that unless a powerful source of light is used, the image will not be distinct.

N. B. As the image thrown on the screen is inverted, the slide is to be inserted in an inverted position in order to get an erect image.

132. The Epi-diascope :—An Epi-diascope (Fig. 201) combines a magic lantern and an episcope. The episcope is a convenient instrument for projecting the images of opaque object like ordinary diagrams, maps, pictures, photographs, etc. on a distant screen. Light from a strong gas-filled lamp, or arc-lamp A having a concave reflector R at the back, is concentrated by the condenser C on to a map or a diagram in a book placed on a table, called the object table. Light from the object PQ then travels upwards through a focussing lens system L on to a plane mirror M having a silvered surface from which it is reflected and finally focussed on to a screen E.

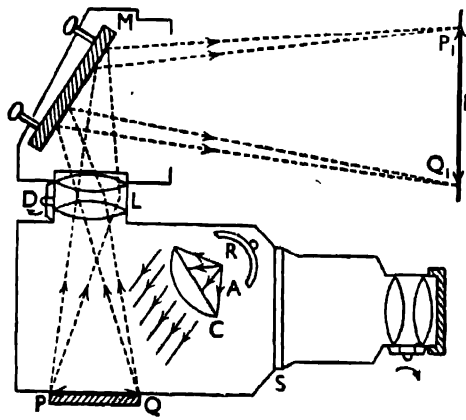


Fig. 201—The Epi-diascope.

The mirror M is placed at 45° with respect to the horizontal. The object PQ is beyond the principal focus of the convergent focussing lens system L and so a real magnified image is produced on the screen E. By adjusting the rack and pinion arrangement D, the distance between L and PQ may be altered until the image P_1Q_1 is made distinct and of the proper size.

When an episcope is also fitted with devices to project lantern slides, like an ordinary magic lantern it is called an **Epi-diascope.**

133. The Photographic Camera :—It is an apparatus by which a permanent image of an object can be taken on a photographic plate or film. Its essential parts are the following—

(i) **A Light-tight Box (BA) (Fig. 202).**—It is made of folding black cloth, leather or paper such that it can be extended or shortened in length according to necessity. The inside of the box must be black or painted black for stopping internal reflections. The box is usually fitted on a tripod stand by means of which it can be set at any desired height or tilted in position. The base of the box is provided with a rack and pinion arrangement (not shown in the figure) by which back of the box at which the plate is placed can be moved in or out.

(ii) **The Photographic Objective (L).**—It is a combination of lenses (L) equivalent to a converging lens and is corrected for chromatic and spherical errors. Its design depends on the purpose for which the camera is used. There is a rack and pinion arrangement D by which the objective can be move forward or backward for purposes of focussing.

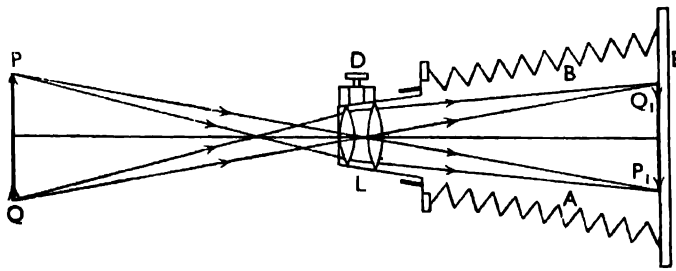


Fig. 202—The Photographic Camera.

(iii) **The Iris Diaphragm.**—It is an adjustable shutter by which the aperture of the lens can be regulated to permit variation in the intensity of the image and improve its definition.

(iv) **The Shutter.**—It permits the time of exposure to be varied. In modern cameras, there is an automatic arrangement for varying the time of exposure from 1/10th to 1/200th second. Such exposures are called *instantaneous*. There is provision for time exposure also whereby the plate can be exposed to light for any length of time according to the judgment of the photographer.

(v) **The Screen (E).**—It is a ground glass plate on which the focussing is first done and is subsequently replaced by the photographic plate.

(vi) **The Plate holder.**—This is a flat light-tight box for housing the sensitive plate. It has a movable shutter which can be drawn out when the plate is to be exposed.

(vii) **The Plate**—It is a plate of glass (or celluloid) on which there is a thin layer of an emulsion of one or more halides of silver in gelatin.

How a Photograph is taken.—The camera is set up at the proper height in front of the object, the shutter of the lens is opened and by varying the distance of the camera from the object, an image is roughly formed on the ground glass screen. Then, by adjusting the distance between the objective and the screen (by altering the position of the screen or the position of the objective or both, by the help of rack and pinion arrangements), an image of the proper size is sharply focussed on the screen. The aperture of the diaphragm is then adjusted for the proper illumination and definition of the image. The camera is then loaded with the plate in the slide. The plate is then exposed to light by drawing out the shutter of the plate. The time of exposure is a matter of art which can be learnt only by experience. It depends on the intensity of the light and the size of the aperture.

No visible image, however, appears on the plate being simply exposed to light. The shutter of the slide-carrier is then closed and the slide is removed to a dark room where the plate is kept immersed in a chemical solution, called the **developer.** The silver salts in those parts where they have been acted on by the incident light rays are now gradually reduced by the *developer* to the metallic state. When the picture is satisfactorily revealed, the plate is carefully washed with water so that even the last trace of the developer may be washed out. The plate is next washed in a solution of **"hypo"**, called the fixer solution. Until this is done the plate is not free from the action of light. The hypo (hypo-sulphide of soda) solution washes away the silver salt from the parts of the plate not affected by light. By repeated washing in water, the last trace of the hypo is removed, after which the plate is dried up in a current of air. The plate thus *developed* and *fixed* is called the **negative**, since the bright parts of the image are depicted black on the plate and *vice versa*. To print the positive (called the photograph), the film-side of a sensitised paper, similarly coated as the plate, is held in contact with the negative in a suitably constructed frame and light is passed through the negative to act on the film for an appropriate length of time. The image on the paper is then *developed* and *fixed* as before. The paper is afterwards thoroughly washed and dried. This is the photograph. The method is called **contact printing.**

***134. Photographic Objective** :—The objective of a good photographic camera should conform to certain requirements as given below :

(*a*) The objective must be achromatic both with respect to the violet and the yellow regions of the spectrum. This is necessitated by the facts that the photographic plates are most sensitive to the violet rays and that the condition of sharp focussing is to be visually

checked with the help of the image cast on a ground glass screen which evidently requires the yellow portion of the spectrum.

(*b*) The objective must have a definite *depth of field* and a certain *depth of focus*. The *depth of focus* is defined as the variations in the image positions which can be tolerated without incurring any objectionable lack of sharpness. So the distance over which the photographic plate can be moved without materially affecting the sharpness of the image is referred to as the depth of focus of the camera lens. Again the *depth of field* is the maximum allowable distance over which the object may be moved so that a sharp image is cast on the photographic plate placed within the depth of focus. Both these factors depend on (*i*) the focal length of the camera lens, (*ii*) the aperture of the lens and (*iii*) the distance of the object from the lens. So by stopping down the lens aperture these factors can be increased.

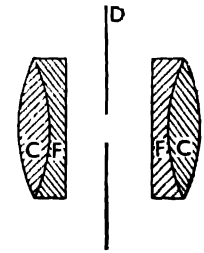


Fig. 203

(*c*) The objective should be so constructed that the image cast on the photo-plate is free from distortion. So in order to avoid distortion various types of objectives consisting of two different systems of lenses have been devised. The diaphragm placed judiciously between these two systems produces two opposite types of distortion (barrel and pin-cushion types) in the two systems which neutralise each other.

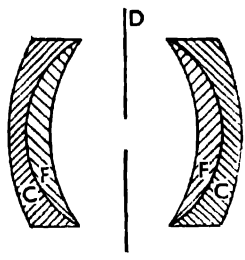


Fig. 204

Fig. 203 shows such an objective generally known as the **portrait lens**. It consists of two systems each of which is a combination of one double convex lens of dense crown glass *C* and a plano-concave lens of light flint glass *F*. The plane faces of the systems are directed towards each other and the diaphragm is placed in between. This objective is not totally free from distortion and is a rapid lens. Fig. 204 shows another type of objective known as the *rapid rectilinear lens*. Here each of the two systems consists of a convexo-concave lens of dense crown glass *C* and a convex meniscus lens of light flint glass *F* put together. The concave surfaces of the two systems face each other and the diaphragm is placed in between. The system is free from distortion and is truly achromatic. An *anastigmat* is a different type of objective which is free from most of the defects. It consists of a concave lens of light flint glass, a convex lens of dense crown glass, a convex meniscus of crown glass and a concave meniscus of flint glass all combined together.

(*d*) The time of exposure should be also considerably small in a good camera. It evidently depends on the amount of light in the image formed by the camera lens which in its turn is controlled by the effective aperture of the diaphragm used in the camera. The quantity of light in the image formed at the centre of the photographic plate is, by the law of inverse square, proportional to $\frac{1}{f^2}$ when the object is at a comparatively large distance. Moreover, it is proportional to the area of the aperture through which light enters, *i.e.* to $\pi\frac{d^2}{4}$ where d is the diameter of the aperture. So the illumination of the image formed on the plate is proportional to $\frac{d^2}{f^2}$. The quantity $\frac{d}{f}$ is known as the **relative aperture** of the camera lens. Lenses with the same value of d/f will produce equally bright images for a given object. So the factor d/f is very important in compiling the exposure tables. Since the time of the exposure is inversely proportional to the brightness of the image formed on the plate, it is proportional to $\frac{1}{(d/f)^2}$, or, to $\frac{f^2}{d^2}$. The factor f/d is generally knows as the **speed of the lens.** If the aperture of a lens be 1 inch and its focal length 8 inches, its relative aperture is $\frac{d}{f}=\frac{1}{8}$. Again if another lens be of aperture 2 inches and of $f=16$ inches, its relative aperture is again 1/8. Both of them will have the same light gathering power and same exposure will be required in both the cases. Lens-apertures are marked according to their relative apertures. A lens having a relative aperture 1/8 is marked as $f/8$, one having a relative aperture 1/4 is marked $f/4$, one having a relative aperture $\frac{1}{n}$ is marked as f/n and so on. This fraction f/n, which actually indicates that the diameter of the aperture is $\frac{1}{n}$th part of the focal length is generally known as the *f-number* or the *stop-number.* Since the time of exposure is proportional to f^2/d^2 and $d=\frac{f}{n}$, the time of exposure is proportional to $f^2\times\frac{n^2}{f^2}=n^2$. So the smaller the value of n, the smaller will be the time of exposure or more rapid will be the lens.

135. Telephoto lens :—In a photographic camera, the size of the image formed on the ground glass screen or the plate depends on

the focal length of the camera objective because the magnification is,

$$m = \frac{f}{f+u}.$$

When the distance (u) of the object is large m is sufficiently reduced. So to increase the magnification and hence the size of the image, the focal length of the objective should be made considerably large. But in that case the plate should also be placed at a distance nearly equal to this large focal length from the objective. So the camera box becomes rather unwieldy and the objective can neither be properly corrected for aberrations and distortions. But by suitably combining a short focus convex lens with a concave lens placed at a distance, it is possible to get an effective focal length which is as large as that necessary for the required magnification. Such a combination is known as the *telephoto lens* which can be fitted in a camera of ordinary size.

Fig. 205 shows an arrangement of a telephoto lens. The rays from the distant object fall on the convex lens A and an image is expected to be formed at p'. But the concave lens B, placed between p' and A such that Bp' is less than its focal length, casts a real image finally at p on the plate. The rays meeting at p appear to have deviated at PQ and hence the equivalent focal length of the system becomes Qp. If Bp' is increased, Bp will also increase ; but such increase should be limited by the maximum distance to which a plate can be shifted in a camera box of adjustable size. Both the constituent lenses are made achromatic but the combination is not free from other defects. The image formed by such a lens is rather flat showing very little perspective.

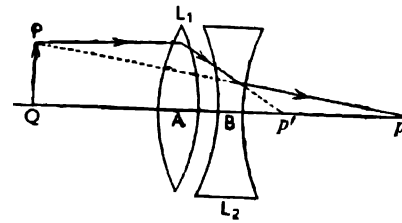


Fig. 205

136. Fixed Focus Cameras :—Ordinary cheap types of box cameras require no preliminary focussing at all. The lens is here fixed to the camera box and its position is not adjustable. The sensitive film is placed at a distance from the lens which is very nearly equal to the focal length of the lens. The objects placed within a range from infinity to about two metres cast sufficiently well defined images on the film. This is effected by sufficiently reducing the aperture of the lens. By using the camera lens at a very small relative aperture the desired depth of focus over the above wide range is secured in such a camera.

137. Eye and vision.:—The eye-ball (Fig. 203) is nearly spherical in shape, and, within lin its, is capable of turning in its socket. It is a nature's optical instrument.

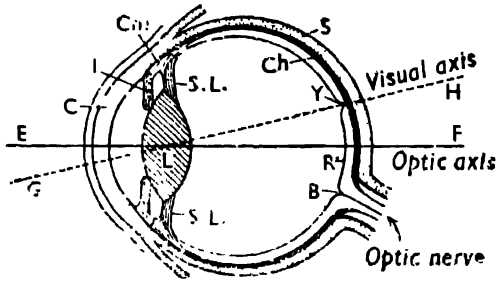


Fig. 206—The Human Eye.

A human being has two similar eyes, and has the advantage of binocular vision. The eye ball is complex in structure. Its exterior is formed by a white membrane *S*, called the **sclera**. The front portion (*C*) of the sclera is transparent and more convex than the rest. This part is known as the **cornea**. Internal to the sclera is a dark-brown membrane (*Ch*) which is known as the **choroid**. The anterior continuation of the choroid is a seat of muscles, known as the **ciliary muscles** (*Cm*). From this ciliary body a diaphragm, circular in form, hangs and is perforated in the centre by an aperture whose size can vary by an involuntary action of the ciliary body. Its function is to regulate the size of the aperture. Its colour gives the colour of an eye. This diaphragm *I* is called the **iris**. Behind the iris is suspended the focussing lens *L* of the eye. It hangs from the inner surface of the ciliary muscle by means of a few fibres, called the **suspensory ligaments** (*S.L.*). The lens is bi-convex (more convex at the back), transparent, and is composed of different layers of different refractive indices. The space in between the cornea in front and the iris is called the **anterior chamber** of the eye, while the space between the iris and the suspensory ligaments is called the **posterior chamber**, and they are filled up by a salt solution known as **aqueous humour**. On the internal back-portion of the choroid is stretched a delicate purple-red membrane having a network of minute structures, called the **rods** and **cones**, which receives the light waves, and the optic-nerve-fibres, which carry the light sensations to the visual centre of the brain. This light sensitive membrane (*R*) is called the **retina**. The *rods* are more sensitive to light than the cones but the *cones* can recognise the colours and details of an object more distinctly than the rods. On the inner surface of the retina there is a region (Y) about 1 to 2 mm. in diameter, called the **yellow spot** or the **macula lutea**. In the centre of the yellow spot there is a small depression, called the **fovea centralis**. The imaginary line that passes through the optical centre of the eye-lens and the *fovea centralis* is called the **visual axis** of the eye. The yellow spot is the region of highest optical sensitivity in the retina. The imaginary line that passes through the centre of the cornea and the optic centre of the eye lens is called the **optic axis** of the eye. The optic axis meets the retina

at the *posterior pole* of the eye. The optic nerve-fibres enter the ratina about 3 mm. below the posterior pole of the eye. The spot where the optic nerves enter the retina is called the **optic disc.** This disc *B* has no sensitivity and hence is called the **blind spot.** The space between the retina and the lens is known as the **vitreous chamber** and is filled up with a transparent, colourless and gelatinous mass known as **vitreous humour (*V*).**

Action of the eye—Rays from an external object entering the eye suffer refraction mainly at three surfaces, the outer surface of the cornea and the two surfaces of the eye-lens. Then again there is the continuous refraction experienced by the rays in passing from layer to layer of the lens and finally the rays are brought to focus on the retina. The image produced by the eye-lens (convergent) on the retina is real, diminished and inverted. *That we see it erect is due to a process of mental interpretation only, to which all observers are habituated.*

[**Note** : It has already been said that the images formed on the retina are always inverted though we see them erect. This, however, cannot be clearly explained. The explanation is that our mind has learnt by touch and muscular sense from our infancy that an inverted image on the retina means the presence of an erect object. Thus it is only a mental interpretation.]

Resolving Power of the Eye :—Since a human eye is essentially an optical instrument, like all other optical instruments, it must have a definite resolving power. It has been shown that the smallest visual angle subtended between two points at the eye so that they may appear as two distinct points is 0·000235 radians for the mean ray of the visible light. For all practical purposes this angle is taken as one minute of an arc (*i.e.* 0·000291 radians).

138. **Power of Accommodation** :—An eye is said to be normal when in a state of full relaxation it can focus on the retina objects at an infinite distance. When such an eye looks at a near object, the image should be formed beyond the retina, but the eye by virtue of an inherent power (which acts involuntarily), called its **power of accommodation**, can form the image of the object on the retina. According to Helmholtz, the mechanism of accommodation is that the ciliary muscles automatically contract, drawing forward the choroid and relaxing the suspensory ligaments ; this diminishes the tension of the lens-capsule and allows the inherent elasticity of the lens to increase its convexity. Chiefly the anterior surface of the lens changes in curvature. The degree of accommodation obviously will vary with the distance of the object under view.

Thus accommodation *is that property of the eye-lens by which its effective focal length is automatically altered to suit the act of viewing distant or near objects.*

Far-Point of the Eye—The most distant position up to which the eye can see, when fully relaxed, is termed the *far-point*. For the normal eye it should be at infinity. But actually the far-point is only a distant point (which varies from person to person) for an eye.

Near-Point of the Eye—For every eye there is a limit to its power of accommodation. This power ceases when the object is brought to a certain minimum distance from the eye. The nearest position upto which a small object can be distinctly seen employing the maximum amount of *accommodation*, is referred to as the *near-point* of the eye. It can be determined for an eye by noting the shortest distance at which a man can read the smallest test-type with the other eye closed.

Least Distance of Distinct Vision.—It is the distance of the *near-point* from the eye. For a normal eye it is about 10 inches (*i.e.* 25 cms.).

139. Defects of Vision:—The common defects of vision are the following—(a) **Long-sight** (or Hypermetropia); (b) **Presbyopia**; (c) **Short-sight** (or Myopia); and (d) **Astigmatism.**

(a) **Long-sight** (or Hypermetropia).—For the normal eye, the distance of the near-point is about 10 inches (25 cms.) and for a long-sighted person it is greater. **A long-sighted eye cannot see near objects distinctly**, but if the accommodation is normal there is no difficulty about distant objects. Rays from the normal near-point are brought to a focus behind the retina (Fig. 207). The causes are: (*a*) the eye-ball is too small, or (*b*) the focal length of the lens is too large.

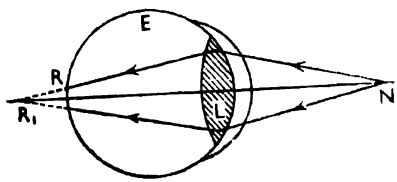


Fig. 207—A long-sighted Eye.

The remedy is to *interpose* a convex lens so that the focal length of the combination is shortened and by that the image is formed on the retina (Fig. 208). Usually it is found in the case of a hypermetropic eye that parallel rays from distant objects are focussed on the retina. In order that rays from the normal near-point N can be focussed on the retina, the converging lens L_1 of the spectacles should be of such power as to make the rays from the object appear to come from the near-point N_1 of the defective eye.

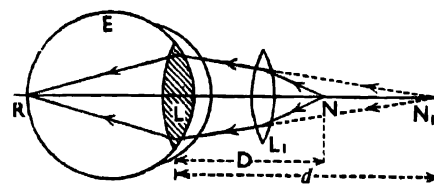


Fig. 208—A corrected long-sighthed Eye.

Let v=distance of retina from the eye-lens, f_e=focal length of the eye and d=distance of the near-point of the long-sighted person found by trial.

Hence, $\frac{1}{v}-\frac{1}{d}=\frac{1}{f_e}$ ... ... (1)

Suppose a lens of focal length f_1 should be interposed to bring the near-point of the defective eye to a position 25 cms. from the eye. So we have,

$\frac{1}{v}-\frac{1}{25}=\frac{1}{f_e}+\frac{1}{f_1}$; or, $\frac{1}{v}-\frac{1}{25}=\frac{1}{v}-\frac{1}{d}+\frac{1}{f_1}$ ... from (1)

Or, $\frac{1}{f_1}=\frac{1}{d}-\frac{1}{25}$. As $d>25$, $\frac{1}{d}<\frac{1}{25}$ and, therefore, f_1 is $-ve$.

This gives the focal length of the spectacles lens required, and the negative sign of f_1 shows that the **lens must be convex.**

(b) **Presbyopia.**—This is another form of long-sight which is due to old age. This is called *Presbyopia* and is sometimes also called **Far-sight.** The crystalline lenses of the eye lose elasticity gradually with age, and the accommodating power of the ciliary muscles decreases. Thus a short-sighted eye in childhood tends to become normal in after-years, but the defect of long-sightedness is sure to increase gradually.

Examples

1. A long-sighted man can see clearly at any distance more than 10 ft. What kind of spectacles lens should be used to enable him to read print placed 10″ from his eye ?

Ans. The distance of the near-point of the man is 10 ft. or 120″. So, we have, if v=distance of retina from the lens of the eye, $\frac{1}{v}-\frac{1}{120}=\frac{1}{f_e}$. Again if f=focal lenth of the spectacles lens, $\frac{1}{v}-\frac{1}{10}=\frac{1}{f_e}+\frac{1}{f}=\frac{1}{v}-\frac{1}{120}+\frac{1}{f}$;

or, $\frac{1}{f}=\frac{1}{120}-\frac{1}{10}=-\frac{11}{120}$ $\therefore$ $f=-10{\cdot}9''$ (approx.).

Hence, the spectacles lens should be convex and of focal length 10·9″ (approximately).

2. A long-sighted person has a minimum distance of distinct vision of 50 cms. What kind of lens must be used in order to reduce this distance to 25 cms. What should be the focal length of the lens ?

Ans. Here u=50 cms. Hence $\frac{1}{v}-\frac{1}{50}=\frac{1}{f_e}$, where f_e is the focal length of the eye-lens. Again, if f_1 is the focal length of the spectacles,

$$\frac{1}{v}-\frac{1}{25}=\frac{1}{f_e}+\frac{1}{f_1}=\frac{1}{v}-\frac{1}{50}+\frac{1}{f_1}. \quad \therefore \quad \frac{1}{f_1}=\frac{1}{50}-\frac{1}{25}=-\frac{1}{50}.$$

$\therefore$ $f_1=-50$ cms.

Thus the spectacles lens should be convex and of focal length 50 cms.

(c) Short-sight (or Myopia).—A short-sighted person cannot see distant objects distinctly. The rays from a distant object are brought to a focus in front of the retina R (Fig. 209); so the far-point is nearer than infinity but as to near-vision there is no difficulty if the accommodation is normal, though the near point N_1 may be much closer than 25 cms. The causes of the defect may be—(*i*) the eye-ball is too elongated (*ii*) the focal length of the eye is too short.

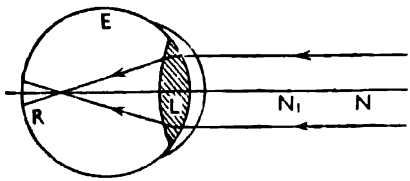


Fig. 209—A short-sighted Eye.

Let d = the distance of the farthest point F upto which the short-sighted person can see distinctly (Fig. 210). The point, in the case of a normal eye, should be at infinity.

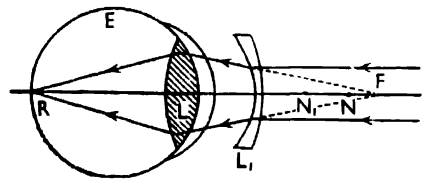


Fig. 210—A corrected short-sighted Eye.

It is necessary to *interpose* a concave lens L_1 such that parallel rays from distant objects will, after refraction through the lens, appear to diverge from F, the far-point of the defective eye and will thus be focussed on the retina by the eye-lens L.

As before, we have, $\frac{1}{v}-\frac{1}{d}=\frac{1}{f_e}$ ... ... (1)

By interposing the lens, the far-point would be at infinity.

So, $\frac{1}{v}-\frac{1}{\infty}=\frac{1}{f_e}+\frac{1}{f}=\frac{1}{v}-\frac{1}{d}+\frac{1}{f}$ ... from (1)

or, $\frac{1}{f}=\frac{1}{d}$ $\left(\because \frac{1}{\infty}=0\right)$. $\therefore f=d$.

Hence the focal length of the spectacles lens should be equal to the distance of the farthest point upto which the short-sighted person can see distinctly and the positive sign of f shows that the lens must be concave.

Examples

1. A short-sighted man who can read clearly when the print is not more than 3 inches from his eye requires spectacles to enable him to see distant objects. What kind of lenses does he need and what must be the focal length? Draw as accurately as you can the paths of the rays of light from a distant object through the lens of the man's eye (a) without the spectacles, (b) with the spectacles.

Ans. Here, the farthest point=3 inches. So, $\frac{1}{v}-\frac{1}{3}=\frac{1}{fe}$.

But if f be the focal length of the lens to be used, we have,

$\frac{1}{v}-\frac{1}{\infty}=\frac{1}{fe}+\frac{1}{f}=\frac{1}{v}-\frac{1}{3}+\frac{1}{f}$; or, $\frac{1}{f}=\frac{1}{3}$. $\therefore$ $f=3$ inches.

The man will evidently require a concave lens of focal length 3 inches. (For diagrams see Fig 209 & 210)

2. *A short-sighted man can read printed matter distinctly when it is held at 15 cms. from his eyes. Find the focal length of the glasses which he must use if he wishes to read with ease a book at a distance of 60 cms.*

Ans. $\frac{1}{v}-\frac{1}{15}=\frac{1}{fe}$, and $\frac{1}{u}-\frac{1}{60}=\frac{1}{fe}+\frac{1}{f}=\frac{1}{v}-\frac{1}{15}+\frac{1}{f}$,

$\therefore$ $\frac{1}{f}=\frac{1}{15}-\frac{1}{60}=\frac{3}{60}=\frac{1}{20}$. $\therefore$ $f=+20$ cms.

So the glasses of the spectacles must be concave and of focal length 20 cms.

(d) Astigmatism.—This defect of the eyes is usually due to irregularity in the curvature of the vertical and horizontal sections of the cornea ; the curve generally is more pronounced in the vertical section than in the horizontal with the result that horizontal and vertical lines at the same distance will not be in focus at the same time. Such an eye, when looking at a network, may be able to see clearly, for instance, the horizontal wires, while vertical wires may be indistinct or curved. **A cylindrical or sphero-cylindrical lens** is used to remove this defect. The defect may differ in degree in the two eyes.

140. The Advantages of Two Eyes (Binocular Vision) :—As with two ears we hear only one sound, so by our two eyes, when fixed on any object we see only one object. Though each of the eyes forms an image on its own retina, the brain integrates the two different images into one image. One advantage of having two eyes is the *power of estimating distance* correctly. It may have been noticed that the threading of a needle is very difficult when one eye is kept closed. To illustrate it further, suspend a small curtain ring at some distance with its plane in the line of sight. Now hold a long stiff wire, bent at one end, and try to insert the bent end through the ring keeping one eye closed. It will be found more difficult to do it when one eye is closed than when both eyes are open. This difference is due to the fact that the simultaneous vision by the two eyes gives a marked facility in estimating distance.

Further due to the difference in position of the two eyes with respect to an object, the right eye sees more of the right side and the left eye sees more of the left side. When the object is viewed simultaneously

by both the eyes, the two images which are slightly different overlap on each other. The resultant impresssion gives a combined idea of *depth* and *solidity*.

141. Persistence of Vision :—Visual impressions, however momentary, received on the retina of the eye persist for about one tenth of a second even after the stimulus is removed ; so if a series of impressions at intervals of more than ten per second falls on the retina, the eye will not be able to distinguish between them and a continuous impression will be produced and this is the reason why the Newton's disc, if rapidly rotated, appears greyish white which results from the overlapping of the colours on the disc.

Again, if there be a picture of a bird on one side of a piece of card-board and a picture of a cage just on the opposite side, then on rapidly revolving the card-board the two impressions blend and the bird will appear to be inside the cage due to persistence of vision. For the same reason the red end of a burning splinter, when whirled round, gives the impression of continuous red circle.

142. Cinematography :—The Cinematograph is a projection lantern specially adopted for exhibiting pictorially on a screen the movements of objects. Its working, known as cinematography, depends essentially upon the principle of persistence of vision. If photographs are taken of a moving object at intervals of about $\frac{1}{10}$ of a second, then the discontinuous pictures, when projected on a screen at the same rate, will fuse together and produce and an illusion of continuity.

In the cinematographic film there are numerous pictures of same object in different succeeding positions taken by means of a special camera (motion-picture camera). These pictures are rapidly moved before a projection lantern, the arrangements of which are similar to those of an optical lantern. The pictures on the film come in front of the condenser to the position occupied by the slide of the optical lantern. About twenty pictures on the film pass in front of the condenser per second, and as each of these arrives in position the film stops for $\frac{1}{10}$th of a second by a clock-work arrangement and passes on to the next picture after a brief dark interval. The images of these rapidly moving pictures are thrown on a screen with an interval (of about $\frac{1}{10}$th of a second) between each pair during which the film is not illuminated, but as the pictures follow one another very rapidly they represent some continuous incident or story due to persistence of vision, as the impression of one picture does not vanish before the second is received ; and this goes on one after another. Thus, in cinematography, we see not the *moving* pictures, but a rapid succession of *stationary* pictures.

Working.—A common form of film is a 16 mm. celluloid film on which the pictures are positively printed. Such films are made up in

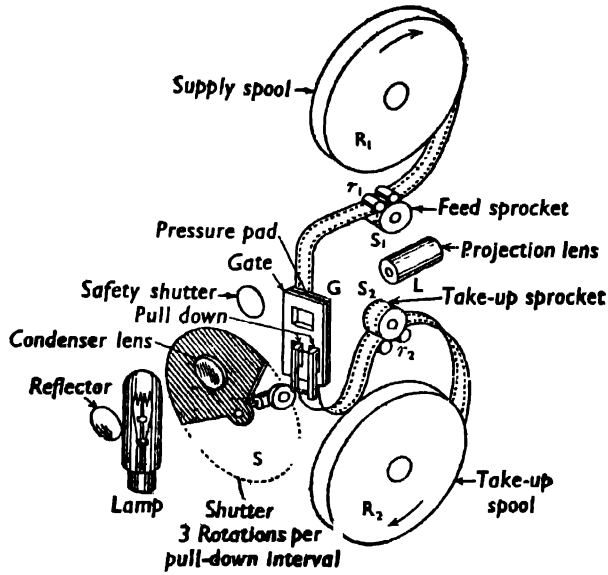


Fig. 211—The Cinematograph.

reels of 50 feet. The film is drawn from a reel box R_1, called the *supply spool*, by means of a uniformly driven sprocket S_1, called the *feed sprocket* (Fig. 211); the film is kept in engagement with S_1 by means of the pressure roller r_1. The film passes through the *pressure pad and gate G* in front of an aperture, as shown in the picture : the film is strongly illuminated by light passed through this aperture from an optical lantern ; the lantern, as usual, consists of a high power lamp, or a powerful carbon arc with a reflector at the back and a condenser lens in front. An intermediate sprocket, shown in the picture as *pull-down*, pulls the film through the gate at rated interval. A revolving *shutter S*, placed between the condenser and the aperture cuts the light off when picture is replaced by the next in succession. The safety shutter, shown in the picture, may be used in times of any derangement. The *take-up sprocket* S_2 (with the help of its pressure roller r_2, collects the used film and passes the same to the other reel box R_2) called the *take-up spool*. The projection lens L forms on a distant screen a *real magnified* image of the picture through which the beam of light is passed.

143. Cinema Tricks :—Sometimes curious tricks are seen to be performed on the cinema. Many natural phenomena are seen to be

accelerated and sometimes retarded. The seed of a plant takes a long time to grow, but the photographs of its growth can be taken at intervals over a comparatively long period of time on a single film, and when it is thrown on the screen at the ordinary rate we get the impression of very rapid growth. On the other hand, snaps of such things as running, high jumping, strokes in cricket or tennis can be taken at 160 to 200 per second and then thrown on the screen at the ordinary rate of 16 per second producing what is called a slow-motion-picture.

144. Heliograph and Heliostat :—A heliograph simply consists of a plane mirror suitably mounted in order to reflect sun light from one place to another several miles away. This instrument is used for the transmission of message. The mirror is tilted in order to cover or uncover it, according to necessity, so that the observer at a distant station may note the duration and the regularity of the flashes of light (according to a given code) from which the message is to be constructed.

The heliostat is only a heliograph mirror by which a reflected beam can be sent to a particular direction all day long, suitably mounting the mirror on a frame driven by a clock work.

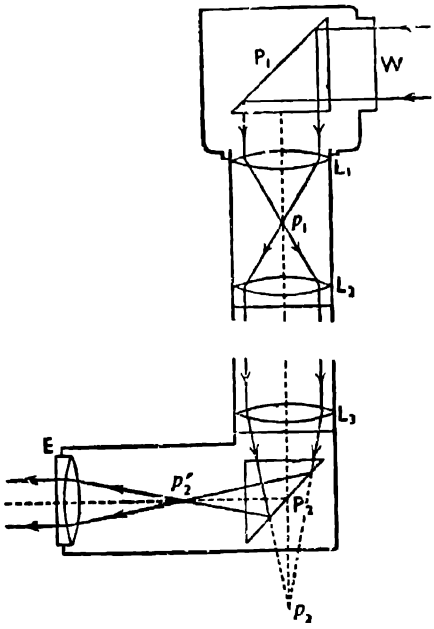


Fig. 212

145. The Periscope :— Periscopes are used in submarines to get a view of the objects lying on the surface of water. The whole assembly of optical parts fitted in a vertical bronze tube is represented in Fig. 212. The parallel rays from a distant object enter the tube through a plate glass window *W*. They are then totally reflected by the right angled isosceles prism P_1 and are made to form the first image at p_1 with the help of the convergent lens L_1. The rays from p_1 fall on the achromatic plano-convex system of lenses L_2 such that after reflection they emerge out parallelly. These parallel rays then traverse a considerably long distance in the vertical tube and ultimately fall on

the achromatic convergent lens L_3. L_3 would have cast an image of the object at p_3. But a second total reflecting prism P_2 diverts the rays and the image p_2 is formed in front on the eye-piece E, which finally magnifies the image. In order to get the view from any direction, the upper part of the tube can be rotated about the axis of the tube.

Questions (Chapter VIII)

1. Show that the magnifying power of a reading lens is $\left(1+\frac{D-a}{f}\right)$, where D is the least distance of distinct vision, a the distance between the eye and the lens and f the focal length of the lens. What conclusions do you draw about the best position of the eye ?

2. A compound microscope is adjusted for viewing the distant image of an object. If the distance of the object from the object glass is now slightly increased, explain what re-adjustment of the instrument would be necessary for obtaining a distinct image again. Will the magnification be the same as before ?

3. The focal lengths of the objective and the eye-piece of an astronomical telescope are 10 in. and 1 in. respectively. The telescope is focussed on an object 5 ft. from the objective, the final image being formed 10 in. from the eye of the observer. Calculate the length of the telescope and the magnification produced by it.

[*Ans.* Length $=\left(12+\frac{10}{11}\right)''$, magnification $=2{\cdot}2$].

4. A simple astronomical telescope is made from two convex lenses of focal lengths 100 cms and 5 cms respectively. Find the magnifying power when the instrument is used to view a distant object, if the final image is formed (*a*) at a great distance, (*b*) at a distance of 25 cms. from the eye-piece.

[*Ans.* (*a*) 20, (*b*) 21]

5. Describe the different parts of a modern microscope. On what factors does its magnifying power depend ? (C. U. 1952; C. U. H. '48, '52)

6. Describe an astronomical telescope. Explain why astronomical telescopes are fitted with objectives of a large diameter. (P. U. 1950 S)

7. (*a*) Describe with a neat diagram, the optical parts of a compound microscope, and show the paths of the principal rays. Derive a formula for the magnifying power. (C. U. 1963, '66, '68)

(*b*) Obtain an expression for the magnifying power of a compound microscope. (P. U. 1951)

8. Describe the construction and use of (*a*) Sextant, (*b*) Photographic Camera, (*c*) Telescope, reflecting and refracting. (C. U. 1941, '49)

9. The objective of a microscope has a focal length of 1" and the eye-piece a focal length of 10/9". The lenses are fixed 4" apart and the microscope is focussed on an object so as to form the final image 10" from the eye-piece. Calculate the magnifying power of the microscope and the position of the object from the objective. (C. U. 1941)

[*Ans.* $u=15''$ and $m=20$]

10. Explain the dispersion produced by a single lens and show how the defect may be corrected ? (C. U. 1952)

Why is such correction unnecessary in the case of a single convex lens used as a magnifier ? (*cf.* C. U. 1947)

11. Explain how a telescope can be made from a concave mirror and a convex lens. Illustrate your answer with diagrams showing the paths of rays.

12. Describe with the help of a neat diagram, the construction and principle of operation of an astronomical telescope and calculate its magnification.
(Utk. U. 1970 S.)

13. Describe the construction of an epi-diascope, indicating the function of each of its component parts. Also trace the courses of rays through the instrument.

14. What spectacles are required by a person who cannot see clearly objects at distances greater than 10 cms. ?

[*Ans.* Man short-sighted, a concave lens of $f = 10$ cms. required.]

15. Write a short note on human eyes. (*cf.* Utk. U. '66, C. U. 1951)

16. What are the principal defects of vision ? Explain how they are rectified with the help of spectacles.

17. A student with defective eye-sight can see clearly noting that is farther from his eyes than 50 cms. What is the number, kind and focal length of the correcting lens that will enable him to see easily and clearly distant objects ?

[*Ans.* Concave lens, $f = 50$ cms. ; or, Power $= -2D$]

18. A long-sighted person can see distinctly only objects which are at a distance of 50 cms. or more ; find the power of the spectacles which will enable him to see distinctly objects at a distance of 25 cms.

[*Ans.* $f = -50$ cms. ; Power $= 2$ Dioptres]

19. Draw a diagram showing the structure of the eye. Describe how it functions and hence show how the principal defects arise. What are the advantages, if any, of having two eyes ? (C. U. '1930)

20. (*a*) Why does an eye-piece system consists of two lenses instead of only one ? (*cf.* C. U. 1958)

(*b*) Describe the construction and properties of a Ramsden's eye-piece.
(*cf.* Raj. U. 1952 ; Pat. U. 1945 ; Luck. U. 1945)

21. What are the advantages of a compound eye-piece ? Describe the construction of Ramsden's eye-piece giving a neat diagram. Locate the positions of principal foci and principal points. Find the focal length of the equivalent lens.
(Del. U. 1968)

22. Describe Ramsden's and Huygens' eye-pieces and explain their uses. Show that the Huygens' eye-piece satisfies the condition of minimum spherical aberration and achromatism.
(*cf.* C. U. 1942, '58 ; P. U. 1950, '52 ; Utk. U. 1952, *cf.* Bom. U. '53)

Calculate the focal length and position of the cross-wire in the Huygens' eye-piece. (Agra U. 1944)

23. Describe the eye-piece that is generally used with an astronomical telescope and explain its usefulness.

Trace a pencil of rays from a distant object through such a telescope fitted with an eye-piece and adjusted for normal vision. (C. U. 1939, '45, '53, '56)

24. (*a*) Discuss the respective merits of Ramsden's and Huygens' eye-pieces.
(*cf.* Luck. U. 1954)

(*b*) Explain fully how the errors due to spherical aberration have been overcome in the construction of object glasses of high power microscopes.
(*cf.* Utk. U. 1951 ; Agra U. 1948)

25. A Galilean telescope is adjusted for normal vision. Calculate its magnifying power. What change in the adjustment is necessary for a short-sighted person and what will be the magnifying power when the least distance of distinct vision is D ? (C. U. 1943)

26. Give a graphical arrangement of the optical parts of a prism binocular. (C. U. 1943, '51, '52)

27. Explain the function of the eye-ring. Show that the magnifying power of the telescope is equal to the radius of the objective glass divided by the radius of the eye-ring. (Raj. U. 1950)

28. Sketch the optical parts of a refracting telescope. In what why is it superior to a reflecting telescope ? (C. U. 1932)

29. Explain what is meant by the term "field of view" of an optical instrument. Find an expression for the field of view of an astronomical

30. Define the magnifying power of a telescope. Give a neat diagram of an astronomical telescope showing the paths of the principal rays. How is its magnifying power for infinity practically determined ? (C. U. 1935)

31. Describe with necessary diagrams the optical parts of an astronomical telescope fitted with a micrometer eye-piece. How can you use it to measure the distance between two stars ? In what respects does an opera-glass differ from an astronomical telescope ? (C. U. 1931)

32. Write notes on :—

(*a*) Photographic Camera (C. U. '68) ; (*b*) Defects of vision and their remedies (C. U. '68) , (*c*) Compound microscope. (C. U. '67)

CHAPTER IX VELOCITY OF LIGHT

146. Velocity of Light :—The velocity of light has been determined by astronomical as well as terrestrial methods. Some of these important determinations are given below.

147. Astronomical methods : —

(a) Römer's method.—The velocity of light was first determined by a Danish astronomer, named Römer, in 1676. He observed the eclipses of one of Jupiter's satellites. The planet Jupiter has got many satellites revolving round it in the same way as the moon revolves round the earth. Moreover, their orbits lie nearly in the same plane as the orbit of Jupiter round the sun. So each satellite becomes eclipsed

when in course of revolution it passes into the shadow of the planet cast by the sun. The interval between two consecutive eclipses of the *innermost satellite* of Jupiter is 42 hours 48 seconds. That is, this is the time required for one complete revolution of this satellite round Jupiter.

As a satellite moves round Jupiter, it takes a constant time to complete one revolution, for it moves with uniform speed. Römer kept the innermost satellite of Jupiter in observation using an astronomical telescope. He continuously recorded for over an year the successive instants of time, as he observed them from the earth, every time this satellite entered into the umbral shadow of Jupiter cast by the sun and became eclipsed. An eclipse takes place once in every complete revolution of the satellite round Jupiter. Römer found that the observed time interval between successive eclipses all round the year was not constant.

If light takes no time to travel from one point to another, then the light emitted by the satellite before an eclipse just occurs will be received by an observer on the earth at the same time irrespective of the relative positions of the satellite and the earth, and the time interval between two successive eclipses will always be constant. But Römer found it not constant. This could be so, Römer argued, if light takes a time to travel from one place to another, *i.e.* if light has a finite velocity. For the instant of time when an eclipse of the satellite will be observed from the earth will then depend on the distance of the earth from the satellite at the moment.

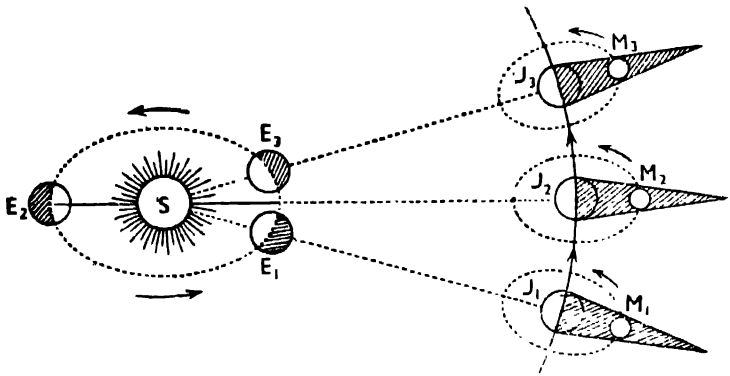


Fig. 213—Römer's Method.

Consequently, if the eclipses are observed from the earth, the time interval between successive eclipses will vary as the relative distance between the earth and the satellite will change. The variation of this time interval will be the smallest or the greatest when the actual

distance between the earth and the satellite will change the least or the most between two successive eclipses.

Jupiter takes 11·86 years to complete one revolution round the sun. Let E_1, E_2 and E_3 be the positions occupied by the earth in succeeding times while moving in its orbit round the sun during a little over one year and let J_1, J_2 and J_3 be the positions of Jupiter in its own orbit during corresponding instants of time. Suppose at E_1 and J_1, the earth and Jupiter are nearest to each other while they are farthest from each other at E_2, J_2 during the revolution of the earth round the sun. The first position, E_1, J_1 where the sun, the earth and Jupiter are in the same straight line with the earth in the middle is called a position of **Conjunction.** The second position E_2J_2 where they are in the same straight line again but with the sun in the middle is called a position of **Opposition.** Let the next position of conjunction be at E_3J_3.

Römer observed that the time interval between successive eclipses was shortest at about the position E_1 of the earth and it increased in succeeding months reaching a maximum value at a position midway between E_1 and E_2 and gradually decreased to a minimum again at the position of *opposition* E_2; then this interval increased again in succeeding months till it became a maximum again at a position midway between E_2 and E_3, decreasing again to a minimum at E_3, the next position of *conjunction*. He found that if the eclipse at E_1 is taken as the first, the eclipse at E_2 will be the 113th and that at E_3 the 225th.

Let θ be the actual interval between two successive eclipses. If n eclipses take place during the time the earth moves from the first position (E_1) of conjunction to the position of opposition (E_2), the actual interval between the occurrence of the first and the nth eclipse is $(n-1)\theta$. The interval (T_1), which will be observed from the earth's surface, will, however, be given by,

$T_1=(n-1)\theta+\frac{E_2J_2}{C}-\frac{E_1J_1}{C}$, where C is the velocity of light. This will be so, because an eclipse will be observed on the earth later by a time equal to $\frac{\text{distance between Earth and Jupiter}}{\text{velocity of light}}$ after its actual occurrence; it should again be noted that first eclipse is observed when the earth is at E_1, and the last eclipse when it is at E_2.

Again, n eclipse will occur as the earth moves from the position (E_2) of opposition to the second position (E_3) of conjunction again, and so the time interval (T_2) between the first and the nth eclipse, as observed from the earth, will be given by,

$T_2=(n-1)\theta+\frac{E_3J_3}{C}-\frac{E_2J_2}{C}$, for the same reasons.

But $E_2J_2 - E_1J_1 = D = E_2J_2 - E_3J_3 =$ (practically) the diameter (D) of the earth's orbit round the sun, assuming the orbit to be circular.

$$\therefore \quad T_1 = (n-1)\theta + \frac{D}{C}, \text{ and } T_2 = (n-1)\theta - \frac{D}{C}.$$

$$\therefore \quad T_1 - T_2 = \frac{2D}{C}, \text{ whence } C = \frac{2D}{T_1 - T_2}.$$

Römer found $(T_1 - T_2)$ to be 33 min. 12 secs.; D was taken by him to be 191×10^6 miles. His value of velocity of light in air was thus $= \frac{2 \times 191 \times 10^6}{33{\cdot}2 \times 60} = 191415$ miles/sec. $= 192{,}000$ miles/sec. (approximately).

The accuracy of Römer's method was limited by the accuracy with which the diameter of the earth's orbit was known in Römer's time.

Sources of Errors:—(*i*) The earth's orbit is actually elliptical, with the sun at one of the foci, while Römer assumed it to be circular.

(*ii*) As there was no precision chronometer in Römer's time, the time records of his experiment were not strictly accurate.

(b) Bradley's Aberration method.—Since light travels with a definite velocity, C, the direction of a star when seen from the earth will be different according as the earth directly moves towards the star or moves at a finite angle to the line joining the earth and the star. If the earth E moved directly towards the star S, the path of the light rays from S would remain unaltered along the direction SE. This would have also been the case if the earth were fixed in position. But the earth itself is moving round the sun with a velocity v. The direction of the rays coming from S appears to be slightly altered due to this motion of the earth. If θ be the actual altitude of the star its value will be apparently altered by a small angle $\delta\theta$ due to the alteration of the direction of the rays referred to above. To find an expression for $\delta\theta$ let us combine the velocity of light C represented vectorially by

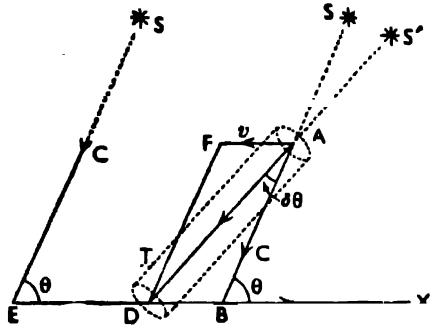


Fig. 214

AB with a velocity equal and opposite to v represented vectorially by $.1F$. The diagonal AD of the parallelogram $AFDB$ evidently represents the relative velocity of the star in direction as well as in magnitude. So the star will appear to be situated at S' on the line DA produced. So the apparent altitude of the star as measured by a telescope T will be $\angle ADB$ while its true altitude is $\angle ABX=\theta$. Hence the change in altitude $\delta\theta=\angle DAB$ and the apparent altitude $\angle ADB=\theta-\delta\theta$. In ΔADB,

$$\frac{BD}{\sin\delta\theta}=\frac{AB}{\sin(\theta-\delta\theta)}\ ;\ \text{or,}\quad \frac{v}{\sin\delta\theta}=\frac{C}{\sin(\theta-\delta\theta)}.$$

But as $\delta\theta$ is very small, it can be neglected compared to θ and $\sin\delta\theta$ can be replaced by $\delta\theta$.

$$\therefore\quad \frac{v}{\delta\theta}=\frac{C}{\sin\theta},\ \text{or,}\ C=v\times\frac{\sin\theta}{\delta\theta} \qquad \ldots \qquad \ldots \quad (1)$$

Thus from the above expression (1) for the change in altitude, we may find out the velocity of light C, if $\delta\theta$ is known, θ being directly measured by the telescope (approximately) and v being known from astronomical data. To determine $\delta\theta$, once the value of the apparent altitude is determined by the telescope when the earth is at E such that the directions of v and C are at right angles. This altitude is given by $(\theta-\delta\theta)$. Again after six months when the earth moves to the position E' where v and C are mutually perpendicular again, the altitude which is now $(\theta+\delta\theta)$ is measured. The difference of these two measurements gives $2\delta\theta$. Thus $\delta\theta$ being known C can be easily computed.

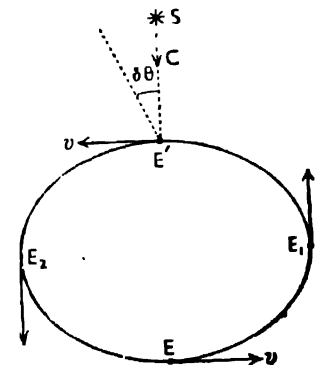


Fig. 215

(**Note** :—At positions E_1 and E_2 of the earth v is parallel to C and as such there is no change in altitude of S at these positions.)

Terrestrial Methods :—

(a) Fizeau's Method.—Fizeau was the first to measure the velocity of light (1849) over terrestrial distances. The principle of his method was to send a light signal to a distant station from which it travelled back and to calculate the velocity by noting the interval of time taken. The diagram of his apparatus in a simplified form is given in Fig. 216. A beam of light from a bright source S passes through the lens L_1 and is brought to a focus at the plane of the toothed wheel

D, after being reflected by a plate of glass P placed at an angle of 45° with the horizon. The toothed wheel rotates in a plane at right angles to the beam of light reflected from the glass plate P. The wheel has a large number of teeth equally spaced along its circumference, the opening between any two teeth being equal to the space occupied by a tooth. As the wheel is rotated, the light is alternately intercepted by a tooth and transmitted through the next opening. The plane of the disc is at the principal focus of a lens L_2, so that the transmitted rays emerge from L_2 as a parallel beam, which, after traversing a distance of a few miles, arrives at the lens L_3 and is brought to a focus at the

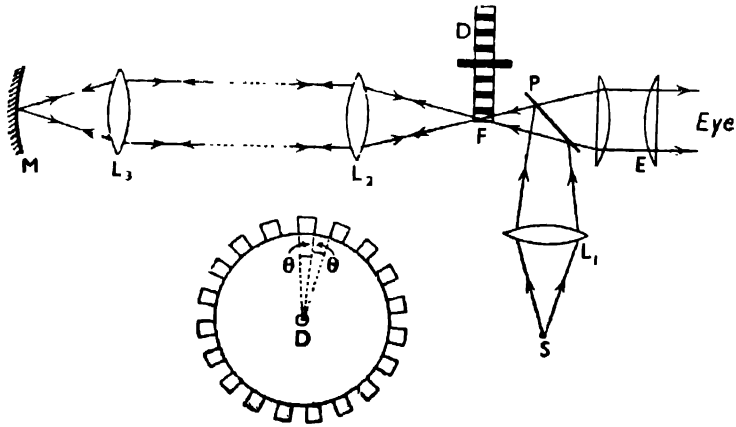


Fig. 216—Fizeau's Method.

surface of a concave mirror M whose centre of curvature is at the centre of the lens L_3. Then the rays are reflected back and retraverse their path as a parallel beam, and if the light is not obstructed, the rays will be reflected by P to S, some being transmitted to the eye through P and the eye-piece E. The observer will then see the image of S.

The wheel D can be rotated at a great speed about its axle. It is possible by rotating the wheel, and so adjusting its speed, that when a beam of light after passing through the opening between two teeth of the wheel comes back to the wheel after reflection from the mirror M, it finds a tooth in place of the opening and thus gets stopped. The result of this will be that no image of the source will be seen. If the speed be doubled, the light will pass through one opening and will fall on the next opening after reflection from M and the image will be visible. If light travels with a finite velocity, then the speed of rotation of the wheel will affect the image seen, and if light travels instantaneously, however great the speed of rotation

may be the image will always be visible. Fizeau found that the speed of rotation affected the visibility of the image, and so he concluded that light travels with a finite velocity. If the speed be so increased that more than 10 images are seen in one second, then due to persistence of vision a continuous image will be seen. But Fizeau so adjusted the speed that the light travelled from the plane of the wheel to M and back again in the time taken by the next tooth to come into the position of the opening, and thus light was cut off from the observer at E.

Calculations.—If light travels with velocity C and if it takes time T to travel from the plane of the wheel to the mirror M (a distance equal to x, suppose) and back, $C = 2x/T$.

Let n be the revolutions of the wheel made per second. Then the angle turned through by the wheel per second, *i.e.* the angular velocity of the wheel $= 2\pi n$.

The time taken by the wheel to turn through an angle θ which is subtended by an opening at the axle of the wheel $= \theta/2\pi n$.

When the light is cut off for the *first time* for a particular speed of the wheel,

$$T = \theta/2\pi n. \qquad C = \frac{2x}{\theta/2\pi n} = 4\pi nx/\theta.$$

If m = number of teeth in the wheel, and if an opening is equal in dimension to one tooth,

$$2m\theta = 2\pi \text{ ; or, } \theta = \pi/m.$$

$$\therefore \quad C = 4\pi nx/\theta = \frac{4\pi nx}{\pi/m} = 4mnx.$$

If during the time in which light travels from the first station to the second and back, the pth disappearance occurs, then $T = (2p-1)\theta/2\pi n_1$ where n_1 = number of revolutions of the wheel per sec. for this new adjustment.

$$\text{But } \theta = \pi/m \qquad T = \frac{(2p-1)}{2mn_1},$$

$$\therefore \quad C = 2x/T = 4mn_1x/(2p-1).$$

In Fizeau's experiment, the wheel had 720 teeth and made 12·6 revolutions per second when the image of the source was eclipsed. So the time for one revolution of the wheel was 1/12·6. The teeth and openings were of equal width and so they together numbered (2×720). Therefore, the time taken by a tooth to occupy the position of the preceding opening

$$= \frac{1}{2 \times 720 \times 12{\cdot}6} \text{ second.}$$

The distance between the plane of the wheel and the concave mirror M was 8633 metres ; so the light travelling from the plane of the wheel

to M and back traversed a distance of (2×8633) metres. Therefore, the velocity of light, according to this experiment,

$$= \frac{2 \times 8633}{\frac{1}{2 \times 720 \times 12{\cdot}6}} = 3{\cdot}13 \times 10^{10} \text{ cm. per sec.}$$

Advantages and Disadvantages :—

Advantages.—(1) It is a terrestrial method and the time of observation is small.

(2) All the quantities involved in the calculation are measurable and no assumptions are necessary. In these respects this method is superior to Romer's method.

Disadvantages.—(1) The primary difficulty for Fizeau was in regard to the production and measurement of a high speed of rotation of the wheel. Moreover, it was difficult to determine the speed at which the exact extinction of image took place.

These difficulties are avoided by recording the speed by an automatic speed recorder and by recording the speed once just before the extinction and then just after the extinction. The mean of these two records gives the speed at extinction.

(2) The light, when not passing through an opening, was reflected by the intercepting tooth back into the field of view causing a general illumination of the field of view. This made the image less distinct.

(3) A large open-air space, about 4 miles in Fizeau's experiment, is necessary and so this experiment cannot be done in a laboratory. Moreover, due to absorption of light in the open-air space and loss of light by reflection from the glass-plate P, the image received by the eye E is faint.

The disadvantages (2) and (3) are sufficiently removed by taking a strong source of light and by bevelling the teeth of the wheel and blackening the inner wall of the wheel as well as the teeth.

(b) Foucault's Method (A Laboratory Method).—Wheatstone conceived the idea of such an experiment, and subsequently it was taken up by Arago, but the success of the method goes to Foucault. The method is as follows :—

Sunlight passing through a rectangular aperture S, falls upon a plane parallel plate of glass P (Fig. 217). The transmitted light falls on the achromatic convergent lens L. Then it travels a distance d of about 20 metres and falls on a plane mirror Q which is rotated at high speed by an air-turbine round an axis at right angles to the plane of the paper. This plane mirror is at the centre of curvature of a concave mirror M of radius of curvature R. So whatever is the position of Q, the light that falls on M from Q, falls normally and is reflected back along the same path.

If Q is stationary, when the light reaches it again after reflection from M, the rays travel back to P and are mostly reflected from it

to form a real image of S at E at the side. If Q rotates rapidly, it moves through an appreciable angle θ by the time the beam leaves Q and returns to it from M, and consequently the light is reflected in

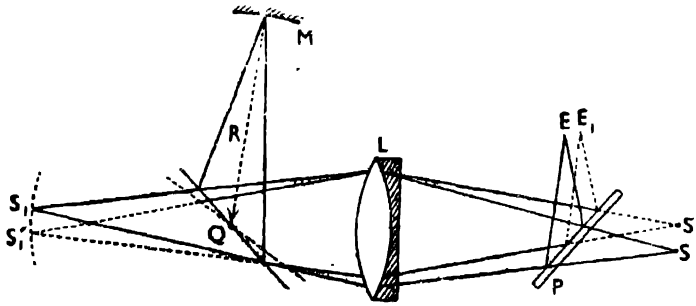


Fig. 217—Foucault's Method.

the direction of the dotted rays to form an image at E_1, in the observing eye-piece at the side (by light reflected from P) and an image S' by the light transmitted through P where $SS' = EE_1$. The displacement EE_1 ($=x$, say) of the image is measured by a micrometer eye-piece.

Calculation :—

Let a = distance of L from the slit S, d = distance of L from the mirror

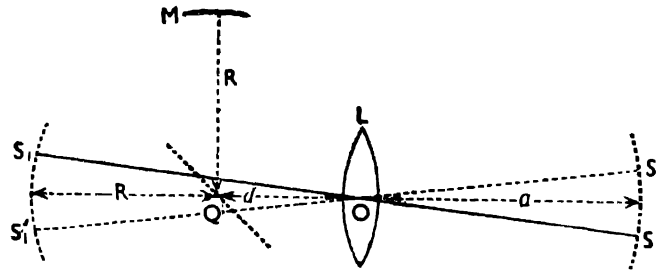


Fig. 218

Q (Fig. 218). The light returning from M being reflected from Q appears to come from a virtual point behind Q equally distant as M. Thus the pencils forming the images at E and E_1 (Fig. 217) appear to come from sources S_1 and S_1', where $QS_1 = QS_1' = R$ and the line joining S to the optical centre O of L passes through S_1. So also the line joining S' to S_1' passes through the optical centre O of the lens L.

From Fig. 218, $\frac{SS}{S_1S_1'} = \frac{a}{R+d}$ approximately, since the angle subtended at the lens L by the two pencils is small.

Or, $\frac{x}{S_1S_1'} = \frac{a}{R+d}$. Now, if a reflecting surface turns through θ, the beam reflected from it turns through 2θ, and so the angle

$$S_1QS_1' = 2\theta = \frac{S_1S_1'}{R};$$

or, $S_1S'_1 = 2\theta \times R$, $\therefore \quad x = \frac{a}{R+d} \times (2\theta \times R)$,

$$\text{or,} \quad \theta = \frac{(R+d)x}{2 \times a \times R} \qquad \ldots \qquad \ldots \quad (1)$$

If n = number of revolutions made by the mirror per second its angular velocity, $\omega = 2\pi n$. Let the light travel from Q to M and back in time t. Hence the velocity of light (C) will be given by,

$$C = \frac{2R}{t} \qquad \ldots \qquad \ldots \quad (2)$$

In the time the mirror Q rotates through θ, where

$$\theta = \omega t = 2\pi n \times t = \frac{(R+d)x}{2a \times R}, \text{ from (1).}$$

$$\therefore \quad t = \frac{(R+d)x}{2a \times R} \times \frac{1}{2\pi n} = \frac{(R+d)x}{4\pi a\, n.R}.$$

$$\therefore \quad C = \frac{2R}{t} = \frac{2}{(R+d)x} \times 4\pi.a.n.R = \frac{8\pi.\ a.\ n.\ R^2}{(R+d)x}.$$

This expresses C in terms of quantities which can be measured.

In Foucault's experiment the displacement $EE_1 = (x)$ of the image was observed to be 0·7 mm. and the velocity of light was found on calculation to be $2{\cdot}98 \times 10^{10}$ cms./sec.

Importance of Foucault's Experiment.—By putting a tube filled with water placed between Q and M, Foucault was able to show that the velocity of light is less in water than in air. This finding served as a great support of the wave theory of light, according to which the velocity of light in air should be greater than in water, for the *R.I.* of water relative to air is greater than unity. The importance of the method, it should be noted therefore, is that the velocity of light in different media may be conveniently determined by putting the same between Q and M. Such determinations decidedly proved that the emission theory of light, according to which the velocity of light in a denser medium should be greater, is wrong.

Merits and Defects of Foucault's Experiment.—The velocity of light in any transparent medium other than air may also be measured by this method. Moreover, the experiment may be carried out within a laboratory. M. Cornu has criticised this method because it depends on the following two assumptions—

(*i*) The laws of oblique reflection from a mirror, moving with a velocity small compared to that of light, are the same as those for a stationary mirror.

(*ii*) The laws of reflection of rays forming a real image, which moves transversely to the direction of the rays, are the same as when the image is stationary.

Moreover, the brightness of the image in Foucault's experiment is diminished very much, unlike in Fizeau's experiment, if the distance QM is increased in order to get an enlarged displacement of the image. This is because, when the mirror Q turns, the image sweeps over M and the light is intercepted and returned by M only for a small fraction of the time in which a complete rotation of Q takes place. Again, in Foucault's experiment the displacement EE_1 of the image is very small and cannot be measured with sufficient accuracy. This is the chief defect of Foucault's method.

(c) **Michelson's modification of Foucault's method**—Foucault's original method does not yield very accurate results because the deflection of the image here is only 0·7 mm. This deflection can, however, be increased by increasing the distance between the source

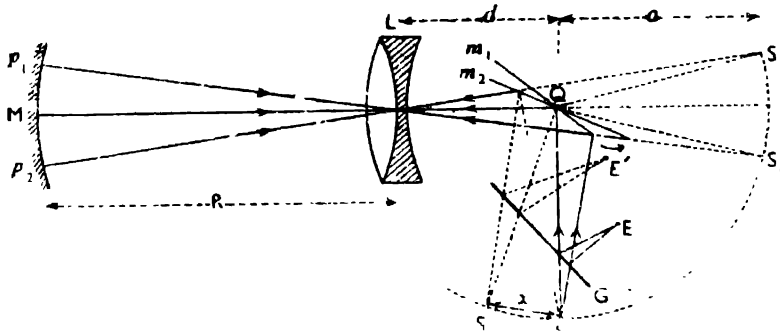


Fig. 219

and the lens and also the distance R, between the concave mirror and the rotating mirror. But an increase in R decreases the brightness of the image formed because light forming the image is reflected from the rotating mirror for that small fraction of a revolution during which the rays from it strike the concave mirror and this

fraction decreases as R increases. Michelson showed that the brightness of the image may be kept constant by placing the lens L between the concave mirror M and the rotating mirror Q and making the focal length of L so long that the image of the source S is formed on the surface of the concave mirror. The arrangements of Michelson are shown in Fig. 219. Here light from a strong source S passes through a plane glass plate G inclined at 45° to the rays and falls on the rotating mirror Q. As the mirror during its rotation is displaced from m_1 to m_2 through an angle θ, the light reflected from it forms images of the source at p_1 and p_2 due to refraction through L. p_1 and p_2 are two extreme points of M and may be considered to be the images of S_1 and S_2 which in their turn are the images of the source in Q in the two displaced positions. In the displaced position the image of S is actually formed at S' and as S is a strong source the distance SS' is measured in a perpendicular direction as EE' by a micrometer eye-piece with the help of the glass plate G as shown in Fig. 219.

As the rotating mirror Q makes one complete revolution the images of S in Q (*i.e.* S_1, S_2 etc.) describe a circle with Q as centre and $SQ=a$, as the radius (shown by dotted curve). While the image of S moves along the arc S_2S_1, it contributes light to form the image at S' or E'. For the fixed aperture p_1p_2 of M if S_1S_2 is kept constant, the brightness of the image will remain the same.

From Fig. 219, $\dfrac{S_1S_2}{p_1p_2}=\dfrac{a+d}{R}$ where $R=LM$, and $d=LQ$.

Thus S_1S_2 can be kept constant for a given value of a, by increasing R and d such that the ratio, $\left(\dfrac{a+d}{R}\right)$, remains constant. This is done by increasing the focal length of L so that the above ratio remains constant and hence the brightness of the image also constant.

To derive the working formula for the experiment let us suppose that θ is the angle that Q describes while S shifts to S' or E to E'. Then the angle SQS' or $\angle S_1QS_2$ is $2\theta=\dfrac{S_1S_2}{a}$. But $S_1S_2=SS'=EE'$ $=x$. Hence, $\theta=\dfrac{x}{2a}$.

The time required to describe this angle is $\dfrac{\theta}{\omega}=t$, where ω is the angular velocity of Q revolving at the rate of n revolutions per second. Thus $\omega=2\pi n$.

$$\therefore \quad t=\frac{\theta}{2\pi n}=\frac{x}{4\pi na}.$$

As Q describes angle θ, light from Q goes from Q to M and comes back so that the image S' becomes fairly steady. So if C be the velocity of light,

$$C = \frac{2(R+d)}{t} = \frac{8\pi na(R+d)}{x}.$$

From this relation C can be calculated. In Michelson's actual experiment R was 600 metres and x was 133 mm. The value of C calculated was $2{\cdot}9985 \times 10^{10}$ cm./sec.

The superiority of this method over the original method of Foucault lies in the fact that here the shift of the image S' from S (*i.e.* x) is about 200 times increased by increasing R but without appreciably sacrificing the brightness of the image. This larger shift ensures a more reliable and accurate value of C.

Example

In Foucault's original experiment the distance between the concave mirror and the rotating mirror was 20 metres. The converging lens was placed at a distance of 1 metre from the source, which was also the distance between the lens and the rotating mirror. The rotating mirror was rotated at the speed of 438 revolutions per sec. to get a deflection of the image of 0·7 mm. Calculate the velocity of light from the data supplied.

Ans. We know, $C = \frac{8\pi anR^2}{(R+d)x}$, notations being usual.

From the data supplied,

$a = 1$ metre $= 10^2$ cm.,

$n = 438$ per sec.,

$R = 20 \times 10^2$ cm ; $d = 10^2$ cm. ; $x = 0{\cdot}07$ cm.

$$\therefore \quad C = \frac{8 \times 3{\cdot}14 \times 10^2 \times 438 \times 4 \times 10^6}{21 \times 10^2 \times 7 \times 10^{-2}} = 2{\cdot}99 \times 10^{10} \text{ cm. per sec.}$$

(d) (i) Michelson's Octagonal Mirror Method.—In 1926 Michelson devised a very accurate method for the determination of the velocity of light—a method which is more accurate than that of Foucault. His arrangement is as follows :

Light from a strong source S passes through a convex lens L which converts it into a parallel beam [Fig. 220(*a*)]. This beam on coming out through a slit E is reflected from a face of an octagonal mirror R which is uniformly rotated at high speed by an air turbine, the exact speed of rotation being measured by a stroboscopic comparison with an electrically maintained standard tuning fork. Then after successive reflections from the stationary plane mirrors M_1 and M_2, it falls on a concave mirror C_1, the mirror M_2 being at its focus. The beam reflected by C_1 is received by a distant concave mirror C_2 placed co-axially with C_1. The light reflected from it is reflected back to it again, as shown

in the figure, by the plane mirror M_3 placed at the focus of C_2. From C_2 the parallel beam travels to C_1 again from where, after successive reflections from the stationary plane mirrors M_4 (placed at the focus of C_1) and M_5, it falls again on a face of the octagonal mirror. Reflected from this face, this beam enters into the observing telescope T after being totally reflected by the prism P. The optical path from

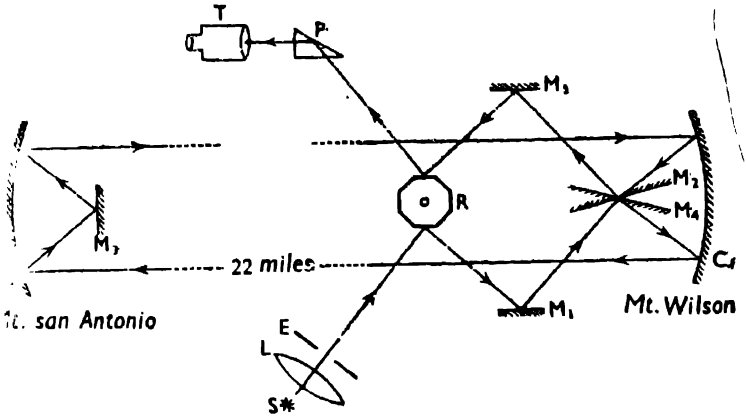


Fig. 220 (*a*)—Michelson's Experiment.

the slit E to the mirror M_3 is arranged to be equal to that from M_3 to T.

As R rotates, the image in the telescope T is displaced. But if the speed of rotation of R is so adjusted that the position of one face is occupied by the next in the time during which the light travels the optical path E to M_3 and back, the image in the telescope will be found at the same position where it was formed when the octagonal mirror was at rest.

The time taken is that of $\frac{1}{8}$th of a revolution of the octagonal mirror. The effect will be the same, if the speed of revolution is doubled, etc. If ω = angular velocity of the octagonal mirror, d = optical distance from E to M_3, C = velocity of light, then for the first speed,

$$\frac{1}{8}\times\frac{2\pi}{\omega}=\frac{2\times d}{C}\text{; or, }C=\frac{8\omega d}{\pi}.$$

If n = number of revolutions per second of the octagonal mirror,

$$\omega=2\pi n.\quad\therefore\quad C=\frac{8\times 2\pi n\times d}{\pi}=16nd.$$

Michelson set up C_1 on Mount Wilson, California, and C_2 on Mount Antonio at a distance of 22 miles. The mirror C_1 was of 30 feet

focal length and 2 feet aperture. His value for velocity of light in vacuum is $2{\cdot}99796 \times 10^{10}$ cms/sec. and the error, he claims, does not exceed 4 km./sec.

(ii) Michelson's One mile evacuated Pipe experiment.—In the Mount Wilson experiments, uncertainty in atmospheric conditions was one of the most important sources of errors in determining the velocity of light in vacuum. In order to overcome this difficulty, Michelson planned one experiment in which the light path was in a long evacuated pipe. The experiment was began by Michelson and his collaborators and was eventually completed by Pearson and Pease after Michelson's death in 1931. An iron pipe 1 mile long and 3 ft. in diameter was constructed with carefully sealed joints. By reflecting light to and fro along the pipe, as shown in the figure, an effective light path of 13 km. was obtained. In various determinations, the vacuum was maintained between 5·5 and 0·5 mm. of mercury. The arrangement of the optical system is shown in Fig. 220(*b*).

Light from a source slit S is reflected in one of the 32 faces of the rotating mirror R, and enters into the pipe. After reflection from a plane mirror M_1, it is rendered parallel by a concave mirror M_2. The parallel rays are then incident on another plane mirror M_3, and are reflected forward and backward between the plane mirror M_3 and M_4

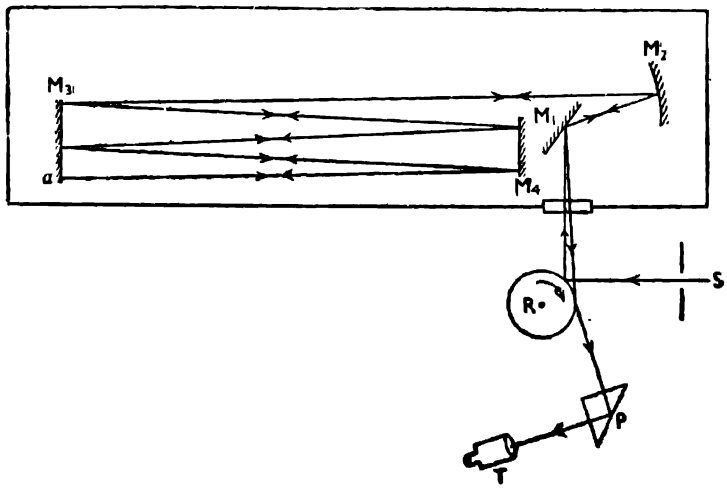


Fig. 220(*b*)

and ultimately made to fall normally on M_3 at a. The light rays then retrace their paths and fall on the adjacent face of the rotating mirror R, from where they are received by a telescope T.

after being totally reflected by the prism P. As before R was rotated at such a speed that during the time of transit of the light, one face of the rotating mirror just replaced the next one. As before, if D is the distance traversed by light from one face of the mirror back to the next one, $\frac{D}{C}=\frac{1}{32n}$; or, $C=32Dn$,

where n is the frequency of revolution of the rotating mirror, for which the image was found stationary for the first time. Making a correction for the residual air in the pipe, the final result for the velocity in vacuum was taken, out of an average of 2885 determinations as 299,774 km. sec.$^{-1}$ with an average deviation of 11 km. sec.$^{-1}$ from the mean.

(e) **Karolus and Mittelstædt method with Kerr Cells.**—The velocity of light was determined by Karolus and Mittelstaedt in 1928 in the laboratory by using two Kerr cells as the optical shutters. The principle of the experiment is the same as in Fizeau's experiment, a beam of light being periodically interrupted by an arrangement of two Kerr cells combined with two nicols.

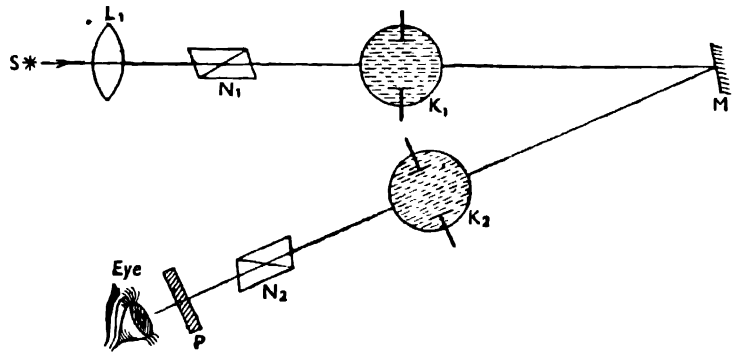


Fig. 221

Light from a source S is condensed by the lens L_1 and passed through a nicol N_1. A Kerr cell K_1 is so placed in front of N_1 that the plane polarised beam from N_1 passes through K_1 with its direction of vibration inclined at an angle 45° to the lines of force in K_1. The emergent light is elliptically polarised. After several reflections from mirrors (only one of which is shown as M in Fig. 221) arranged in a corridor of about 40 metres in length, the light falls on the second Kerr cell K_2 which is crossed with respect

to K_1, *i.e.* the directions of lines of force in the two are mutually perpendicular. If the electric fields in the two cells are identical, the phase difference between the two components of the elliptically polarised light passing through K_2 will be annulled. Thus on emergence from K_2 the beam will be once again plane polarised. The nicol N_2 through which the beam now passes being initially crossed with respect to N_1, no light will finally come out of N_2. This complete quenching can be observed on a screen placed at P. But this complete extinction is not ordinarily obtained because light travels with a finite velocity. So a definite time is required by light to pass from K_1 to K_2. During this interval the field in K_2 changes considerably with respect to K_1. So complete extinction is not obtained. By slowly adjusting the frequency of the applied field, the period of alternation of the field applied to K_1 and K_2 is made exactly equal to the time required by light to pass from K_1 to K_2. Thus the fields in the two cells are made exactly identical and a complete quenching is secured in nicol N_2. If ν be the frequency of alternation of the field applied, its period is $\frac{1}{\nu}$. Again if the optical path between K_1 and K_2 be D and the velocity of light C, then,

$$\frac{D}{C}=\frac{1}{\nu}; \quad \text{or,} \quad C=D\nu \qquad \ldots \qquad (1)$$

If n alternations occur before light passes from K_1 to K_2,

$$\frac{D}{C}=n\times\frac{1}{\nu}; \quad \text{or,} \quad C=\frac{D\nu}{n} \qquad \ldots \qquad (2)$$

From either of these relations (1) and (2), C can be determined.

In the actual determination by Karolus and Mittelstaedt, a steady bias p.d. of 6000 volts was applied to the Kerr cells on which was impressed an alternating p.d. of amplitude 2000 volts. The alternating p.d. was applied by a valve oscillator whose frequency could be accurately determined by the beat method with the help of a quartz oscillator of known frequency. The Kerr cells were filled with nitro-benzene for which the Kerr effect is very prominent. The result obtained by this method was $C=299778\pm 20$ km. per sec.

***(f) Anderson's method with a Kerr Cell.**—Anderson in 1940 determined the velocity of light with considerable precision. In his method also a Kerr cell is used in order to interrupt the wave train so that the method may be considered to be a modernised

version of Fizeau's method. A beam of light from the source S (Fig. 222) is allowed to pass first through a nicol n_1 and then through a Kerr cell K. A steady bias voltage of about 5000 volts was applied to this Kerr cell on which was superimposed a high frequency alternating voltage of about 1000 volts from an oscillator circuit. As a result a continuous train of light waves was transmitted through the second nicol n_2 which was crossed with respect to n_1. The intensity of this wave train varies about a mean value with a frequency equal to that of the alternating component of the applied field. Let the wave train transmitted through the second nicol consist of modulated pulses of length l and spacing l. These pulses fall on the half-silvered glass plate G inclined at 45° to the direction of the incident pulses and are divided into two parts. One part follows a shorter path to the mirror M_1 and thence to the photo-cell P through G. The other part follows a longer route to mirrors M_2, M_3, M_4, M_5 and back to G wherefrom a portion is reflected into the photo-cell P. If the distance from G to M_5 is D and that from G to M_1 is d, the pulses received by the photo-cell will have a path difference $2(D-d)$. They will give a minimum response in P when this path difference is an odd multiple of the pulse length l (because it corresponds to $\lambda/2$ in unmodulated waves). This condition is secured by adjusting the positions of M_1 and M_5 so as to have a minimum response in the photo-cell. If ν be the frequency of the oscillator and C the velocity of light, then

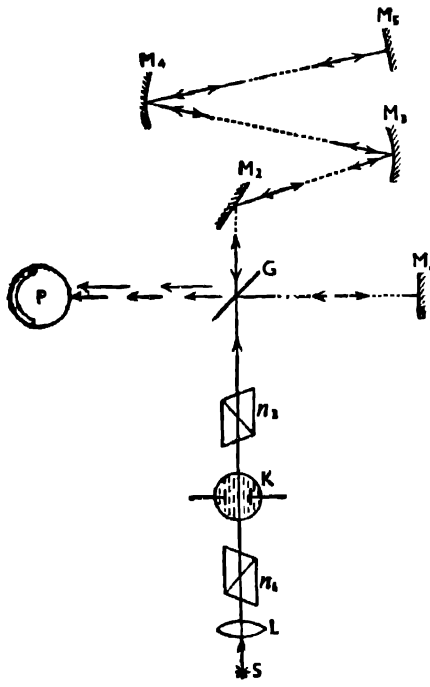


Fig. 222

$$\frac{C}{\nu}=2l.$$

Thus the condition for minimum i

$$2(D-d)=(2n+1)\,l=(2n+1)\,\frac{C}{2\nu},$$

where n is an integer. For computing n, a known standard value of C is accepted and n is calculated to the nearest integer.

Then, putting this value of n in the above expression an accurate value of C is found. The result obtained by Anderson for the velocity of light in vacuum is $C = 299776 \pm 14$ km./sec.

* **Recent Methods of determining 'C'.—**

During the second world war, the radio-location or radar method of finding the enemy targets necessitated an exact knowledge of the velocity of the electromagnetic waves. In radio-location, the hitherto calculated or accepted value of C was found to be somewhat lower. So in 1949 Aslakson used the radar method itself for accurately determining the velocity of electromagnetic waves. For this purpose two stations at a known distance was chosen and the time of transit of the electromagnetic waves for this distance was accurately determined. From his experimental data, C was found to $299{,}792 \pm 2{\cdot}4$ km./sec. In 1951, a subsequent determination gave $C = 299{,}764 \pm 1{\cdot}9$ km./sec.

In 1950 Essen in England and Hanson and Bol in U.S.A. determined C by the cavity resonator method and obtained results in excellent agreement with those obtained by the radar method. In 1951-54 Froome by using microwave interferometry estimated the value of C which is also in good agreement with the radar data. All these determinations show a definite deviation from the values obtained by optical methods. In the post-war period, a few accurate optical determinations have been made, of which the method adopted by **Bergstrand** yields a value of C practically identical with the value obtained by the radar method. **Bergstrand's Method** is briefly described in the following paragraphs.

The main difficulty in Anderson's method in the alignment of the beams entering the photo-cell has been avoided judiciously in Bergstrand's method. Here by using a Kerr cell light from the source S is modulated by a high frequency crystal controlled oscillator, O [Fig 222(a)]. This light is then reflected from M and made incident on the photo-cell P which gets its operating voltage from O. Thus the sensitivity of P varies with the same frequency as the intensity of the incident light. As the rapid light pulses from S take a definite time in traversing the distance $SMP = D$, a phase difference would occur between the voltage applied to the photo-cell P and fluctuations in the intensity of the incident light. The current passing through the meter G depends on this phase-difference.

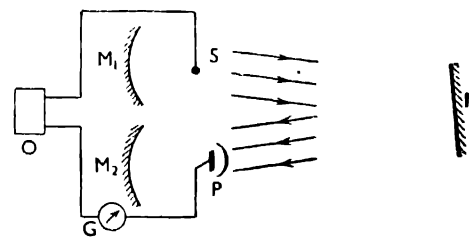


Fig. 222 (a)

The experimental arrangements in Bergstrand's method are diagrammatically shown in Fig. 222(*b*). S is a lamp serving as the source of monochromatic yellow light ; L_1 is a system of lenses condensing light from S on to a point near the Kerr cell K ; N_1 and N_2 are two crossed nicols. The light rays emerging from this system are made parallel by means of the lens L_2. The plates of the Kerr cell K are fed with a high frequency voltage from a quartz oscillator O_1 (8·33 Mc/sec.). O_2 is a low frequency bias voltage oscillator (50 cycles/sec.) and P is a photo-cell. Light from L_2 reaches P via M_1 M_2 M_3 M_4 M_5. M_2 and M_4 are concave spherical mirrors ; M_3 is a plane mirror , M_1 and M_5 are plano-concave lenses silvered on the plane face. M_1, M_2, M_3 form the sender and M_4, M_5 and the focussing lens L_3 the receiver. The spherical aberration of M_1 and M_4 are balanced by that of M_2 and M_5 and the chromatic aberration of M_1 and M_5 does not come into play as monochromatic light is used in the experiment.

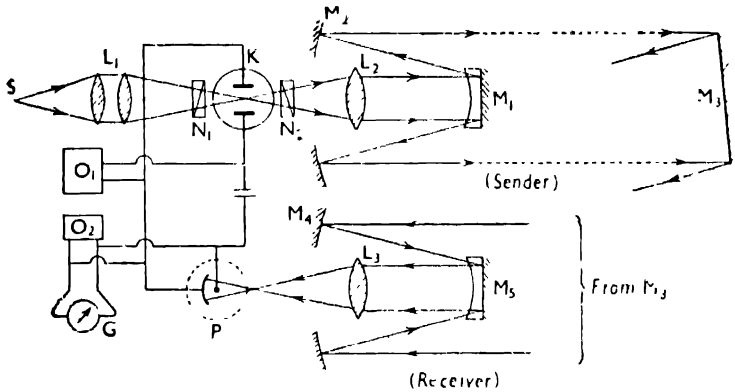


Fig. 222(*b*)

If the total effective light path be $2D$, the phase-lag between the source and the receiver for the modulated wave is $\omega.\frac{2D}{C}$, where C is the velocity of light and $\omega = 2\pi f$, f being the frequency of modulation. If this phase-lag be $\pi/2 - n\pi$, where n is an integer, the current in the current-meter G is zero. Hence the condition of zero current can be written as,

$$\omega\left(t - \frac{2D}{C}\right) = \frac{\pi}{2} - n\pi,$$

where t represents the total time of uncertainty arising out of the lag between the application of a voltage to K and the emergence of light from N_2, the transit time of electrons in the photo-cell and

the time of voltage transference from Kerr cell to the anode of the photo-cell.

$\therefore \quad D = K + \frac{2n-1}{8}\lambda \left(\because \quad \omega = \frac{2\pi C}{\lambda} \right)$, where K is a constant depending on t, which can be eliminated from two observations with two different values of D, say D_1 and D_2. Here λ is evidently the modulated wavelength.

Then, $(D_2 - D_1) = \frac{N\lambda}{4}$, where N is another integer. As N is an integer, it can be found from the approximate values of λ and $(D_2 - D_1)$. Now from a knowledge N and an accurate value of $D_2 - D_1$, λ can be accurately determined. Thus knowing the frequency of modulation, C can be determined from relation $C = \lambda f$. Bergstrand took $D_1 = 90$ meters and $D_2 = 7000$ meters and $f = 8{,}332{,}230$ cycles/sec. Though in the first determination Bergstrand's values of C did not approach appreciably the value obtained from radar experiments, his later determinations in 1950-51 yielded value of C equal to $299{,}793{\cdot}1 \pm 0{\cdot}25$ km./sec. which is in excellent agreement with the radar and other recent methods. In 1972, the National Bureau of Standards in U.S.A. determined C with the most modern equipment and got a value of $C = 299{,}792{,}456{\cdot}2$ metres per sec. Here the experimental error is supposed to be only 1% of the errors in the previous accurate estimations.

The Velocity of light in a Moving Medium.—

In 1859, Fizeau found the effect of the motion of the medium on the velocity of light by an arrangement [Fig. 222(c)] resembling the Rayleigh refractometer. Light from a narrow source S reflected from the inclined

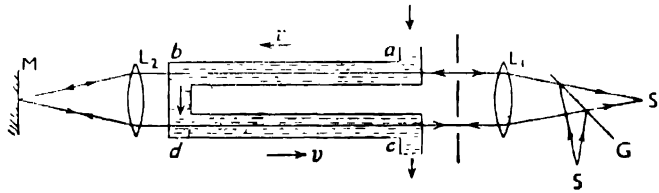


Fig. 222(c)

glass plate G is collimated by the lens L_1 and is made to pass through the water circulating system to the plane mirror M and back as shown in the figure. As is evident from the arrangements, a double slit pattern of fringe system will be formed at S'. The beam of light travelling along ab and returning along dc always travels in the direction of water flow. But the other beam travelling along cd and returning along bc travels in a direction opposite to that of water flow. So if the velocity of light be different in the two cases—one in the direction of a moving medium and the other in the opposite direction—a phase difference will arise and there will be a shift of the fringes at S_1. The fringe shift does not correspond to the Newtonian addition of velocities.

This was also latter on corroborated by experiments of Michelson. The result was first explained by assuming a kind of ether-drag by the moving water, while light propagates with a constant velocity with respect to other. But the assumption of ether-drag could not be established by the famous experiment of Michelson and Morley. From the standpoint of relativistic ideas, the results were, however, subsequently explained without any further assumptions.

148. Wave velocity and Group velocity :—We know that the light wave is simple harmonic in character and can be represented by the displacement equation,

$y = a \sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$ where λ is the wavelength of the wave and the velocity of this phase wave (*i.e. wave velocity* or phase velocity) is given by $V = \frac{\lambda}{T}$. But in almost all methods of determination of the velocity of light we actually observe a kind of signal or group consisting of a sharply bounded train of waves. So the velocity that we measure is, in most of the cases, the velocity of this signal which is known as the *group velocity* and not the velocity of the simple harmonic wave. The formation of this group or signal is due to the dispersion of the composite light as it passes through a dispersive medium. The component waves differing slightly in their wavelengths will combine together to give rise to a group or train of waves whose velocity (*i.e.*, group velocity) is actually determined in most of the experiments.

Let us consider two waves of wavelengths λ and $\lambda+d\lambda$ and periods T and $T+dT$ with the same amplitude a to be traversing a dispersive medium. Their displacements at any instant t are given by,

$$y_1 = a \sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$$

and, $y_2 = a \sin 2\pi\left(\frac{x}{\lambda+d\lambda} - \frac{t}{T+dT}\right)$.

∴ The combined displacement is given by,

$$y = y_1 + y_2 = a \sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) + a \sin 2\pi\left(\frac{x}{\lambda+d\lambda} - \frac{t}{T+dT}\right)$$

$$= 2a \sin \pi\left[\left(\frac{x}{\lambda} + \frac{x}{\lambda+d\lambda}\right) - \left(\frac{t}{T} + \frac{t}{T+dT}\right)\right] \cos \pi\left[\left(\frac{x}{\lambda} - \frac{x}{\lambda+d\lambda}\right) - \left(\frac{t}{T} - \frac{t}{T+dT}\right)\right]$$

$$= 2a \sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) \cos \pi\left(\frac{x d\lambda}{\lambda^2} - \frac{t dT}{T^2}\right)$$

$$= 2a \cos 2\pi\left(\frac{x d\lambda}{2\lambda^2} - \frac{t dT}{2T^2}\right) \sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) \quad \ldots \quad \ldots \quad (1)$$

This resultant displacement represents the group or signal that is propagated in the medium. The cosine factor in expression (1) represents the modulation of the sine wave which is shown in Fig. 223 as the envelope of the higher frequency carrier wave. Since in the group wave the effective wavelength and period are respectively $\frac{2\lambda^2}{d\lambda}$ and $\frac{2T^2}{dT}$, the group velocity can be written as

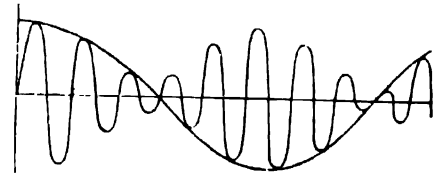


Fig. 223

$$U=\frac{\lambda^2}{T^2}\times\frac{dT}{d\lambda}. \quad \ldots \quad \ldots \quad (2)$$

We know that the wave velocity $V=\frac{\lambda}{T}$; or, $T=\frac{\lambda}{V}$.

$$\therefore \quad \frac{dT}{d\lambda}=\frac{V-\lambda\frac{dV}{d\lambda}}{V^2}$$

So from relation (2) we have, $U=V^2\left(\frac{V-\lambda\frac{dV}{d\lambda}}{V^2}\right)=V-\lambda\frac{dV}{d\lambda}$ ... (3)

In the various methods for finding the velocity of light, a definite group is separated from the continuous train of waves either by a toothed wheel, or rotating mirror or a Kerr cell. So the velocity measured is the group velocity U. In Bradley's aberration method no such group is separated whereas in Römer's method there is the question of sorting out the groups during the discontinuous process of observation of the eclipses. But both the astronomical methods yield almost equal values of the velocity measured—though one gives the wave velocity and the other the group velocity. This is because of the fact that both the determinations being in space, $\frac{dV}{d\lambda}=0$ there. So V practically coincides with U.

In the optical region, V in a transparent medium generally increases with λ so that $\frac{dV}{d\lambda}$ is positive. Hence the group velocity is less than the wave velocity in a transparent medium. Michelson measured the velocity of light in air and reduced it to velocity in vacuum. He then found the velocity of light in water as also that in carbon disulphide by Foucault's method as referred to in Art. 147, Terrestrial method (*b*). From the

ratio of these velocities he found that the refractive indices $\left(\mu = \frac{C}{V}\right)$ of water and CS_2 are respectively 1·33 and 1·77. The μ of water fairly tallies with the value obtained from the direct determination, but μ of CS_2 is found to be at variance with the experimental value of 1·64. The anomaly may be explained away if we remember that the velocities measured in water and in CS_2 are group velocities. In the case of water since $\frac{dV}{d\lambda}$ is negligibly small U and V become almost the same and hence the value of μ does not show much difference. But CS_2 being a highly dispersive medium $\frac{dV}{d\lambda}$ is fairly large and the determine velocity of light (U) is much less than V. So $\frac{C}{U}$ is greater than the actual $\mu \left(= \frac{C}{V}\right)$.

149. The value (C) of Velocity of Light and its importance .— The value of C that is accepted to be the most accurate is $(2·9986 \pm 0·003) \times 10^{10}$ cm /sec Birge in 1941 estimated the weighted mean value of C to be $299,776 \pm 4$ km. sec $^{-1}$, whereas in 1944 Dorsey suggested a new weighted mean of $299,773 \pm 10$ km. sec.$^{-1}$. The most generally accepted value, however, is 3×10^{10} cm./sec. or 186000 miles/sec. (1) It is considered to be one of the universal constants in Physics. (2) Since all electromagnetic waves travel with a velocity C, the frequency or the wavelength of any electromagnetic radiation can be evaluated from the relation $C = \lambda \times \nu$. (3) The mass and momentum attending a quantum of radiation moving with a velocity C are respectively given by $m = \frac{E}{C^2}$ and $p = \frac{E}{C}$. (4) According to the theory of relativity the mass of a body moving with a velocity v comparable with C is given as $m = m_0 \Big/ \sqrt{1 - \frac{v^2}{C^2}}$ where m_0 is the rest mass of body. (5) The constant C serves as the linking factor between the e.m. and e.s. system of units.

150. Light Year :—Distances between stars are enormously large. So in astronomy such distances are measured in terms of a very large unit, called the *light year*. It is equal to the distance through which light travels in one year, *i.e.*, $186000 \times 60 \times 60 \times 24 \times 365$ miles.

Questions (Chapter IX)

1. Describe carefully Fizeau's method of determining the velocity of light. (C. U. 1936, '38, '47 ; Punj. U. 1945 , Ban. U. 1949 , Gau. U. 1951)

2. Describe in details a laboratory method of determining the velocity of light. Mention some of the defects and state how they are minimised. (C. U. 1952, '54, '58)

3. Explain how the measurement of velocity of light in water shows that the emission theory of light is wrong.
(C. U. 1952 ; Hons. '33 , Pat. U. 1949 ; Bom. 1950)

4. Describe and explain Michelson's rotating mirror method of finding the velocity of light and compare its merits and demerits with those of other methods. (Del. U. '67 , C. U. '63 ; Bihar U. 1954)

5. Mention different methods available for determining the velocity of light and describe in details Foucault's rotating mirror method. Discuss the merits and demerits. (Punj. U. 1951 ; Pat. U. 1949 ; Ban. 1950)

6. Write a short note on the determinations of the velocity of light.
(C. U. 1942, '48, '51)

7. Mention different methods that have been used for the determination of the velocity of light and describe the one you consider to be the most accurate.
(Del. U. 1952 , Luck. U. 1951)

8. Give a critical account of some of the modern methods of measuring the velocity of light accurately. What is the best value ?
(C. U. H. 1939 , *cf.* Agra U. '47)

Compare it with the older methods of Fizeau and Foucault.
(Punj. U. 1950)

9. Describe a method of directly measuring the velocity of light in a transparent medium. How has it been shown that the velocity of light depends on the nature of the medium ? (C. U. 1931)

10. Describe in details Michelson's method of determining the velocity of light in (*a*) air, (*b*) liquid. (C. U. H. 1950)

11. In an actual experiment (on velocity of light by Fizeau's method), the wheel has 150 teeth and an equal number of spaces of equal width. If the distance of the mirror be 12 km , at what speed (in revolutions per minute) of the wheel the first eclipse occurs ? ($V = 3 \times 10^{10}$ cm. per sec.) (C. U. 1947)

[*Ans.* 2500 rev./min.]

12. In Fizeau's experiment, the wheel has 720 teeth and the distance of the mirror from the wheel is 8000 yds. The first eclipse occurs at a speed of 13·75 rev. per sec. of the rotating wheel. Calculate the velocity of light.

[*Ans.* 180,000 miles per sec. nearly]

13. Explain what are meant by 'wave velocity' and 'group velocity'. Find out an expression for the group velocity in terms of the phase velocity. What type of velocity do you determine in the experiments devised for finding the velocity of light ?

14. In Michelson's experiment, the distance between the slit and the revolving mirror is 28 ft and the mirrors are separated by 2000 ft. If the mirror is rotated at the rate of 258 revs. per sec., calculate the displacement of the image. ($C = 3 \times 10^{10}$ cm. per sec.)

[*Ans.* 11·24 cm.]

15. Describe Michelson's rotating mirror method for determining the velocity of light. (Utk. U. 1966 ; Del. U. 1969)

16. Describe an accurate method of determining the velocity of light.
(C. U. 1965)

151. Photometry :—The science of measurement of light is called *photometry* (*photos*—light, *metron*—a measure) and the instruments with the help of which such measurements are carried out are known as *photometers*. As the subject of artificial lighting is of gradually increasing importance now-a-days, various methods of accurately determining the illuminating powers of sources, the illumination at a point of an illumination surface and other factors on which the luminous flux and illumination depend have been devised. All such methods of qualitative and quantitative measurements on the properties of luminous flux and illumination are dealt with in subject of *photometry*.

152. Definitions of Photometric terms :—

(i) **Luminous Efficiency.**—We know that only a small portion of the total energy radiated by a source of light is capable of producing *visual sensation*. The ratio of this portion producing visual sensation to the total energy radiated is known as the luminous efficiency of the source.

(ii) **Luminous flux.**—As a beam of light is a stream of energy, it is quite natural to associate the idea of flux or rate of flow of energy with it. Hence *the luminous flux through an area is defined as 'the rate of passage of radiant energy evaluated with reference to its visual effect'*.

The *total luminous flux* from a source indicates the total amount of visual energy emitted by the source in all possible directions per sec.

(iii) **Luminous Intensity.**—If a body is a luminous source, it would appear as more or less intense according as it emits more or less luminous flux. A convenient measure of this property is the amount of luminous flux radiated by the body per unit solid angle. If the body is an extended one, this property would depend also on the direction of flow of the luminous flux. So in specifying the light-giving property of a body there must be reference to the direction of the flow of the radiated flux. The term *'luminous intensity in a particular direction'* is used to convey the above meaning and is *defined as the amount of luminous flux given out by a body per unit solid angle* (*i.e.*, per steradian *which is the unit of solid angle*) *in that direction*. The above definition applies equally to self-luminous or

non-luminous bodies. Formerly, the expressions '*illuminating power*' or '*intensity of emission*' of a source were used to mean what is described above as *luminous intensity*.

If Q is the amount of luminous flux emitted by a body uniformly in all directions, its *luminous intensity* in any direction is $I=\frac{Q}{4\pi}$, since 4π is the total solid angle round the body.

(iv) **Illumination or Illuminance.**—The *illumination or illuminance* at a point of a surface is defined as the amount of luminous flux incident on unit area round that point. Since the term flux inherently indicates a rate of flow, illumination also implies the flow of luminous energy per unit area per sec. Illumination, in general, does not signify any particular direction and the definition includes the flux incident on the surface from any direction whatsoever. Again since we are here considering only the flux incident on the surface and not what happens to it after incidence, it becomes evident that illumination is independent of the nature and colour of the surface and must not be confused with the brightness of the surface which depends on the visual impressions created by an illuminated surface. If a white piece of paper and a black piece be in turn placed at the same distance from the source at equal inclination they will have identical illumination but will not appear equally bright.

If a hollow sphere of radius r encloses a point source of light at its centre, the amount of luminous energy falling per second per unit area of the internal surface of the sphere will be $\frac{Q}{4\pi r^2}$, where Q is the total luminous energy emitted by the source per second. So the illumination E_0 at any point of the spherical surface is $E_0=\frac{Q}{4\pi r^2}$.

But $\frac{Q}{4\pi}=I_0$ represents the luminous intensity of the source.

$$\therefore \quad E_0=\frac{I_0}{r^2} \qquad \ldots \qquad \ldots \qquad (1)$$

This relation (1) is valid only for normal incidence of light *i.e.* E_0 refers to *normal illumination* only.

(v) **Oblique illumination and Lambert's cosine law.**—If the incident light falls obliquely on a surface its illumination is known as the oblique illumination which bears a definite relation with the normal illumination as has been shown by Lambert.

If a small surface AB (Fig. 224) of area ds inclined at an angle θ to the direction of incidence of the light rays from the source O be considered, then the illumination E at the point P on the surface (for oblique incidence) will be given by $E = \frac{dQ}{ds}$ here dQ is the luminous energy contained within the cone AOB ($\because$ ds is a small area).

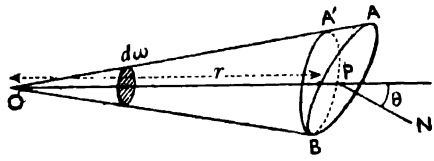


Fig. 224

If the area $A'B = ds \cos\theta$ be normal to the direction of flux considered, the normal illumination would have been,

$$E_0 = \frac{dQ}{ds \cos\theta}\ ;\ \text{or,}\ E_0 \cos\theta = \frac{dQ}{ds}.$$

$$\therefore\ E = E_0 \cos\theta.$$

In other words, if a surface is inclined to the direction of luminous flux at an angle θ, the illumination of the surface will be reduced in the ratio $\cos\theta : 1$. This is *Lambert's cosine law of illumination.*

Again since $E = \frac{I}{r^2}$ and $E_0 = \frac{I_0}{r^2}$, we can write $\frac{E}{E_0} = \frac{I}{I_0}$. But $\frac{E}{E_0} = \cos\theta$ from Lambert's law. Hence, $\frac{I}{I_0} = \cos\theta$ also.

$$\therefore\ I = I_0 \cos\theta.$$

Hence the *cosine law* also holds good in the case of luminous intensity, *i.e., in the case of emission.*

(vi) Brightness or Luminance.—As has been already referred to, surfaces having identical illumination may not appear equally bright to the observer because of differences in the nature and the colour of the surface. *The brightness of a surface in a given direction* is defined as the ratio of the luminous intensity of an element of the surface to the area of the element projected on a plane normal to the given direction.

Let I_0 be the luminous intensity of an element illuminated surface, ds (Fig. 225) in a direction normal to the element ds. From the definition of brightness, we get the brightness B_0 in

the normal direction as B_0 $\frac{I_0}{ds}$. Now if the surface element be viewed in a direction making an angle θ to the normal to ds and if I_θ be the corresponding luminous intensity, then the brightness in the direction θ is given by $B_\theta = I_\theta/ds \cos\theta$ since $ds \cos\theta$ here represents the element projected on the plane normal to the direction.

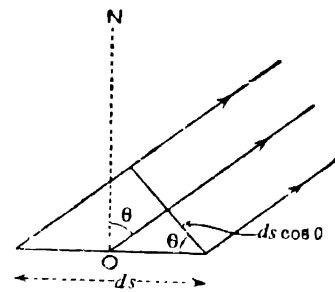


Fig. 225

We know that if E_0 and E_θ be illumination of the surface in normal and the oblique directions, then $I_0 = E_0 r^2$ and $I_\theta = E_\theta r^2$.

$$\therefore \quad B_0 = \frac{E_0 r^2}{ds} \text{ and } B_\theta = \frac{E_\theta r^2}{ds \cos\theta}.$$

But from Lambert's law, $E_\theta = E_0 \cos\theta$;

or, $E_\theta/\cos\theta = E_0$. $\quad \therefore \quad B_0 = \frac{E_0 r^2}{ds} = B_\theta$.

Thus the brightness of a surface is independent of the direction from which it is viewed.

Luminous Intensity, Illumination and Brightness compared :— *Luminous intensity* refers to a source and is measured by the luminous flux emitted by it per unit solid angle but the *illumination* refers to the incident luminous flux per unit area of a receiving surface. Luminous intensity may sometimes refer to an illuminated surface when it acts as a source or an emitter. Brightness of a source or illuminated surface refer to a subjective impression created by the source of the illuminated surface and is measured by the ratio of the luminous intensity to a normal element of surface. As has been already stated two pieces of black and white paper held at equal distance and at identical orientation to the flux will have the same illumination but will never appear equally bright.

***153. Apparent brightness of an object when seen with a naked eye :—**The apparent brightness of an object is defined as the quantity of light falling on unit area of the image formed on the retina. As the sensation of brightness is a visual impression, it is decided by the illumination of an image formed on the retina. If the illumination is greater the optic nerves will get a better response and as such the brightness will increase.

Let A = area of an extended object O, S = area of the pupil of the eye E, α = area of the retinal image R, u = distance of O from eye-lens E and v = distance between E and R (Fig. 226).

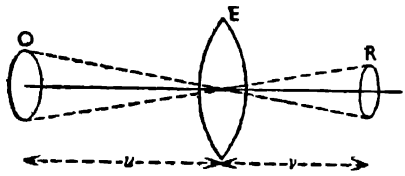


Fig. 226

If B be the brightness of the object O in the direction of the pupil of the eye, then from definition of brightness, $B=\frac{I}{A}$, where I is the luminous intensity of the source or object. $I=B.A$. If the pupil of the eye subtends a solid angle at O equal to $\omega=\frac{S}{u^2}$, the luminous flux falling on the pupil from O is given by $Q_1=I\omega$, because I represents the luminous flux per unit solid angle.

$$\therefore \quad Q_1=BA\omega=\frac{BSA}{u^2}.$$

This flux casts an image of area α on the retina. So the illumination of the image so formed is $\frac{Q_1}{\alpha}=\frac{BSA}{\alpha u^2}$ ... ... (1)

But since A and α represent areas, from Fig. 226.

$$\frac{A}{\alpha}=\frac{u^2}{v^2}.$$

$\therefore$ From relation (1), $\frac{Q_1}{\alpha}=\frac{BS}{v^2}$ ... ... (2)

So the illumination of the retinal image is $\frac{B.S}{v^2}$. Now as v is a constant for the eye, the illumination of the retinal image and hence the *apparent brightness of an extended object* is independent of the distance of the object from the eye. Its brightness appears to be the same at all distances if the intrinsic brightness of the object remains the same, the other factor, the area of the aperture of the pupil, remaining practically constant for a normal eye.

If the source is a point object, the image cast on the retina will also be a point image. So the brightness of the object will depend on the luminous flux concentrated by the pupil on the retina which in its turn depends on the solid angle subtended by the pupil at the position of the object. So as the object recedes from the eye the above solid angle subtended being smaller, the smaller will be the luminous flux concentrated on the retina by the pupil. So the *brightness decreases as the point object recedes from the eye.*

***154. Apparent brightness of an image (of an extended object) in air as formed by a lens.—**

Let B = the brightness of the extended object O.

A = the area of the object O, I = luminous intensity of the object, S = area of the surface of the lens (L), α = area of the image O_1 formed, u = object distance (OL), v = image distance (LO_1), $\omega = \frac{S}{u^2}$ = solid angle subtended by L at O and $\omega_1 = \frac{S}{v^2}$ = solid angle subtended by L at O_1 (Fig. 227).

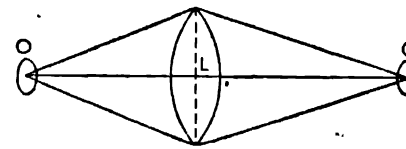


Fig. 227

Supposing that the lens does neither absorb nor reflect any flux incident on it, the luminous flux incident on the lens is given by $Q_1 = I\omega = B.A.\omega = \frac{B.S.A.}{u^2}$. This luminous flux is contained within the solid angle ω_1 of the image. Hence the luminous intensity I_1 of the image is given by $I_1 = \frac{B.S.A.}{u^2} \Big/ \omega_1 = \frac{B.S.A.}{u^2} \Big/ \frac{S}{v^2} = B.A.\frac{v^2}{u^2}$.

$\therefore$ The brightness of the image $B_1 = \frac{I_1}{\alpha} = B.\frac{A}{\alpha}.\frac{v^2}{u^2}$.

But $\frac{A}{\alpha} = \frac{u^2}{v^2}$. $\therefore$ $B_1 = B.\frac{u^2}{v^2}.\frac{v^2}{u^2} = B.$

Thus the *image appears as bright as the extended object* itself and the brightness does not depend on the lens aperture. Hence it is not possible to increase the brightness of the image of an extended object by means of a lens or a telescope when the image is formed in air.

If the image is cast on a screen the illumination of the image can be shown to be proportional to the brightness B of the source and the area of the lens aperture S. (See last article)

***155. Brightness of sky and stars as seen through a telescope.—** If, instead of an extended object, a point object like a star be viewed through a lens or a telescope, its apparent brightness will depend on the luminous flux concentrated by the telescope or the lens on the eye. So the bigger the concentrating lens (*i.e.* the objective of the telescope) the more will be the luminous flux entering the pupil of the eye and the brighter the star will appear. If the star is viewed with a naked eye the flux concentrated on the retina will be proportional to the area

of the aperture of the pupil, *i e* to πr^2 when r is the radius of the pupil of eye But when a telescope objective, whose radius of aperture is R is used, the light concentrated and finally reaching the retina (supposing no loss from any part of the instrument) will be proportional to πR^2. So the apparent brightness of the star increases in the ratio R^2/r^2. So the stars normally invisible to the naked eye can easily be seen through a telescope. Even during day time when the background of the extended sky as seen by a telescope appears none the more brighter the stars may be seen against this background as their brightnesses will be many times enhanced by the telescope.

156. Standards and Units :—The four basic terms in photometry are *luminous flux, luminous intensity, illumination and brightness*. For measurement and comparison it is necessary to have a reference standard of at least one of these quantities and also a suitable unit for it. In photometry, the primary standard maintained is that of 'luminous intensity'. To realise this standard practically various prototypes have been proposed and used uptil now. They may be classified under two heads, (*a*) flame standards, and (*b*) temperature standards.

(a) **Flame Standards.**—The following three flame standards are historically the most important—

(*i*) The Standard Candle, (*ii*) Vernon Harcourt Pentane Lamp and (*iii*) Hefner Standard Lamp.

(i) **The Standard Candle**—It is a candle made of spermaceti wax $\frac{7}{8}$ inch in diameter, weighing one-sixth of a pound and burning at the rate of 120 grains per hour.

The illuminating power (luminous intensity) of this candle in the horizontal direction was the old British standard for illuminating power and its illuminating power was called unit illuminating power and designated as one candle-power (C P.). So the term *candle-power*, used almost universally in photometry, owes its name to the standard candle.

Note—This standard was subsequently found not to fulfil the requirements of a *true* standard, for its illuminating power was found to vary, though slightly, with factors such as pressure, temperature, humidity, CO_2-content of the air, the shape of the wick etc. So it stands abolished today as *standard* though the name candle-power still remains to represent the luminous intensity of a light source.

(ii) **Vernon Harcourt Pentane Lamp.**—This lamp superseded the old British standard candle and is still used as the British standard of luminous intensity, and until January, 1948, served also as the *international standard.*

By this lamp (Fig. 228) a flame is produced by burning the vapour of pentane oil (a light volatile oil derived from paraffin) and the lamp has no wick in it.

The illuminating powers of other sources were used to be referred to this lamp as a primary standard until January, 1948, and its illuminating power was taken as 10 c.p., for a flame of standard specification.

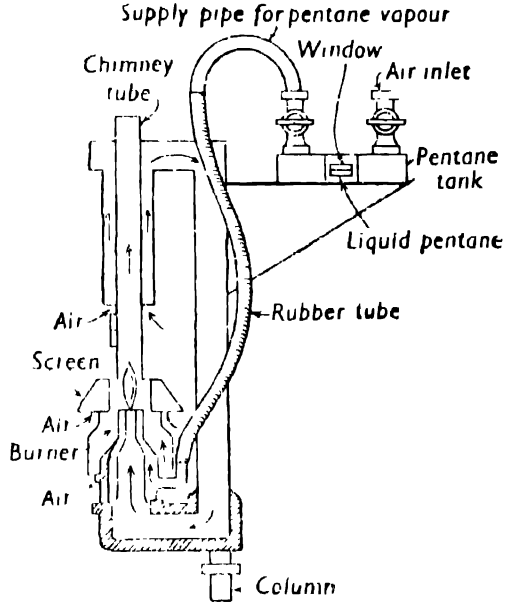


Fig. 228 – The V. H. Pentane Lamp.

(iii) **Hefner Standard Lamp.**—The lamp (Fig. 229) is the German standard of illuminating power. A specially constructed wick of untwisted cotton is used in this lamp and the fuel used is a pure grade of amyl acetate. When properly adjusted, the illuminating power of the flame, reduced to standard conditions of pressure, humidity and CO_2-content, is one *Hefner candle-power.* One Hefner candle-power is equal to 0·9 old British candle-power.

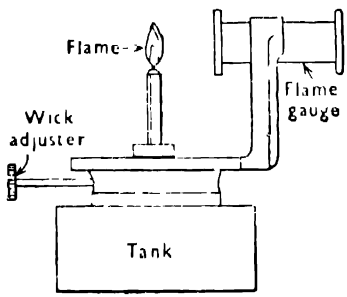


Fig. 229 The Hefner Lamp.

(b) **Temperature Standard :—** This is the most recent standard and its working principle lies in the fact that a body at high temperature emits luminous flux. The *luminous intensity* of a source of **freezing platinum** constitutes this standard. This standard was introduced on January 1, 1948, by the Comite' Inter-

national des Poids et Measures, and is now the internationally accepted standard.

The temperature standard or the primary standard as it is commonly called consists of a Planckian or full radiator made of fused thoria and is a cylinder 5 cm. long and 0·25 cm. in internal diameter (Fig. 230). This cylinder contains powdered fused thoria to a depth of about 1·5 cm. This is placed vertically inside a fused thoria crucible packed with pure platinum. The cylindrical cavity radiator along with the thoria crucible is contained in a pot made of a highly refractory material the intervening space being filled with powdered thoria (unfused) to provide adequate thermal lagging. The whole arrangement is heated by a high frequency induction furnace. By carefully controlling the supply of energy to the furnace the platinum in the crucible may be kept in the molten state at its freezing point. The lid of the radiator is pierced by a hole 0·15 cm. in diameter which acts as the primary standard. One-sixtieth of the luminous intensity of this black-body source at the freezing point of platinum is called a *Candela* which is used as the unit of luminous intensity.

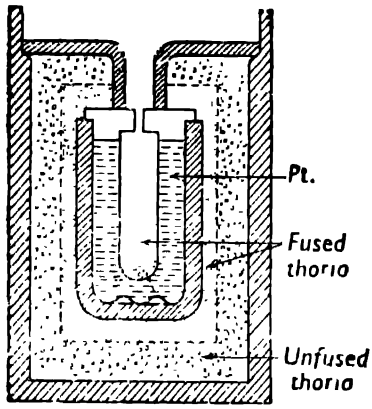


Fig. 230

Units

Unit of Luminous Intensity.—The modern unit for it is called the **candela** which is $\frac{1}{60}$ of the *luminous intensity* of the new international primary standard (freezing platinum).

If it is stated that luminous intensity of a source is 25 candelas, it means that the stated luminous intensity is $\frac{25}{60}$ of the luminous intensity of the primary standard.

[N.B.—Relation between '*Candela*' and the old unit '*candle-power*'. The primary standard has been found to have a luminous intensity of about 59 units of the original candle-power per square centimetre of the source. Thus one candela equals $\frac{59}{60}$ of one c. p. (original)].

Depending on the design of a source, the luminous intensity of a source varies in different directions and so the following two terms

have been found to be important in photometry from practical considerations.

Mean Horizontal Candle-Power (M.H.C.P.)—It is the average value of the luminous intensity (measured in candle-powers) of a source of light in all directions in the horizontal plane passing through the source of light.

Mean Spherical Candle-Power (M.S.C.P.)—It is the average value of the luminous intensity (measured in candle powers) of a source of light in all directions round the source, and is therefore the same as the candle power of an ideal source of light which radiates the same luminous flux *uniformly* in all directions.

Unit of Luminous Flux.—It is called the **Lumen** which is defined as the amount of luminous flux emitted per unit solid angle by a source having a luminous intensity of one *Candela*.

Thus the total flux emitted per second by a source of luminous intensity of one candela is 4π *lumens*.

Unit of Illumination.—Here the unit is defined as that illumination (formerly called *intensity of illumination*) which is produced on a surface of *unit area* placed normally to the direction of flux at a *distance of unity* from a source emitting *one lumen of luminous flux*.

In the above definition, the unit is called one *foot-candle* if the unit area is taken as one square foot and the unit distance as one foot. It is called a *metre-candle* (or *lux*), if unit area is taken as one square metre and unit distance as one metre. It is called a *centimetre-candle* (or *phot*) if unit area is taken as one square centimetre and unit distance as one centimetre.

The brightness or luminance in any direction is measured by the number of candelas per unit area in that direction. The unit is one *Stilb* (*i.e.* 1 candela/cm.^{2}) or one *Nit* (*i.e.* 1 candela per sq. metre). *Lambert* is another type of unit of brightness which is defined as the brightness of a surface when the total luminous flux emitted per second in all directions on one side of a square centimetre of the emitting surface is one lumen.

Conversion Table

1 metre candle = 1 lux = 1 lumen per sq. metre.
1 centimetre-candle = 1 Phot = 1 lumen per sq. cm.
= 10^4 lux.
1 foot-candle = 1 lumen per sq. ft. = 10·764 lux.
So, Lumens = foot-candles × area in sq. ft.
= metre-candles × area in sq. metre
= centimetre-candles × area in sq. cms.

It may be noted that the *illumination* on the earth's surface due to the *Full Moon* is approximately equal to 1 foot-candle, and that due to the sun is 60,000 foot candles. The average eye can see conveniently and without strain if the object it is looking at has an illumination of 3 to 6 foot-candles.

157. Principle of Photometry:—In photometric measurements, we generally compare the luminous intensities (*i.e.* Illuminating Powers) of two sources and if the luminous intensity of one of them be known, that of the other may also be determined. In actual photometric measurements various methods are adopted. But in most of these methods, the two sources A and B (Fig. 231) are made to illuminate a surface S either in between them or on the same side of both The illumination of this surface due to the source A is then made identical with that due to B This is called *matching*. If I_1 and I_2 be the luminous intensities of A and B respectively and if r_1 and r_2 be the distances of S from A and B, then illuminations at S due to these two sources are respectively,



Fig. 231

$$E_1 = \frac{I_1 \cos\theta_1}{r_1^2} \text{ and } E_2 = \frac{I_2 \cos\theta_2}{r_2^2}.$$

So when exact matching has been observed, $E_1 = E_2$.

$$\therefore \frac{I_1 \cos\theta_1}{r_1^2} = \frac{I_2 \cos\theta_2}{r_2^2}.$$

When the flux is incident on S normally from both the sources, $\cos\theta_1 = 1 = \cos\theta_2$.

$$\therefore \frac{I_1}{r_1^2} = \frac{I_2}{r_2^2}; \text{ or, } \frac{I_1}{I_2} = \frac{r_1^2}{r_2^2} \quad \ldots (1)$$

Relation (1) represents the general relation of most of the photometric measurements.

158. Different types of Photometers:—

(a) Rumford's Photometer,—In this photometer an opaque rod C is vertically fixed in front of a ground glass screen S (Fig. 232). Two lights A and B whose luminous intensities are to be compared are placed in front of the rod, and their distances from the screen are so adjusted that the two shadows A_1, B_1, which are formed side by side on the screen corresponding to them, are made equally dark. Both the shadows being equally dark, the illumination on the screen (*i.e.* in the position of the shadows) due to both the sources must be the same because the shadow cast by the

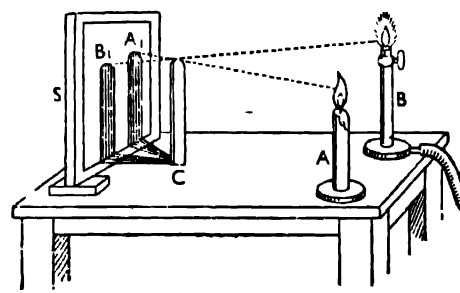


Fig. 232

rod cutting off rays from one source is illuminated by the other source.

If I_1 be the luminous intensity of source A and r_1 be the distance of A from the shadow cast by B, I_2 be the luminous intensity of the source B and r_2 be its distance from the shadow cast by A, then the illumination on the screen due to the source $A = \frac{I_1}{r_1^2}$ and that due to the source $B = \frac{I_2}{r_2^2}$.

Since the illuminations are equal, we have,

$$\frac{I_1}{r_1^2} = \frac{I_2}{r_2^2} \text{ ; or, } \frac{I_1}{I_2} = \frac{r_1^2}{r_2^2}.$$

That is, the luminous intensities of the two sources of light are directly proportional to the squares of their distances from the screen at the position of balance.

(b) **Bunsen's Grease-spot Photometer.**—In this photometer (Fig. 233), on an optical bench EF the two sources of light, A and B, are placed on opposite sides of a white paper screen C. The screen

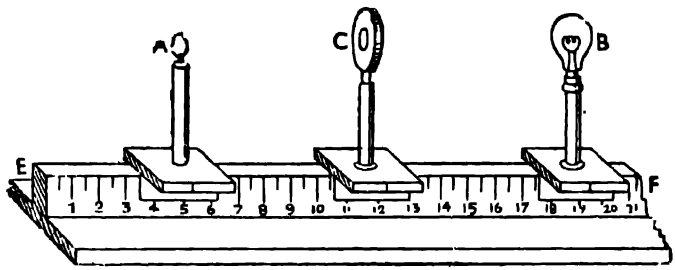


Fig. 233—Bunsen's Photometer.

C, called the photometer head, consists of a piece of white paper with a spot of grease or oil in the middle. The heights of the two sources of light and the grease-spot are so adjusted that their centres are on the same horizontal line. The grease-spot being more translucent than the rest of the paper allows more light to pass through it, and so, if the screen is illuminated by one of the source only and observed from the same side, *the spot* sends back less light to the eye by reflection than the rest of the paper and so the *spot appears darker than the rest of the paper.* When the observer looks at it from the other side, *i.e.* by transmitted light, *the spot appears brighter* than the rest. The same thing happens

in case only the other source is used. If now the screen is illuminated by both the sources, one from each side, and the distances of the two sources from the grease-spot are so adjusted that the spot is made equally bright on either side, *i.e.* both sides look alike, the illumination at the screen due to both the sources is equal. If r_1 and r_2 be the distances of the two sources having luminous intensities I_1 and I_2 from the screen, we have,

$$\frac{I_1}{I_2} = \frac{r_1^2}{r_2^2}.$$

Improved Photometer Head.—In Bunsen's photometer the screen with the grease-spot is often provided with two plane mirrors, inclined at equal angles on the two sides of it, so that an observer may see both the faces of the grease-spot at the same time by reflection from the two mirrors as in Fig. 234, each face of the screen being looked at by one eye. This arrangement helps the observer in making his judgment of equal illumination by observing both the surfaces simultaneously without movement of the eye.

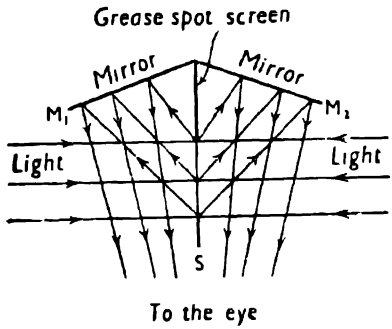


Fig. 234

Theory of the Bunsen Photometer.—Let q_1, q_2 be the quantities of light falling per second on unit area of the screen C (Fig. 233) from the sources A and B respectively, q_1 and q_2 evidently representing the illumination of C due to A and B respectively. Suppose a is the fraction of unit quantity of incident light diffusively reflected from the greased portions of the screen. Then $(1-a)$ will represent the fraction that should be transmitted. But, owing to the partial opacity of the spot, exactly this fraction will not be transmitted. Let $k(1-a)$ be transmitted when k is a constant, less than unity, depending on the power of absorption of light by the grease.

Hence, the total quantity of light reaching the eye, on the side of the source A, from unit area of the greased surface $= q_1 \times a$ by reflection $+ q_2 k(1-a)$ by transmission. Similarly, the quantity of light reaching the eye on the other side $= (q_2 \times a) + q_1 k(1-a)$.

$\therefore$ At the position of balance, we must have,

$$q_1 a + q_2 k(1-a) = q_2 a + q_1 k(1-a),$$

$$\therefore \quad q_1 \{a - k(1-a)\} = q_2 \{a - k(1-a)\}, \qquad \therefore \quad q_1 = q_2.$$

Considering the ungreased portion of the screen, the same result may also be arrived at. The only difference in the two cases will be in the values of the constants a and k, and the total quantity of light reaching the eye on either side of the screen in this case will be greater. Therefore, at the position of balance, q_1 being equal to q_2, we have, $q_1 = q_2 = I_1/r_1^2 = I_2/r_2^2$, where I_1, I_2 are the luminous intensities of the lamps, and r_1 and r_2 the respective distances of the lamps from the photometer head at the position of balance.

To verify the law of Inverse Squares with the help of the Bunsen's Photometer.—Take four candles (arranged close together) on one side and one candle on the other. Proceeding as suggested in the above experiment it will be found that the row of four candles will require to be twice as far from the grease-spot as the single candle for equal illumination.

Then, we have, $\frac{I_1}{I_2} = \frac{4}{1} = \frac{r_1^2}{r_2^2}$ $\therefore$ $\frac{r_1}{r_2} = \frac{2}{1}$.

Similarly, if two electric bulbs of different luminous intensities, say I_1 and I_2 (which are known), are fitted on the stands at the two sides of the spot, and their positions adjusted for equal illumination of the spot, it will be found that the ratio of I_1 and I_2 equals the ratio of the squares of their respective distances from the spot; the law is thus verified.

(c) **Lummer-Brodhun Photometer**:—The Lummer-Brodhun photometer is extensively used in accurate measurements on Photometry based on the visual observation. The attached figure (Fig. 235) shows an arrangement of the photometer. A is a screen of some pure white material like barium sulphate. The two surfaces of A are illuminated by two sources S_1 and S_2, whose luminous intensities are to be compared. P_1 and P_2 are two isosceles right-angled prisms and D is a combination of two right-angled prisms which are in contact only in the central region of their bases with Canada balsam. The outer region of the bases are separated by a thin film of air. The whole arrangement is to be enclosed in a chamber blackened inside. T is a telescope for receiving light from D through a window in the chamber. First let us suppose that the rays from S_1 only fall on the left face of A. They are then scattered from A and some of these scattered rays (1, 2, 3) fall on the prism P_1 and next on the left component of D. The rays 1 and 3 here get totally reflected at the glass-air interface of the base but the ray 2 enters T directly without any deviation. So in the telescope the central region becomes brighter and the outer portion comparatively darker. Again if we suppose that the rays from S_2 only fall on the right face of A. The scattered rays 4, 5, 6, falling on P_2 get

reflected therefrom and finally fall on D. The ray 5 passes out straight through the central region and the rays 4 and 6 are totally reflected from the outer region of the base of D. They are finally received by T. So in the telescope the central region is darker and the outer region appears bright. Now if both S_1 and S_2 are simultaneously effective and if their distances from A are properly adjusted, the illumination at the central region which is purely due to the rays from the left face of A can be made exactly equal to that at the outer region which is contributed by the rays from the right face of A. When the matching is perfect the field of the telescope becomes uniformly illuminated. When such matching condition is attained the illuminations on both the faces of A are identical. Hence if I_1 and I_2 be the luminous intensities of S_1 and S_2 at distances r_1 and r_2 from A, then from the principle of photometry it follows that $\frac{I_1}{I_2}=\frac{r_1^2}{r_2^2}$. Thus I_1 and I_2 can be compared.

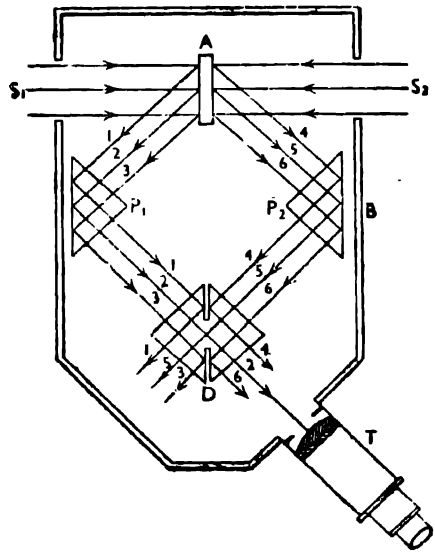


Fig. 235

If the diffusing powers of the two faces of A are not the same, the result may be faulty. To compensate for this A is rotated through 180° and the experiment repeated. The mean of these two sets gives a more accurate result.

(d) Guild's Flicker Photometer : The arrangements in a flicker photometer are shown in the attached figure (Fig. 236). Light from a comparison source S_0 is made to fall normally on a screen A of magnesium oxide by means of a right-angled isosceles prism P. A portion of light diffused from A is allowed to fall on the circular hole H of the sighting tube T, in a direction of 45° with the surface of A. The inner surface of the sighting tube T is whitened and is illuminated uniformly by a lamp, the diffused light from which enters T through the diffusing plate D. This gives a field of constant illumination. W is a rotating sector disc, the sectors and openings of which are accurately made 90° [Fig. 236(*a*)]. The right hand faces

of the sectors are coated with magnesium oxide. This sector W is rotated about a horizontal axle at an adjusted speed with the help of an electric motor. While a sector intercepts the rays from A to H, it will allow in its turn light from the source S_1, as diffused from its right hand face, to enter H. When an opening replaces this sector during the rotation the light from A enters H. So as the rotation continues the eye at E will have flickering sensation due to different illuminations in the sectors and in the openings at

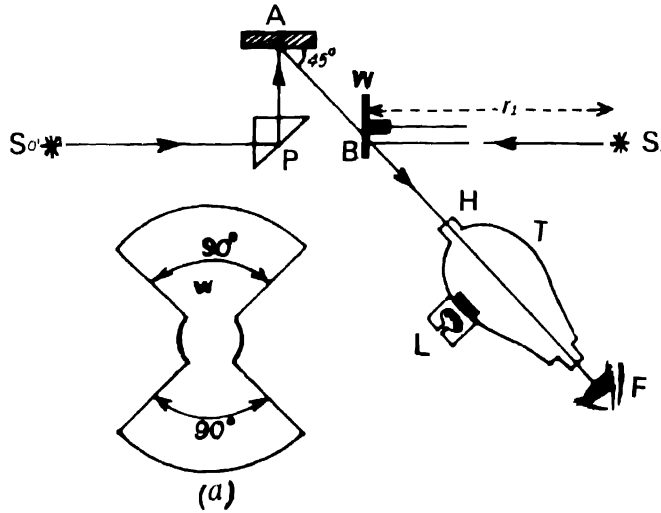


Fig. 230

B. Now by adjusting the distance (r_1) of S_1 from B, this flickering effect can be done away with The speed of W is adjusted to the minimum value corresponding to no flickering sensation. At this stage the illumination at B due to S_0 and S_1 are identical. The second source S_2 is now taken in place of S_1 and its distance (r_2) from B is again adjusted till for the same speed of rotation of W no flickering occurs. Hence the illumination at B due to S_2 is same as that due to S_0. If I_1 and I_2 be the luminous intensities of S_1 and S_2 respectively, then from the condition of matching, we get,

$$\frac{I_1}{I_2} = \frac{r_1^2}{r_2^2}.$$

With the flicker photometer the luminous intensities of sources emitting light of different colours can be satisfactorily compared.

159. Applications of Photometry :—

(a) Determination of Percentage of Light transmitted by a Glass Plate :— A suitable photometer is set up as usual between two

sources of light and the distance between the sources are adjusted until the illumination at the photometer head are equal. The glass plate is then placed between the photometer and any of the two sources. So the illumination (as seen on the photometer head) due to the source of light near the plate is diminished, and so it will be necessary to move this source more towards the photometer in order to make the illuminations again equal.

If r_1 be the initial distance of this source from the photometer and r_2 the final distance when the glass plate is interposed, the intensity of illumination in the beginning is $\frac{I}{r_1^2}$, when I is the luminous intensity of the sources, while the final value is $\frac{xI}{r_2^2}$, where x is the fraction of light transmitted by glass plate. The other source being at the same place as before, these illuminations are equal, that is,

$$\frac{I}{r_1^2}=\frac{xI}{r_2^2}; \quad \text{or,} \quad x=\frac{r_2^2}{r_1^2}.$$

Hence the percentage of light transmitted by the plate is $100x=\frac{100r_2^2}{r_1^2}$.

(b) Determination of the Percentage of light reflected by a reflector :—Here also a suitable photometer is taken and the illumination produced on the photometer head A due to a source S_1 at a distance r_1 from A is matched with that due to the comparison source S_0. Then S_1 is shifted to the position S'_1 and the reflector R is made to reflect the rays from it on to the photometer head. By properly adjusting the positions of S'_1 and R, the illuminations on A are again exactly matched. Let $(a+b)$ be the effective distance of S'_1, I be the luminous intensity of S_1 (*i.e.* also of S'_1) and x be the fraction of light reflected from R. Since in both the cases the matching is done against the same illumination, we have,

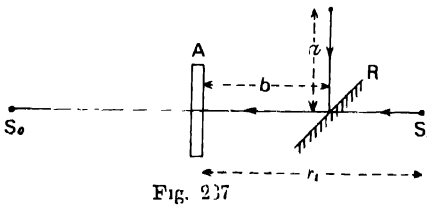


Fig. 237

$$\frac{I}{r_1^2}=\frac{xI}{(a+b)^2};$$

$$\text{or,} \quad x=\frac{(a+b)^2}{r_1^2}.$$

Hence the percentage of light reflected from the reflector R is given by,

$$100x = \frac{100(a+b)^2}{a}.$$

(c) **Measurement of mean spherical candle power of a source :** —The mean spherical candle power of lamp can be conveniently measured with the help of the *integrating photometer* (Fig. 238) devised by Ulbright. The lamp L is enclosed in a hollow sphere painted white on the inner surface with Zinc oxide. The sphere is fitted with an opal glass window at V, which is screened from the direct rays from L by a stop S. The diameter of the sphere is many times the dimensions of the lamp or the window. The light from L after several diffused reflections from the inner surface of the sphere reaches V. The illumination and the effective brightness of V depends on the total luminous flux originating from L and is thus proportional to the mean spherical candle power of the lamp. The apparatus is, however, to be calibrated with reference to a standard lamp of known photometric properties. This is done in the following way.

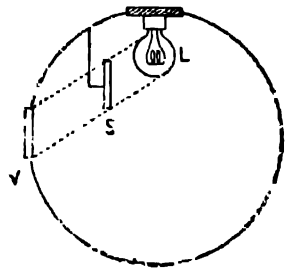


Fig. 238

The standard lamp is mounted on a photometer and its candle power (luminous intensity) in any specified direction is found by matching against a lamp of known candle-power. This determination is done in various directions of the standard lamp. On a graph paper showing the polar co-ordinates r and θ, distances proportional to the measured candle-powers are set out radially in the proper directions. If such curves be drawn for all planes through the axis of the lamp, they will form the *polar-surface* of the lamp.

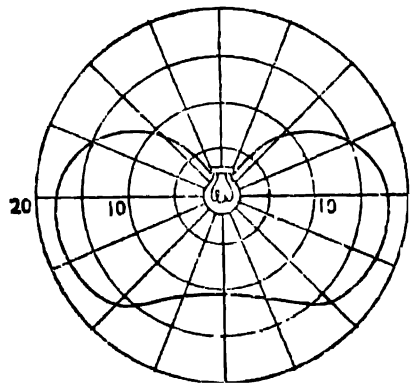


Fig. 239

In Fig. 239, the *polar curve* for a particular plane passing through the

axis of the lamp is shown. If a sphere of volume equal to that enclosed by the above polar surface be drawn, its radius gives the *mean spherical candle-power* of the standard lamp is in question. The actual evaluation is rather tedious but once the M. S. C. P. is known for a standard lamp, it may be used for calibrating the integrating photometer. The calibrated instrument may then be used for subsequent evaluations of the M.S.C.P.'s of other lamps.

160. Measurement of Illumination :—

(a) Weber Illumination Meter.—The illuminometer due to Weber consists of a wide tube B fitted with the test plate A at one of its ends. To the other end of B are attached two tubes D and E and a Lummer-Brodhun cube P is placed at the junction of these three tubes as shown in Fig. 240. An opal glass plate is placed at O and a standard lamp of known luminous intensity can be slided in the tube D with a rack and pinion arrangement. The position of the lamp L in D can be read off from a scale attached to the sliding rod R. The light from the place whose illumination is to be measured is directly received on A and the illumination at the central region of P is matched against that at the outer region due to the standard lamp L by adjusting the position of the latter. The scale on R is directly calibrated with respect to the illumination at A due to a lamp of known luminous intensity. Hence the position of L in D as read out on R directly gives the illumination at A due to any other source.

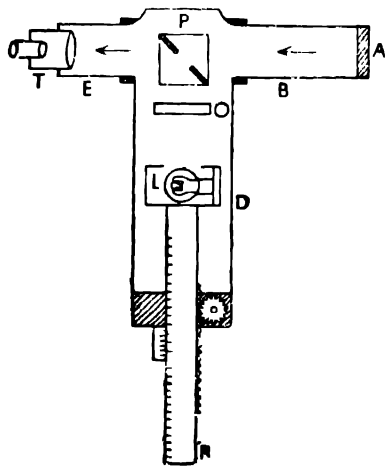


Fig. 240

(b) Photovoltaic Illuminometers :—In the modern forms of illuminometers photovoltaic cell are used. Here the cell consists of a layer of cuprous oxide (Cu_2O) formed on a surface of metallic copper. The boundary between the oxide layer and the metallic plate is known as the *barrier plane* and the cell is consequently sometimes called a barrier-layer type of photoelectric cell (Fig. 241). A metal film S [Fig. 241(a)] in the form of a ring is sputtered on to the Cu_2O layer in order to make the electrical contact with the external circuit perfect. When light falls on the Cu_2O layer of the cell electrons emitted from the Cu_2O layer penetrate the barrier plane

and reaches the cu-plate. So the Cu_2O layer becomes the $+ve$ electrode and the cu-plate the $-ve$ electrode of the external circuit when it is connected to a galvanometer through an adjustable

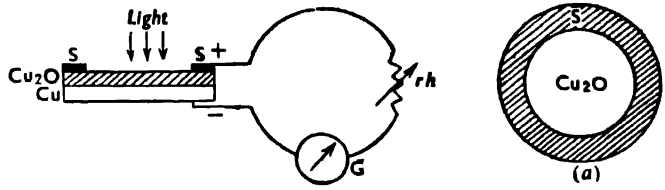


Fig. 241

resistance. The current in the galvanometer is directly proportional to the illumination on the surface of the Cu_2O-layer. So the galvanometer is generally calibrated in foot-candles or other suitable units to give the illumination on the Cu_2O-layer directly. The range of the instrument can also be varied by changing the resistance in the external circuit.

The modern exposure-meters for photographic works are also based on the same principle. Here the galvanometers are so calibrated as to give stops and exposure times directly.

161. Luminous Efficiency of a source :—As has already been mentioned *the luminous efficiency* of a source is defined as the ratio of the visual energy to the total energy emitted by a source. There are, however, several ways of expressing the efficiency of a source. The *overall efficiency* or simply the *efficiency* of a source indicates the ratio of the light-output in lumens to the power consumed by the source in watts. For a carbon filament lamp this efficiency is nearly 2·6 lumens per watt consumed. The overall efficiency may be divided into two parts. If P watts be the total power input of which the total energy radiated per second is R watts and the luminous flux emitted is L lumens, then the overall efficiency

$$\eta=\frac{L}{P}=\frac{L}{R}\times\frac{R}{P}=\eta_l\times\eta_r$$

where, $\eta_l=\frac{L}{R}$ represents the luminous efficiency of the source in lumens per watt radiated, and $\eta_r=\frac{R}{P}$ represents the radiating efficiency of the source.

For moderate intensities the human eye is most sensitive to a wavelength $5{\cdot}56\times10^{-5}$ cm. in the green. At this wavelength a luminous flux of 621 lumens necessitates the consumption of energy at the rate of one watt. Thus 1 lumen is equivalent to $\frac{1}{621}$ watt $=0{\cdot}00161$ watt. This is sometimes referred to as the (least) *mechanical equivalent of light*. *Absolute Efficiency* of a source is measured as the ratio of the

luminous flux expressed in watts at the above rate to the input power in watts.

162. Sources of Light :—In towns and cities artificial sources of light are generally used for better lighting of streets, buildings etc. Electric lamps and gas lamps are extensively used in most of the towns in India. But in big cities various types of discharge lamps are used for efficient and better illuminations of the streets, the railway stations and the factories etc. The discharge lamps may be distinctly divided into two categories :

(1) **Cold cathode lamps.**—They contain a gas at low pressure between two metallic electrodes connected to a high voltage supply and produce a diffuse glow (of positive column) filling the tube which may be bent to any shape (*vide* Current Electricity : Discharge through gases). The colour of the light depends on the gas used, e.g. neon—red, argon—blue, helium—yellowish, hydrogen—pink, etc. Such lamps are generally used for advertisement purposes.

(2) **Hot Cathode lamps.**—In a hot cathode lamp the cathode is generally cylindrical in form and is made of thoriated tungsten. A heating coil is inserted inside. The pressure of an inert gas (Neon or Argon) inside is generally about 10 mm. It operates at a comparatively low voltage. The filament or heating coil emits electrons when heated and the discharge becomes more efficient. Generally some quantity of Na or Hg is put inside the tube which evaporates during the discharge and finally the characteristic strong sodium or mercury light is emitted by the lamp.

The fluorescent lamps are also extensively used now-a-days for various lighting purposes. They are essentially hot cathode lamps and work both on A.C. and D.C. The glass tubes of these lamps contain mercury vapour at low pressure and a little argon which helps in striking the tube. The inner wall of the tube is painted with a suitable fluorescent powder like zink silicate, calcium tungstate, etc. When the tube is lighted with the help of a starting switch, the ionised mercury vapour gives out visible and ultra-violet light and the latter effectively produces fluorescence in the fluorescent coating of the tube. The wattage consumed in these tubes are comparatively low while the luminous intensity of the tubes are much higher than the ordinary lamps. (For details see Current Electricity).

Examples

1. In a grease-spot (Bunsen) photometer light from a lamp with a dirty chimney is exactly balanced by that of a candle distant 10 cms. from the spot. When the chimney is cleaned the candle has to be shifted by 2 cms. to obtain a balance. Calculate the percentage of light absorbed by the dirty chimney.

Ans. Let P_1 be the luminous intensity of the lamp with the dirty chimney, P_2 that with the clean chimney, and d the distance of the lamp from the grease-spot. Then, we have, $P_1/d_1^2 = 1/10^2$ ... ... (1)

Next the distance of the candle becomes (10 − 2) cms. when the chimney is cleaned. Hence, we have, $P_2/d_1^2 = 1/8^2$ ... ... (2)

From (1) and (2), $\frac{P_1}{P_2} = \frac{8^2}{10^2} = \frac{64}{100}$, $\therefore P_1 = \frac{64}{100} P_2$.

Hence the dirty chimney absorbs $(1-\frac{64}{100})$ or, $\frac{36}{100}$ of the total light. So the percentage of the light absorbed $=\frac{36}{100}\times 100=36\%$.

2. A photographic print is found to be satisfactory when the exposure was for 15 seconds at a distance of 2 feet from a 16 candle-power lamp. At what distance must the same paper be held from a 32 candle-power lamp in order that on exposure of 20 seconds will give the same result ?

Ans. The amount of the light falling perpendicularly upon unit area of the photographic plate in one second, *i.e.* the illumination at a distance of 2 feet from the 16 c. p. lamp $=16/2^2$; hence the amount of light falling in 15 seconds $=(16/2^2)\times 15$.

Again, if the paper be held at distance d feet from the 32 c. p. lamp, the amount falling in 20 seconds $=32/d^2\times 20$.

In order that the result in both the cases be equal, $32/d^2\times 20=16/2^2\times 15=60$;

or, $\quad d^2=\frac{32\times 20}{60}=\frac{32}{3}$; whence $d=3{\cdot}23$ ft.

3. A book is to be read from a distance of 5 ft. from an electric lamp, for which the best illumination is taken to be 5 ft. candles. Assuming 20% of the light reaching the book is lost by reflection at the ceiling and walls, calculate the candle-power of the lamp which should be used for the purpose.

Ans. Direct rays from the lamp reaching the book provide 80% of the light, whose illumination $=\frac{5\times 80}{100}$ ft. candles $=4$ ft.-candles.

If x be the candle-power of the lamp required, illumination at a distance of 5 ft. $=\frac{x}{5^2}$ ft.-candles. $\therefore\ \frac{x}{5^2}=4$. That is, $x=100$ c. p. Hence, a 100 c. p. lamp will be required.

4. A 300 c. p. lamp is suspended 5 feet above the centre of a rectangular table 8 ft. × 6 ft. Calculate the illumination at (a) centre (b) at a corner of the table.

Ans. (*a*) Illumination at the centre $\frac{I}{r^2}=\frac{300}{5^2}=12$ foot-candles.

(*b*) Illumination at the corner $=\frac{I\cos\theta}{r^2}$ (from Lambert's law) where r and θ are the polar co-ordinates of the corner point.

To determine these co-ordinates, let us first evaluate the diagonal of the rectangular table. Diagonal $=\sqrt{8^2+6^2}=10$ ft.

$\therefore$ Half the diagonal is the distance of the centre from the corner of the table. This is 5 feet.

$\therefore\ r=\sqrt{5^2+5^2}=5\sqrt{2}$,

$\therefore\ \cos\theta=\frac{5}{5\sqrt{2}}=\frac{1}{\sqrt{2}}$.

$\therefore$ Illumination E at the corner $=\frac{300}{50\sqrt{2}}=3\sqrt{2}$ foot-candles.

5. A 100 c. p. lamp is placed 2 ft. below a horizontal plane mirror which reflects 65 per cent of the incident light. Three feet below the lamp is a horizontal table. Determine the illumination at a point on the table 4 ft. to one side of the vertical through the lamp assuming that the emission is uniform in all directions. (Assume that the coefficient of reflection is independent of the direction of incidence.)

Ans. The illumination at the point due to the lamp itself is $E_1=\frac{I\cos\theta}{r^2}$.

Here $\quad r=\sqrt{3^2+4^2}=5.\quad \therefore\ \ E_1=\frac{100}{25}\times\frac{3}{5}=2{\cdot}4$ ft.-candles.

The image of the lamp is 2 ft. above the mirror and hence $(2+2+3)=7$ ft. above the table. Its luminous intensity is $\frac{65}{100}\times 100$ c.p. $=65$ c.p.

$\therefore$ Illumination at the point due to this image is $E_2=\frac{I_2\cos\theta_2}{r_2^2}$, Here, $r_2=\sqrt{7^2+4^2}=\sqrt{65}$ and $\cos\theta_2=\frac{7}{\sqrt{65}}$.

$$\therefore \quad E_2=\frac{65}{65}\times\frac{7}{\sqrt{65}}=0{\cdot}87 \text{ ft.-candle.}$$

$\therefore$ Total illumination is $E_1+E_2=2{\cdot}4+0{\cdot}87=3{\cdot}27$ ft.-candles.

Questions (Chapter X)

1. Define 'Luminous intensity' and 'illumination' and distinguish between the two terms.

Show how the illumination at a point on a surface depends on the inclination of the surface with the rays. (C. U. 1965)

A 100 c. p. lamp is suspended 20 ft. above the middle of a road 20 ft. wide. Calculate the illumination (*a*) at a point P on the road vertically below the lamp, (*b*) at a point on the edge of the road nearest the lamp and (*c*) at a point on the road 30 ft away from P.

[*Ans.* (*a*) 0·25 ft.-candle, (*b*) 0·18 ft-candle; (*c*) 0·043 ft-candle.]

2. Describe an apparatus for comparing the luminous intensities of two sources of light.

3. Explain Lambert's Cosine Law.

Explain what is meant by a *lambert*. To what does the term refer? Clearly bring out the significance of the photometric term involved in the definition and establish its relation with the luminous intensity and illumination.

4. A draftman's table, which can be rotated about a horizontal axis passing through its centre, is illuminated by a small 100 candle-power lamp suspended vertically above the centre of the table at a distance of 4 ft. Find the reduction of the illumination at the centre of the table when it is tilted from the horizontal position through 30°.

[*Ans.* 13·4%]

5. Two lamps of 10 and 20 c p. respectively are placed 100 cms. apart. Show that there are two positions on the line through the lamps in which a screen may be placed so as to receive equal illumination from each.

6. Explain the principle of a grease spot photometer. What are meant by foot-candles, lux, phot and Nit?

7. How would you compare the luminous intensities of two sources of light?

There are two electric lamps directly above a table, one of 32 c.p. is at a height of 4 feet and the other of 50 c.p. at a height of 10 feet. Compare the illuminations of the table when both lamps are alternately burning.

[*Ans.* 4 : 1]

8. If a photographic print can be made with 4 sec. exposure at a distance of 4 ft. from a 32 c.p. lamp, what exposure will be required if the negative is held at 2 feet from a 16 c.p. lamp?

[*Ans.* 2 secs.]

9. Two lamps balance on a shadow photometer at distances of 60 and 42 cms. from the screen. The stronger lamp is then covered with a glass shade which transmits 80% of the incident light. How far must the other lamp be displaced in order to restore balance?

[*Ans.* 4·95 cms.]

10. A lamp produces a certain illumination on a screen when situated at a distance of 85 cms. from it. On placing a sheet of glass between the lamp and the screen the lamp must be moved 5 cms. nearer to the screen to produce the same illumination as before. What percentage of light is stopped by the glass ?
[*Ans.* 11·42%]

11. Give the theory and method of comparing the luminous intensities of sources of light differing in colours.

12. Write short notes on : (1) Lummer-Brodhun photometer, (2) Fluorescent lamp, (3) Integrating Photometer.

13 What is a flicker photometer ? Explain the working principle of the Guild's flicker photometer.
A foot ball pitch, 120 metres by 60 metres, is to be illuminated for night play by equal banks of 1,000 watt lamps supported on 12 towers, which are distributed around the ground to provide approximately uniform illumination of the pitch. Assuming 40 per cent of the total light emitted reaches the playing pitch and that an illumination of 1,000 lumens per sq. metre is necessary for television purposes, calculate the number of lamps on each tower. The overall luminous efficiency of each lamp is taken as 30 lumens per watt.
[*Ans.* 50]

14. Explain the principle of Lummer-Brodhun photometer and its practical applications. (Del. U. 1969)

15. What would be the apparent brightness, as viewed through a telescope, of the following objects placed at a great distance ; (*a*) point source of light ; (*b*) an extended surface of uniform illumination. (C. U. H. 1937, '43, '52)

16. Explain the principle and construction of the Lummer-Brodhun photometer and discuss its practical application. (C. U. H. 1930, '37, '41)

17. An extended object appears equally bright at all distance and the brightness of its image formed by a refracting system cannot exceed the brightness of the object. Comment on the statement.
Explain how it is possible with the help of a large telescope to see stars, during day time, though they are not visible to the naked eye. (C. U. H. 1943, '52)

CHAPTER XI

SPECTROSCOPY

163. Dispersion :—When a beam of white light incident at one face of a prism passes through it, the beam emerging at the other face is not only deviated towards the base of the prism but is also broken up into different colours. This was first observed by Newton. Suppose, sunlight is allowed to enter into a darkened room through a narrow slit *P* (Fig. 242). The light is allowed to fall on the face *AB* of a glass prism *ABC* placed with its refracting edge *A* parallel to the slit. The emergent light is then allowed to fall on a white screen *S* where a coloured patch is obtained. The colour of one end of the patch is red (R) and the other

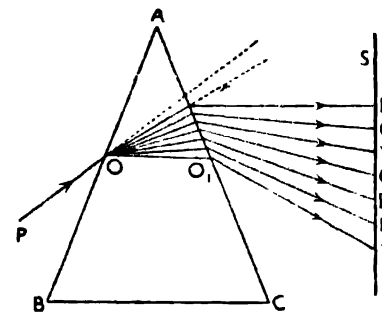


Fig. 242—Dispersion of Sunlight

end **violet (V)**. Besides these, beginning with violet there are five other colours—**indigo, blue, green, yellow** and **orange**, as shown in the figure by the initial letters.

Such phenomenon of breaking-up of white light into its component colours is known as **dispersion**, and the coloured band is called a **spectrum**. In the **solar spectrum**, there are, however, a large number of different tints, each of which shades of gradually into the next, and, in general, **seven principal colours**, violet, indigo, blue, green, yellow, orange and red (*vibgyor*), spoken of as the colours of the spectrum, are clearly distinguishable in it.

If, now, by making small pin-holes in the screen, each of these constituent colours be separated and allowed to be deviated by another prism, it will be found that *violet is the most deviated and red the least*; that different colours occupy unequal spaces in the spectrum, *the violet occupying the greatest and orange the least*; and that each of these colours is simple and cannot be broken up again into any other colour.

Light, which can be decomposed into several colours, is called **compound** or **composite light**, and the light of a single colour is referred to as **simple** or **monochromatic**.

Newton's Conclusions—

(1) **White light** is not simple, but it is a **combination of seven colours.**

(2) The colours can be separated by passing white light through a prism.

(3) The different colours are deviated to different extents. This is what is expressed by saying that the different colours have different **refrangibility**; amongst the visible colours violet is the most refrangible and red the least.

(4) The deviation of the yellow is intermediate between the deviation of the two extreme colours, violet and red. So yellow is called the **mean colour.**

164. Impure and Pure Spectra:—If a single ray of white light could be isolated and allowed to pass through a prism, it would split up into its separate colour constituents producing a pure spectrum; but in practice a single ray cannot be isolated, and even if a very narrow pencil is taken, each ray of that pencil produces a spectrum of its own on the screen with the result that the constituent colours overlap on each other to some extent as in Fig. 243.

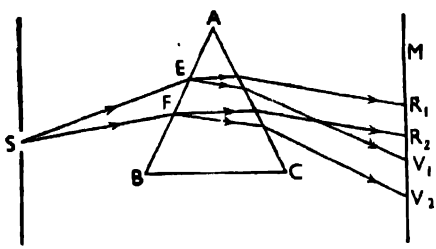


Fig. 243—Impure Spectrum.

Such a spectrum, in which the constituent colours of the different rays are partially superposed on each other, is called an **impure**

spectrum ; and the spectrum in which the colours do not overlap on each other but are separated distinctly into elementary colours is called a **pure spectrum.**

It is obvious from Fig. 243 that if a spectrum be produced by a broad beam it will be impure as there will be much overlapping of colours. The spectra R_1V_1, R_2V_2 are formed by the two extreme rays SE and SF of the incident pencil. The spectra due to the other rays will be formed between them. The widths of R_1R_2, V_1V_2 will depend on the breadth of the incident pencil ; so to obtain a pure spectrum with no overlapping, the first condition is that the incident beam should come through a **narrow slit.**

In a pure spectrum the differently coloured rays should be brought to separate foci on the screen. It may be possible for a convex lens to bring the differently coloured rays to separate foci if they are in parallel groups after emerging from the prism, *i.e.* if all the red rays, all the yellow rays, and so on, come as parallel beams. For this, the incident rays should be parallel. Moreover, all the different spectra will not be in focus on the same screen, unless all the beams forming the coloured images have got almost the same deviation. For this reason the best result is obtained when the prism is set in such a position that the mean colour undergoes minimum deviation is passing through it, or, in other words, the prism should be placed in the **position of minimum deviation** for the mean rays, *i.e.* the yellow rays. The other rays will then be nearly at minimum deviation.

165. Conditions for Formation of Pure Spectrum :—

(*i*) *The slit should be narrow.*

(*ii*) *The prism should be placed in the position of minimum deviation for the mean rays.*

(*iii*) *An achromatic convex lens, i e. a lens which produces no colour effect should be placed between the prism and the screen to bring the emergent rays to focus, and another between the slit and the prism to make the incident rays parallel.*

(*iv*) *The refracting edge of the prism should be parallel to the slit.*

166. Production of Pure Spectrum :—

(a) **First method.**—A source of white light, suppose, illumi-

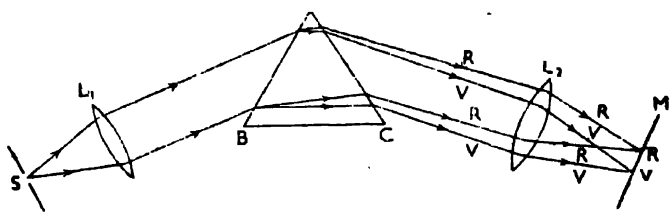


Fig. 244—Pure Spectrum.

nates the narrow vertical slit S (Fig. 244). The slit is placed at the

principal focus of a convex lens in order to have the rays, coming through the slit, rendered parallel. The prism is placed with its refracting edge vertical in the position of minimum deviation for the yellow rays and, hence, approximately for all rays. A second lens brings the differently coloured rays to their different foci (R to V) on the screen M which is at the focal plane of the lens.

(b) **Second method.**—S is a narrow slit strongly illuminated by white light and the prism ABC (Fig. 245) is placed in the position of minimum deviation for the yellow rays, when the rays of any particular colour will appear to diverge from a point from the same side of, and at almost the same distance from the prism as the slit S. Thus the red rays appear to diverge from the point R', the violet rays from V', and all the other rays from points intermediate between R' and V'.

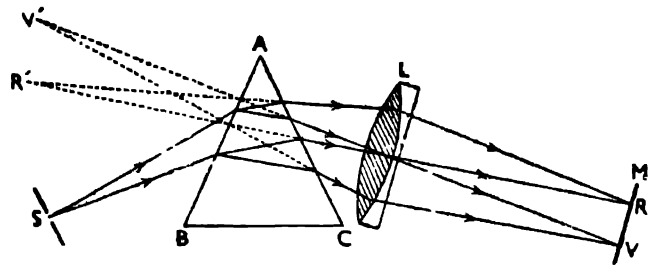


Fig. 245—Production of Pure Spectrum.

If now an achromatic convex lens L is placed between the prism and the screen M at a distance from the slit greater than its focal length, each of the coloured constituents lying between R' and V' will form a separate real image on the screen, and so a real image of the virtual spectrum will be obtained on the screen (Fig. 245). In the real pure spectrum RV thus obtained the different colours will occupy separate positions in the order of their refrangibilities, red being towards the edge and violet towards the base of the prism.

Recombination of Colours.—The seven different colours produced by the dispersion of a beam of white light through a prism may be recombined by the use of seven tiny plane mirrors placed suitably so as to reflect each of the dispersed colours to the same spot on a screen. The resultant colour of the spot will be same as that of the original light showing thereby that the white light is comprised of these seven components only.

The same effect can also be produced by a *Newton's Disc* which is a card board disc mounted on a turn-table. The disc is divided into four quadrants each of which is painted with different colours of the spectrum and the widths of these coloured patches are in the same proportion as in the solar spectrum. When the turn-table is

rapidly rotated, the impressions produced by the different colours overlap and the disc appears greyish white. Here the recombination is a physiological effect caused by the persistence of visual impressions.

167. The Study of Spectrum :—It should be noted that the visible portion of the solar spectrum forms only a small part of the total solar radiations which extend on both ends of the visible part. There is an invisible type of radiations beyond the red of the spectrum, known as **Infra-red** radiations, which we have already called heat radiation, and also there is another invisible type of radiations beyond the violet, known as **Ultra-violet** radiation.

When all known radiations are analysed it is found that there are also other waves smaller than the ultra-violet waves, such as X rays, Gamma-rays, and also waves greater than the infra-red waves, such as Wireless waves. The effects of the different parts of the total spectrum are also different.

168. Effects of the different Parts of Spectrum :—

(a) Luminous Effect.—In defining 'luminous flux', it was stated that this flux is evaluated in terms of the capacity of radiation for producing visual sensation. It is only a small part of the whole spectrum of radiations that is capable of producing this effect, and even over this small part the visual sensation produced is not uniform. The effect is maximum at the yellow-green part of the spectrum (at 5550 Å wavelength) and diminishes on either end of this line. This is shown in Fig. 246, and explains why objects look less bright in red or violet light than in yellow-green light. In the figure, v stands for $\frac{G_0}{G_\lambda} \times v_0$ where, v=visibility at wavelength λ_i, v_0=visibility at $\lambda = 0{\cdot}554\mu$, usually considered as *unity*, G_0= illumination due to the fixed irradiator (watts per sq. cm.) at $\lambda = 0{\cdot}554\mu$, G_λ=illumination due to fixed irradiator at $\lambda = \lambda_i$.

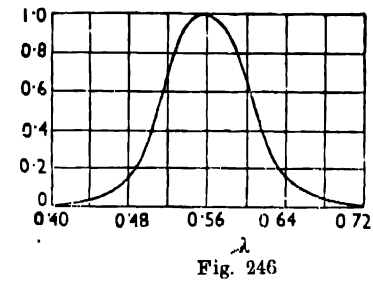


Fig. 246

(b) Heating Effect.—By holding a linear *thermopile* or a *delicate thermometer* with blackened bulb at different parts of the visible spectrum, it can be proved that the *heating effect diminishes from the red to the violet end of the spectrum.* By using a prism of *rock salt, which is transparent to heat radiation*, the increase of the heating effect can be detected up to some distance in the invisible portion of the spectrum beyond the red end of the visible spectrum where it is maximum, and

then a range of heat rays of diminishing intensity may be detected for a distance about seven times the length of the visible spectrum. This portion of the invisible spectrum extending beyond the red end of the visible spectrum is known as the **Infra-red spectrum** and the rays are called **Infra-red rays.** As glass absorbs these rays, prisms and lenses of **rock salt** are used for studying the infra-red spectrum.

In the invisible part of the spectrum beyond the red, the heating effect is great, and so these long waves are called radiant **heat waves,** but it must be *remembered* that all waves carry energy, and, therefore, may produce heat, though in different degrees, when they are absorbed.

(c) **Chemical Effect.**—The chemical effect of the different radiations can be shown by the decomposition of certain salts by the action of the rays of different wavelengths. *This action increases from the red rays to the violet rays of the visible spectrum and extends to a considerable distance beyond the visible spectrum. This portion of the invisible spectrum beyond the violet is called* the **ultra-violet spectrum,** and the rays forming it are called **ultra violet rays.** As glass absorbs ultra-violet rays, prisms of **quartz** (also called rock crystal), or of fluorspar, are used for studying this portion of the spectrum. The rays of this part, on account of the chemical action they produce in decomposing salts of silver, are also known as **actinic rays.**

The ordinary photographic plates which contain silver salts are greatly affected by ultra-violet rays. Red rays have very little action on the photographic plate.

169. Different Kinds of Spectra :— Spectra may be divided into two classes :—(1) *Emission Spectra* ; (2) *Absorption Spectra.*

(1) **Emission Spectra** are the spectra of light emitted by a substance excited under suitable condition and may be divided into three sub-divisions ; (*a*) *Continuous Spectra ;* (*b*) *Line Spectra ;* (*c*) *Fluted (or Band) Spectra.*

(a) **Continuous Spectra.**—The spectra given by an incandescent *solid* gives the colours continuously from red to violet without and break or gap, depending on the temperature of the solid and so the spectrum is called *continuous.*

Liquids and gases under great pressure also give continuous spectra.

A continuous spectrum contains light of all wavelengths. In solids, liquids or compressed gases, the atoms of the neighbouring molecules are very close together so that in the excited state natural vibration frequency of one will be influenced by those of others by way of forced vibration and as a consequence all vibration frequencies and hence all wavelengths are expected.

Examples. The spectra of lime-light, a luminous coal-gas flame, incandescent electric lamp and electric arc are continuous.

(b) Line Spectra.—The spectrum obtained from an incandescent gas or a vaporised substance in flames is not continuous, but consists of a number of bright lines separated from one another by dark spaces, each elementary substance giving its characteristic line or lines whether in combination or not. Thus the line spectrum is the characteristic of atoms and is emitted by a substance in the atomic state only. Most of the lines in the spectra of different elements, when examined by a high resolution instrument, are found to consist of two or more component lines. When there are only two components in a line, it is called a *doublet* and when there are three components in it, it is termed a *triplet* and so on.

The line spectra of all elements exhibit a definite orderlines of arrangements when considered from the standpoint of appearance and wavelengths. According to the characteristic appearance of the lines, the spectra of most of the elements have been classed as *principal*, *sharp* and *diffuse* series. But another type of orderly arrangement is found when the spectra of particularly hydrogen and other hydrogen like atoms (*i.e.* singly ionised helium, doubly ionised lithium, etc.) are arranged according to their wavelengths (λ) or wave numbers $\left(\frac{1}{\lambda}\right)$. This type of *spectral series* was first found by Balmer in the visible hydrogen spectrum which consists of only four lines. This is known as the *Balmer series*. The wavelengths (λ) of these lines can be expressed by the relation, $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$ where R is a constant known as the *Rydberg constant* ($=109677{\cdot}581$ cm.$^{-1}$) and n may have integral values greater than 2. Subsequently other series of hydrogen spectra have been discovered in the ultra-violet and infrared regions. These are Lymann series, Paschen series, Brackett series and Pfund series, $\frac{1}{\lambda}$ of each of the lines of these series may be represented by a similar relation as above. Empirically it was shown by Ritz that the wave number of any line of a spectral series (visible or invisible) can be expressed as the difference of two terms in a way similar to the Balmer relation. Later on Bohr introduced the idea of discrete energy levels of atoms and showed that when an excited atom emits radiation it does so in discrete quanta ($h\nu$) which is equal to the energy difference of two such energy levels. With these assumptions he actually deduced a general expression for the wave number of spectral lines which is found to yield the Balmer relation as a particular case and correctly gives the wave numbers of all the lines of the different spectral series. (Vide, Physical Optics, Art. 111)

Examples. The spectrum of incandescent sodium vapour produced by adding a little common salt to a Bunsen flame (non-luminous), when examined by a spectroscope, gives two deep bright lines in the positions occupied by the yellow part of the white light spectrum, called *the D-lines of the spectrum*, which are characteristic of the metal, sodium (*vide* spectrum chart, Plate I). But the wavelengths of these two lines differ only very slightly; so ordinarily yellow light given by sodium vapour is called monochromatic. Similarly, the spectrum of hydrogen gas produced by passing electrical discharges through it consists of four lines of which three lines, one in the red, one in the green and a third in the violet portion of the spectrum are prominent. The spectrum of lithium salts gives a bright *red* line. The salts of potassium have got *two* prominent lines, one in the *red* and *one* in the extreme *violet* and those of iron have got *a large number of bright lines* in the different parts of the spectrum. Each elementary substance has its own characteristic lines.

(c) **Fluted (or Band) Spectra.**—The line spectrum is characteristic of atoms. Under certain circumstances, a molecule can also be made to emit light *characteristic of the molecule*, depending on the method of excitation. Such a spectrum is characterised by a number of broad luminous bands, each being sharply defined at one edge and gradually *shading off* at the other. On careful examination it has been found that a large number of bright lines are closely packed at the bright end while the spacing of lines is more and more wide at the faint end. Band spectra of a gas may be obtained by enclosing the gas in a Geissler tube at a low pressure and then passing an electric discharge at a comparatively low voltage. When in the solid state, the band spectra of a substance are generally obtained by filling a hole drilled in a pure carbon rod with its powder and then using this rod as the positive electrode of a carbon-arc.

Just as line spectra are due to excited states of atoms, band spectra are due to excited states of molecules. Energy of molecules can have only discrete values called energy states. When a molecule jumps from a higher energy level (E_1) down to a lower energy level (E_2), difference of energy, between the two states is emitted according to Bohr's frequency condition,

$$h\nu = E_2 - E_1.$$

When a diatomic molecule absorbs energy and passes on to a higher energy level, this absorbed energy may be either (1) the energy of the electrons, or (ii) the vibrational energy of the two atoms, or (iii) the rotational energy of the system. All these causes may together contribute to the total absorbed energy also. So during emission, the excess of energy in the excited state will be released giving rise to bands corresponding to the above causes (*i.e.* electronic, vibrational or rotational). In polyatomic molecules the band structure is more complex and the energy contributions are neither fully accounted for.

(2) Absorption Spectra.—If in the path of white light some transparent substances be interposed which absorb some of the

constituent rays of the light, then the spectrum of the transmitted light will be found wanting in the colours of the absorbed rays. Such a spectrum is known as *absorption spectrum.*

Each substance has its own absorption spectrum by which it can be identified. Such spectra may be divided into two subdivisions : (*i*) *Dark-line spectra* ; (*ii*) *Dark-band spectra.*

(i) **Dark-line Spectra.**—*If white light from a hot source is passed through a space filled with a cooler vapour, the vapour will absorb from the white light just those constituents which the vapour itself emits when heated to a high temperature* , so the resultant spectrum is like a continuous spectrum crossed by a number of dark-lines due to the absorption of some of the rays during their passage through the vapour.

Examples

(*a*) Solar spectrum, *i.e.*, the spectrum of *sunlight* is an example of this class. Here a continuous spectrum is crossed by a large number of dark-lines.

(*b*) View the spectrum of an *electric arc* in a spectrometer. It is a continuous spectrum. Now interpose between the arc and the slit sodium vapour by heating sodium chloride (common salt) in a broad non luminous Bunsen flame. *A dark-line appears in the yellow part of the white light spectrum.*

This shows that vapours and gases, which produce bright-line spectra, when emitting light, must, when absorbing, produce dark-line spectra.

(ii) **Dark-band Spectra.**—Instead of interposing sodium vapour as in example (*b*) given above, if the spectrum be viewed through a piece of red glass, then only the red colour of the spectrum will be visible, because red absorbs all the colours except the red. Similiar effects will be produced by using other coloured glasses. A dilute solution of potassium permanganate absorbs the middle region of the spectrum. Hence in the spectrum of white light passing through such absorbing media, *dark bands* or absorption bands, as they are termed, are seen to be present due to some portion being absorbed.

When all the colours of a compound light are equally absorbed in passing through a substance, the case is known as **general absorption**, but the absorption of any particular colour or colours, as in the above cases, is known as **selective absorption.**

170 Some Wavelengths of Ether Waves : Unit of Wavelength :—Of all known ether waves, only a small portion can produce the sensation of light. The wavelength of the longest of these waves is 80×10^{-6} cm. These waves produce the sensation of **red** light. The wavelength of the shortest waves is 40×10^{-6} cm. They produce the sensation of **violet** light. Wavelengths are measured in units, called **Angstrom units** (A. U.), one Angstrom unit being in equal to

10^{-8} cm. Therefore the wavelength of red waves is 8000 Å. U., and that of the violet waves 4000 Å. U. So the wavelength of the visible portion of the spectrum ranges from 7500 to 4000 Å. U. The wavelength of Ultra-violet rays ranges from 4000 to 1000, of X-rays from 1400 to 0·06; Infra-red rays from 8,000 to 40,00,000 Å. U.; longer waves are generally called the Hertzian waves. Hence the following types of radiation are all similar and are arranged in the order—Hertzian waves, Infra-red waves, Visible light, Ultra-violet light, X-rays Gamma rays (Fig. 247). The Angstrom unit is generally used to measure the wavelengths of the *visible* and *ultra-violet* regions. Wavelengths of *infra-red* are generally expressed in microns or millimicrons where 1 micron (μ) $=10^{-4}$ cm. and 1 millimicron ($m\mu$) $=10^{-7}$ cm. Wavelengths of X-rays are generally expressed in X-units where 1 X-unit $=10^{-11}$ cm.

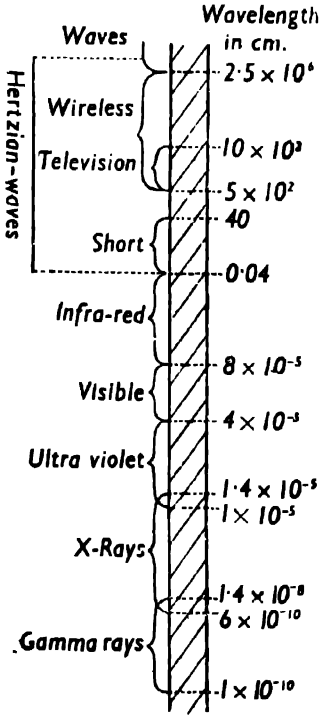


Fig 247

171. Spectra of the Sun and Stars: Fraunhofer Lines:—If a solar spectrum be carefully examined, it will be seen that the whole length of the spectrum is crossed by a large number of dark-lines. Fraunhofer was the first to notice this and he made a systematic study of these lines. He named these lines by the letters of the alphabet *ABCDEFGH*; of which *A*, *B* and *C* are in the red, *D* in the yellow, and so on. These lines are known as **Fraunhofer lines.**

The spectrum chart (Plate I) shows the *bright line spectra* of some elements and the *solar spectrum with Fraunhofer lines.*

It was not until 1861 that Bunsen and Kirchhoff first gave an explanation of the Fraunhofer lines. From experimental studies they enunciated a law which is known as the Bunsen-Kirchhoff law and may be stated as, "*The vapour of an element at a lower temperature selectively absorbs that radiation (light) which it will*

itself emit when at a higher temperature." The sun is assumed to consist of a white-hot solid (or liquid) at the centre, known as the **photosphere**, surrounded by a comparatively cooler atmosphere, called the **chromosphere** in which practically vapours of all the terrestrial elements like oxygen, hydrogen, calcium, sodium, etc., are present. It has already been stated that the comparatively cooler vapour of an element absorbs those light waves which it would itself emit if it were at a higher temperature. So, according to Bunsen and Kirchhoff, the white light emitted by the sun, in its passage through the cooler envelope containing vapours of different elements, is robbed of those rays that can be produced by the elements when incandescent. **Hence the presence of dark-lines in the solar spectrum indicates the presence of some terrestrial elements in the atmosphere of the sun.** The lines appear dark by contrast with other portions of the spectrum and are not really due to absence of light. As an evidence of the correctness of this, it may be cited that during an annular solar eclipse, when the sunlight is cut off by the moon's disc, the solar spectrum becomes *reversed*, the dark lines appearing bright in the absence of the more luminous spectrum. Some of the Fraunhofer's lines are found to less distinct when observations are made at high altitudes (at about 9000 ft. above the sea level). These lines in the solar spectrum arise due to absorption of sunlight by the water vapour. They can also be reproduced in the laboratory by passing white light from an incandescent source through an atmosphere of water vapour. These lines are known as *telluric lines*.

The constitution of stars can also be studied by the spectroscopic method. Each star produces a definite spectrum. On examination it has been found that the spectra of some stars resemble in character the spectrum of the sun. They contain a number of dark-lines which really correspond to elements available on this earth. There are others again which are mere allied to the fluted spectra of some elements. Constitutionally many of the stars are similar to our sun. As many stars are very hot, most of the elements in them are thermally ionised. The spectra of these ionised elements are beyond the visible region and the corresponding radiations are absorbed before they reach the spectrometer slit. So these ionised elements cannot be traced in the spectrum of those stars. This is why the continuous spectrum of the star Orionis is intercepted by a few lines due to hydrogen only.

The spectra of *nebulae* are purely emission spectra consisting of a small number of bright lines. This indicates that the nebulae are wholly gaseous and these gases are also at a low pressure.

172. Spectrometer :—The spectrometer (Fig. 248) is a compact apparatus used for producing a pure spectrum as also observing spectra of various kinds and for making measurement with regard to them. Not

only prismatic dispersion but also diffraction phenomena as observed with a single-slit, a double-slit or a plane grating can be studied with it. The spectrometer consists of the following parts :—

(1) **A prism table (P)** —It is a circular metal plate mounted on a vertical support. The position of the table can be raised or lowered by means of a screw *F*. Just below *P* there is a second circular plate fitted with three levelling screws with the help of which the upper plate (*i.e.* the prism table) can be made exactly horizontal. The prism table

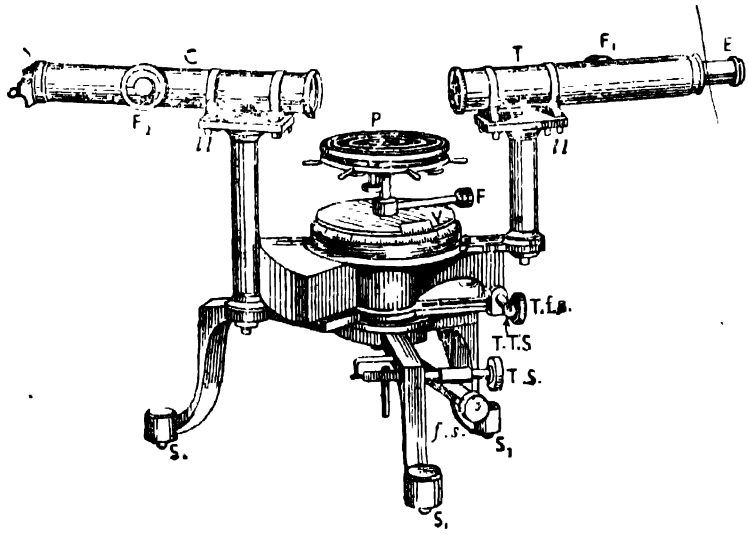


Fig. 248

can be rotated about a vertical axis passing through its centre and the angle of rotation can be read from a circular scale and vernier arrangement *V* as shown in the figure. The vernier is generally fixed to the prism table attachment and the scale to the telescope *T* attachment. In some improved types of apparatus the circular scale is fixed and both the prism table and the telescope have their own verniers. The prism table can be made stationary at any position with the fixation screw (*f.s.*). With the help of the tangent screw (*T.S.*) a slight rotational motion can be imparted to *P*.

(2) **A Collimator (C).**—It consists of a hollow metal tube at one end of which an achromatic converging lens system is fitted. At the other end is inserted a narrower coaxial cylinder at the outer end of which is fitted an adjustable narrow slit *S*. The narrower cylinder can be slided inside the outer one with the help of the

focussing screw F_2 thereby altering the distance between the slit and the collimator lens. The collimator is horizontally placed on a vertical stand attached to the body of the spectrometer. The collimator tube can be given a slight tilt in the vertical plane by the two levelling screws *ll* and it should be so adjusted that its horizontal axis passes through the centre of the prism table.

(3) **A Telescope** (T).—It is an astronomical telescope of small size. Its objective is an achromatic combination of lenses and the eye-piece is of the Ramsden type provided with a cross-wire. The axis of the telescope can be adjusted in the horizontal plane by means of levelling screws *ll*. The axis of the telescope tube should pass through the axis of the instrument and in the horizontal position it should be perpendicular to the latter. The telescope is supported on a rigid vertical stand which can be rotated about the axis of the instrument. This angle of rotation of the telescope can be measured by a pair of verniers (set 180° apart) which can slide along the circular scale. The telescope can be fixed in any position with the help of the fixation screw (*T.f.s.*) and a slight rotational motion can be imparted to it with the help of a tangent screw (*T.T.S.*).

(4) **Tripod Base.**—The whole arrangement described above is mounted on a tripod base, all the three legs of which are provided with levelling screws (S_1, S_2, S_3).

173. Adjustments of the Spectrometer :—For all accurate measurements with a spectrometer the following essential adjustments are to be made.

(i) **Levelling of the instrument.**—It is done in three distinct steps namely, levelling of the telescope, levelling of the collimator and levelling of the prism table.

In levelling the telescope, a spirit level is to be placed on the telescope tube such that the axis of the telescope tube is parallel to that of the spirit level. Now the telescope T is brought in a line parallel to the line joining the two levelling screws S_2 and S_1 (Fig. 248). If the air-bubble of the spirit level is not at the centre, it is to be brought to the centre by adjusting half with levelling screws *ll* of the telescope and half by turning the base screws S_2 and S_1 in opposite directions by equal amounts. Now rotate the vertical upright to which the telescope is attached through 180° so that the axis of the telescope is again parallel to the line S_2, S_1. If the air-bubble of the spirit level is shifted from the centre it is again to be brought in position by adjusting half by *ll* of the telescope and half by S_2 and S_1 again. These operations are repeated a number of times till in both the settings of T parallel to the line S_2, S_1 the spirit level indicates horizontality of the telescope tube.

This adjustment ensures that the axis of rotation of the telescope is perpendicular to the line S_2S_1. Now rotate the upright holding the telescope through 90° so that T and C lie in the same line. If the air-bubble is again displaced it is brought to the centre by adjusting screw S_3 of the base as shown in Fig. 248. After this adjustment the axis of rotation of the telescope becomes vertical and the axis of the telescope itself horizontal.

To level the collimator, the spirit level is placed on C and the position of the air-bubble is adjusted by means of the collimator screws ll (Fig. 248). The better method of levelling the collimator is to illuminate the slit of the collimator by means of a Na-flame and to see the image of the slit in the telescope. If this slit image is not symmetrically placed with respect to the horizontal cross-wire, the collimator is to be properly tilted by means of the screws ll till the image becomes symmetrical. Since the telescope is previously set in the horizontal plane, this symmetrical position of the slit image ensures that the collimator axis is also horizontal.

The prism table is levelled with the help of the three levelling screws attached below its bottom-plate. The spirit level is placed parallel to the line joining two of these screws and the air-bubble is brought to the centre by adjusting any one of these two screws or, if needed, by both of them rotating the same in opposite directions. The spirit level is then placed perpendicularly to the former line of setting and the air-bubble is adjusted by means of the third screw. Thus the top of the table P becomes horizontal.

(ii) **Alignment of the Source** —*This adjustment is essential in all types of spectroscopic works.* Even if all other adjustments are accurately done but if the source is not properly aligned along the axis of the collimator, the spectra become indistinct and measurements are in most cases faulty if not impossible. To align the source the observer is to look along the collimator axis and see that the steady and intense part of the flame is against the slit. In the cases of the sources being Geissler's discharge tubes the most intense part of the discharge should be just in front of the slit. This adjustment requires a bit of trial ; but once the source is properly aligned, the spectra observed are very prominent and the measurements become reliable. It is better to place a slotted screen in between the source and the slit such that the slot in the screen is against the slit. The screen prevents the light of the source from directly reaching the eye of the observer.

(iii) **Adjustment of the Slit.**—By placing a plumb line against the slit, the slit is made exactly vertical by turning it round in its own plane, if necessary. The slit width is then made sufficiently narrow by eye-estimation.

(iv) **Focussing of the Cross-wires.**—The telescope tube is now turned towards an illuminated background. The eye-piece tube

is now slided in or out till the cross-wires are distinctly visible. The telescope tube is then brought in line with the collimator and by means of the focussing screw F_1 a sharp image of the slit is cast in the plane of the cross-wires. The cross-wires, if necessary, may now be set vertically and horizontally in comparison with the slit image which is already made vertical. The width of the slit may now be adjusted so that the slit image is sufficiently narrow but at the same time distinct.

(v) **Focussing for parallel rays** :—The spectrometer may be focussed for parallel rays either by the direct focussing method or by Schuster's method.

(a) **Direct focussing method** :—In this method the telescope of the spectrometer is first directed towards a distant object and its sharp image is cast on the cross-wires of the eye-piece by adjusting the focussing screw F_1 so that there is no parallax between the image and the cross-wires. The telescope is now focussed for parallel rays. The telescope is then turned towards the collimator, the slit of which is properly illuminated by a Na-flame. The image of the slit is then cast on the cross-wire of T by adjusting the screw F_2 of the collimator till there is no parallax between the slit image and the cross-wires. Thus the collimator is also adjusted for a parallel beam and the spectrometer is focussed for parellel rays.

(b) **Schuster's method of focussing.**—In the laboratory dark rooms where distant objects are not available, the spectrometers are generally focussed for parallel rays by the Schuster's method. In this method the slit of the collimator is first illuminated and a prism is placed with its edge vertical and its centre coinciding with the axis of rotation of the prism table. The prism table is then rotated till the angle of incidence is more than 45° (by eye-estimation). Looking from the other side of the prism, the slit image is first received by the naked eye. The prism table is then rotated slowly first in the clockwise direction all along moving the eye in the proper direction so as to receive the slit image. If the image does not turn back from any particular position in that direction, the prism table is, then slowly rotated in the anti-clockwise sense and the slit image is all along kept in view. It will be found in this case that the image first moves in a particular direction and then at a limiting position it becomes almost steady and then on further rotation of the prism table in the same sense, the slit image begins to turn back in the former direction. *This limiting position of the turning back of the slit image is the position of minimum deviation of the prism.* This minimum deviation position is now again adjusted and the telescope is brought in position so as to receive the slit image (in the minimum deviation position) on the vertical cross-wire of the telescope. The telescope is then fixed in this position by the screw *T. f. s.* (Fig. 248) and by means of the tangent screw (*T. T. S.*) it is slightly moved to *the*

greater deviation side. This adjustment shifts the slit image (in the minimum deviation position) to that end of the field of view which is *nearer to the collimator axis.* Since this position of the slit image indicates the angle of closest approach (min. dev. angle), now if the prism table be rotated in any sense—clock-wise or counter-clock-wise the slit image will always move away from the collimator axis (*i e.*, from the direct ray), that is, the slit image will return back to the vertical cross-wire whether the prism table is rotated in a clock-wise or an anti clock-wise sense. Once this adjustment of the minimum deviation position is made at the end of the field of view of the telescope which lies nearer the collimator axis, the focussing for parallel rays can be done in two distinct operations as described below.

(α) *Focussing in the normal position.*—The prism table is so rotated that the *edge* of the prism *approaches the collimator.* Thus the normal on the incidence side approaches the incident ray or the incident ray may be supposed to approach the normal. This position of the prism is called the *normal position.* Here the prism table is rotated only to such an extent that the slit image just coincides with the vertical cross-wire. In this normal position the screw F_2 of the collimator is to be adjusted so that the image is well-defined. [Remember : When the edge of the prism approaches the *Collimator*, adjust the *Collimator.*]

(β) *Focussing in the slanting or inclined position* :—The prism table is now rotated in the opposite sense so that its edge approaches the telescope. Here since the angle of incidence gradually increases and the incident ray becomes more inclined to the normal, the position is known as the *slanting* or the *inclined position.* The slit image will now gradually come back to its minimum deviation position at the end of the field of view and with further rotation of the prism table in the same sense, it gradually returns back to the position of the vertical cross-wire. If the slit image is not well-defined, it is made sharp by adjusting the telescope by means of its focussing screw F_1. [Remember : When the edge of the prism approaches the *telescope*, adjust the *telescope.*]

Now repeat the operations (α) and (β), a number of times, till the slit image on the cross-wires is equally sharp in both the normal and the slanting positions. Though equally well-defined, the images in the two positions are not equally broad—the normal image being always of greater breadth. When this equal sharpness is attained the spectrometer is adjusted for parallel rays.

Note :—(*i*) While rotating the telescope, it should not be so done by holding the telescope tube. The rotation should be given by holding the upright on which the telescope is mounted.

(*ii*) If the spectrometer is a good one fitted with a telescope of even moderate resolution and if the prism used be of flint glass, then after focussing for parallel rays with Na-light, the resolution of two lines (D_1 and D_2) of sodium may be easily seen by simply making the slit width sufficiently narrow. Once this resolution is secured, one may be sure that the spectrometer has been properly focussed for parallel rays. For better resolution and purity of spectral lines, the slit width should be made very narrow without impairing the intensity of the lines.

174. Theory of the Schuster's Method of Focussing :—We know that the angular magnification produced by a prism is given by $m_\theta = -\frac{di'}{di} = \frac{\cos i}{\cos i'} \cdot \frac{\cos r'}{\cos r}$. Now if the prism is in the *normal* position, $\angle i$ is less than $\angle i'$ and the angular magnification is greater than unity. So the emergent rays are inclined at a large angle, *i.e.*, the image of the slit is formed near the prism. By adjusting the collimator this image is shifted to a greater distance so that the rays are less inclined and approach parallelism. Next bringing the prism to the slanting position $\angle i$ become greater than $\angle i'$, and the angular magnification becomes less than unity. So the emergent rays are inclined at a small angle and the image is formed at a large distance. Hence the telescope is used to focus this image distinctly. With this setting of the telescope, when the prism is again taken back to the normal position, the emergent rays are made still more parallel by the collimator and image is shifted further away. Again setting the prism in the slanting position, this distant image is still further pushed away and focussed by the telescope. So by repeating these processes a number of times, the spectrometer is focussed for parallel rays and the emergent rays become exactly parallel.

175. Uses of the Spectrometer.

(A) Measurement of the refractive index of the material of prisms—both thick and thin.—This is done with the help of the relation, $$\mu = \sin\frac{A+D_m}{2} \Big/ \sin\frac{A}{2}.$$

(B) Measurement of the refractive index of a liquid.—This is also done by using the above relation and filling up a hollow prism with the experimental liquid. (See Determination of Refractive Index).

(C) Determination of the wavelengths of spectral lines.—For the accurate determination of the wavelengths of spectral lines, generally gratings are mounted on the prism table of the spectrometer. Then using a light of known wavelength, the grating element is first determined. Now using the grating relation, λ is determined (See Chapter on Diffraction) for a line measuring its angular distance from the central maximum. In order to determine λ with a prism spectrometer, it is to be first calibrated with reference to a number

of lines of known λ's. To do this, the spectrometer is properly levelled and focussed for parallel rays and is then fixed at the position of minimum deviation for the yellow rays of sodium. Now using a suitable Geissler tube emitting a good number of lines of known wavelengths as the source, the deviations of these lines are noted from the telescope vernier. The direct reading is then recorded after removing the prism. Thus angles of deviation of the lines of known λ's are determined. A wavelength—deviation curve (*i.e.*, calibration curve) is then plotted for the particular prism. Now to determine λ of an unknown line, its deviation in the prism (set at minimum deviation for Na-yellow) is found out. Then referring to the above calibration curve, its λ is determined.

If the telescope be replaced by a photographic camera arrangement, the spectrometer becomes a *spectrograph*. In order to find out the wavelengths of unknown lines, their photographs are taken on a plate and slightly above or below those lines, the photographs of some reference lines of known λ's are taken. Then after developing and drying the plate the unknown λ's are determined with the help of a comparator from a knowledge of the wavelengths of the reference lines.

176. Sources of Spectra :—There are four principal methods of exciting a substance so as to emit light from which the characteristic spectra of that substance can be obtained. These methods are as follows :

(i) Flame method.—It is the simplest of all methods. It is suitable for salts of *Na, Li, Thallium, K, Ba, Sr, Ca, Rb* and *Cæsium* etc. By a suitable device the salt under test is made to volatilise in a non-luminous Bunsen flame. The flame acquires the characteristic colour of the particular element whose salt is used and the *flame spectrum* obtained from this flame as the source gives the characteristic lines (or bands if the emission is in the molecular state). Much brighter flame spectra are obtained if the Bunsen flame be replaced by an oxy-hydrogen or oxy-acetylene flame. According to Kirchhoff, even a trace of sodium (or any other element) as small as 7×10^{-11} gm. can be detected by this method and as such it is much superior to the method of ordinary chemical analysis so far as the detection of an element is concerned.

(ii) Spark method.—If two metal rods are connected to the terminals of the secondary of an induction coil and their points are brought near each other, a stream of sparks (sometimes even five or six inches long) passes between them when the induction coil is started. The spectrum of the spark so obtained is called the *spark spectrum* of the metal which is used as the sparking rods. A Leyden jar may also be effectively used in producing a spark spectrum.

The spark spectrum of a solution is obtained by putting it in a test tube to the bottom of which a platinum wire is sealed, the wire not reaching the surface of the liquid inside the tube. The test

tube is corked at the top through which passes a capillary glass tube with a platinum wire passing along its bore. A small portion of the wire should project out the capillary tube the lower end of which should preferably be sealed to the wire. The projecting end of the wire should be slightly above the liquid surface. If the two platinum wires are now connected to the secondary terminals of an induction coil spark passes between the liquid surface and the upper wire. Hence the spark spectra of solution can be produced by this method.

(iii) **Electric Arc Method**—If two carbon rods (Fig. 249, left) are connected to the terminals of a voltage source of about 80 to 200 volts and if the rods are brought together and then slowly drawn apart, a bright discharge passes between them. This is known as the electric arc. During the discharge both the carbon electrodes are consumed by

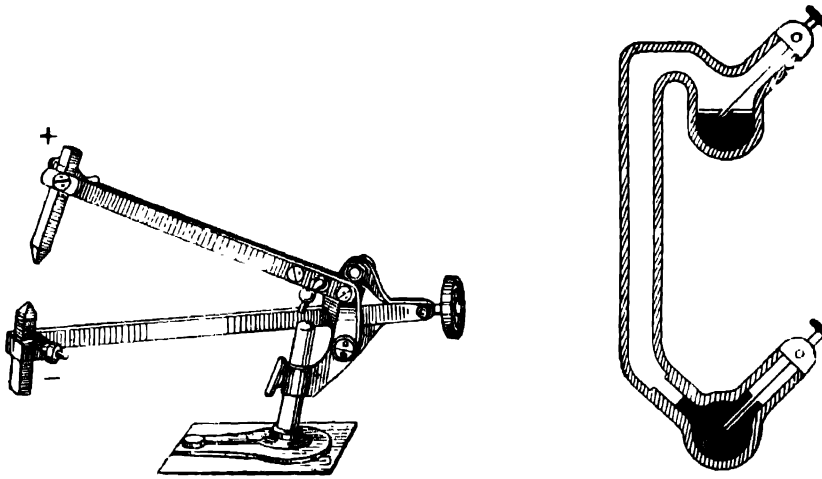


Fig. 249

the arc, the positive one twice as fast as the negative. So the positive electrode becomes hollowed out into a *crater like* depression. If the image of the arc is thrown on the slit of a spectrometer by means of a focussing lens, by adjusting the latter the spectra of the different parts of the arc can be examined in succession. It is found that the spectrum of the incandescent electrode is continuous while that of the burning vapour (*i.e.* arc flame) gives a multitude of lines. In order to study the spectra of metals by the electric arc method a hole is axially bore in the positive carbon rod and it is then packed with the granules of the metal or a suitable salt of the metal. This +ve rod is kept vertically just below the −ve one and an arc is produced between the two, the flame of this arc gives the characteristic spectrum of the metal superposed on an

arc flame spectrum of carbon. A purer spectrum may however be obtained by producing an arc between two rods of the metal whose spectrum is required. But in such cases the current consumed by the arc should be rather small.

A mercury arc lamp (Fig. 249, right) consists of a vacuum tube of fused quartz containing sufficient quantity of mercury in it. The two electrodes are sealed in the tube in the two depressed regions where excess mercury can be kept. When the tube is slightly tilted mercury in the two reservoirs are linked up as a single column. Now a sufficient current is passed through this mercury thread and the tube is gradually made horizontal so that the continuity of the thread breaks. This produces a strong discharge in the tube which is found to be very rich in ultra-violet rays. Another type of *Hg*-arc operates at atmospheric pressure and is started automatically by first evaporating mercury by means of a heating coil. To work with this mercury arc as also with ordinary carbon or iron arcs the eye must be protected from ultra-violet rays by using suitable spectacles.

(iv) **Discharge tube Method**—(*Geissler tube method*)—The discharge tubes or Geissler tubes used for spectroscopic purposes are ordinary glass tubes fitted with electrodes sealed through their walls and containing rarefied gases likes O_2, H_2, N_2, *He*, *Ne*, etc. at a pressure of nearly 1 mm. of *Hg*. When the electrodes are connected to the secondary terminals of an induction coil, the tube lights up. Discharge tubes that can be operated directly from the mains are also available. The nature of the spectra produced by a discharge tube depends on the gas used in the tube.

Note :—Now-a-days *arc lamps of different types* have been put into the market. They give line spectra of *Na*, *Cd*, *Hg*, *Zn*, *Th*, etc. They are superior to vacuum tubes (discharge tubes) inasmuch as they are many times more intense than the latter and can be worked with a series resistance in the lighting circuit. Each lamp consists of a small inner glass tube mounted inside an outer tube which is evacuated to prevent loss of heat. When current is started, a filament gets heated and emits electrons. The filament is then switched off but the voltage applied to the terminals of the tube is retained. The discharge at once sets up in the inert gas in the tube. The substance whose spectrum is required is present in the tube as a deposit on the walls of the tube (or sometimes as a coating on the heating wire itself). This is evaporated by the heat generated and a prominent discharge passes with the characteristic colour of the metal. The intensity of this discharge is so pronounced that the spectrum of the inert gas in the tube is suppressed almost completely.

177. Study of the Ultra-violet and Infra-red regions of the spectrum :—

(A) Ultra-violet rays.—Ritter in 1801 discovered the existence

of ultra-violet rays in a solar spectrum. A photographic plate placed beyond the violet region of the solar spectrum was found to have been exposed, which conclusively proved the existence of an invisible short wave region beyond the violet of the solar spectrum. The *ultra-violet region* as it is commonly called, extends from 3900 Å to about 100 Å. The common laboratory source for producing these rays are the iron or copper arcs and the mercury arc suitably encased in a tube which absorbs these rays to a negligible extent. As these rays are absorbed by most of the commonly used transparent materials and also by air to a considerable extent, these arcs should preferably be vacuum arcs.

For the study of the Ultra-violet rays, the spectrum is generally produced either by a *prism* or by a *grating*.

In the *prism method*, since the radiation has to pass through the prism, the material of the prism should be such as not to absorb the radiation to be studied. On the short-wave side, glass transmits upto 3300 Å.U., U-viol glass upto 2000 Å.U., quartz upto 1850 Å. U., fluorite upto 1200 Å.U. and SiF crystals (artificially prepared from the salt) transmit upto 1100 Å.U. Absorption therefore puts a limit to the region which can be studied by a prism of a particular material. The lenses which are used together with the prism for the production of the spectrum must be also made of the same material as that of the prism in order that the range of study may not be reduced.

In the *grating method*, if a plane transmission grating is used, the absorption once again limits its use. The range is, however, extended without limit by the use of reflection gratings. These may be either (*a*) concave, or (*b*) plane. Since the radiation is not to pass through the material of the grating, it can be used down to the shortest ultra-violet region and in fact even in the *X*-ray region. Such gratings are the only methods of producing spectra beyond the fluorite region (1200 Å.U.).

For the recording of the ultra-violet spectra photographic method is generally adopted. Ordinary photo-plates may be used upto 2200 Å. U. only, because gelatine absorbs radiations of still shorter wavelengths. To obviate this difficulty Schumann invented plates in which the sensitive layer is held together by just a trace of gelatine. These plates, known as Schumann plates, are neither very satisfactory because of the occasional peeling off. It has been found that ordinary plates immersed in some suitable fluorescent oil can be used over the whole of the ultra-violet region. Ultra-violet rays incident on this oil film excite visible rays which affect the plate. The Ilford *Q*-plates, wherein halide crystals project out of the surface of the emulsion, are now-a-days used in ultra-violet spectroscopy. These are more robust than the Schumann plates and speedier than the oil-sensitised ones.

Once the spectral lines are photographed a comparison spectrum is taken by the side of the above lines and then from the knowledge of the

wavelengths of the comparison lines, the wavelengths of the unknown lines are measured with the help of a comparator. In the grating method, however, if the grating elements is previously, known, λ can be calculated from the knowledge of the angles of incidence and diffraction.

As air absorbs all radiations below 1800 Å.U. to study such regions the whole of the path from the source to the recording plate must be in vacuum. So *vacuum spectographs* are essential for the study of ultra-violet rays beyond the quartz region.

Since the whole range of ultra-violet radiations can be conveniently divided into three distinct regions, the apparatus used for the study of these regions are also of three distinct types as shown below.

(a) **Quartz region.**—This consists of near ultra-violet (3970 to 3000 Å.U.) and the short (upto 1850 Å.U.) ultra-violet regions.

Here quartz spectrograph as shown below is generally used. The parts of the instrument are labelled below the figure (Fig. 250). As quartz is a double refracting crystal and also introduces a rotation in the plane of polarisation, fused quartz is preferably used in constructing the lenses and prisms in a quartz spectrograph.

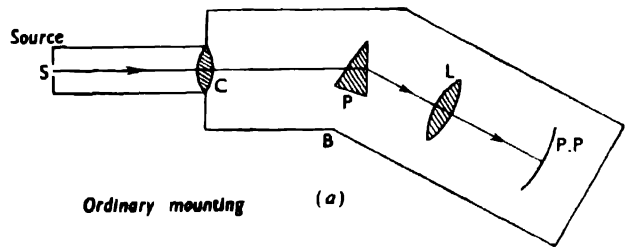


Fig. 250
S = Slit, C = Collimating lens, P = Prism, L = Photographic lens
$P.P$ = Photo-plate, B = Light-tight box.

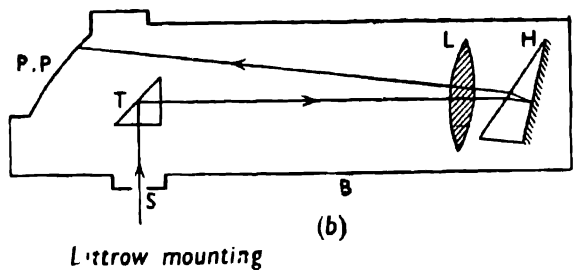
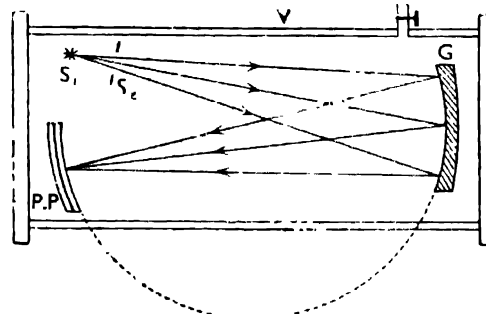


Fig. 250
Quartz Spectrographs
S = Slit ; T = Total reflecting prism ; L = Collimating cum focussing lens
H = Quartz half-prism silvered on the back-side ; $P.P$ = Photo-plate,
B = Light-tight box.

(b) **Schumann region**—This region extends upto 1100 Å.U. As the rays of this region are readily absorbed by air, vacuum spectrographs with fluorite or fluorspar prisms and lenses are to be used for this region. The whole arrangements will be similar to above but they should be made in an evacuated chamber.

(c) **Extreme ultra violet region (Lymann region).**—This region extends from 1100 Å.U. down to the *X*-ray region. Here also the spectrographic arrangements are to be made in vacuum. For producing spectra reflection gratings either plane or concave are to be used. Lymann, Compton and others used concave reflection gratings with Eagle's mounting upto wave-length of 300 Å. U.

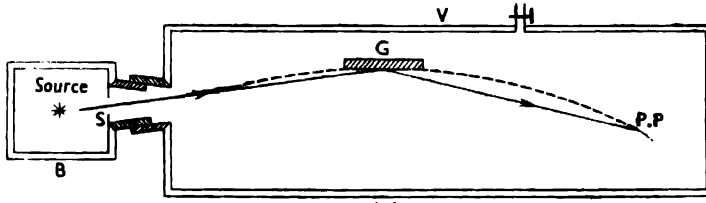


(a) *Eagle mounting*

Fig. 251

V = Vacuum chamber, S_1 = Source, S_2 = Slit, G = Concave grating; $P.P$ = Photo-plate mounted on the Rowland circle.

The arrangements are shown in Fig. 251(*a*). Below 300 Å.U. a plane or a concave reflection grating at grazing incidence is to be used as shown in Fig. 251(*b*).



(b) *Grazing incidence*

Fig. 251

V = Vacuum chamber, L = Slit, B = Vacuum box containing the source and fitted to V by ground cones, G = Concave grating, $P.P$ = Photo-plate.

(B) Infra-red Rays.—Infra-red rays consist of the long-wave heat radiation copiously emitted by hot bodies. These rays are invisible to the eyes and extend beyond the red and of the visible spectrum (*i.e.* 7200Å.U.) upto 400 microns (1 micron = 10^{-4} cm.). The common sources of continuous infra-red radiation are: (1) a black body, (2) Welsbach mantle and (3) Nernst filament lamp. For the detection of the radiation upto 13μ, special infra-red plates

are used for the photographic study. But for the whole infra-red region, in general, radiation instruments like, thermopiles, radiometers, bolometers or radiomicrometers are used according to the suitability of the range. For the production of the spectrum of a definite range first of all the radiations from the source are sorted out for that particular range either by the residual ray method or by the method of focal isolation (see Art. 203 College Physics, Volume II, Heat) and then the spectrum is produced either by a prism or a grating type of radio-spectrometer. The identification of the spectral lines are made by linear bolometers or thermopiles specially constructed for the purpose.

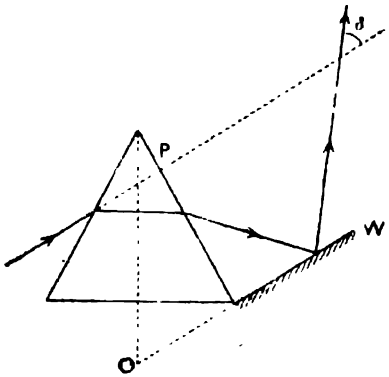


Fig. 252

In prism type of the radio-spectrometer quartz may be used from the visible range upto $3{\cdot}5\mu$, fluorite from 3 to 9μ, rock salt from 15 to 20μ, sylvin, from 15 to 20μ and KBr crystals from 19 to 28μ. Unlike visible spectroscopy, focussing is here done by concave mirrors. Since the source and the position of the detector are fixed in a spectro-radiometer the refracted ray must be deviated by a constant amount. At the same time it is necessary that the beam passes through the prism at minimum deviation. Both these demands are met by using a Wadsworth mirror W on the prism table. It is a plane mirror so placed on the prism table that its plane passes through the axis of rotation O of the table. The median bisector of the vertical angle of the prism P also passes through this axis. It can be shown that in the above setting any ray passing through the prism at minimum deviation undergoes the same total deviation δ (Fig. 252) whatever be the position of the prism. The diagrammatic sketch of a prism type of radio-spectrometer is shown in Fig. 253. Here S is the source, T the detecting thermo-

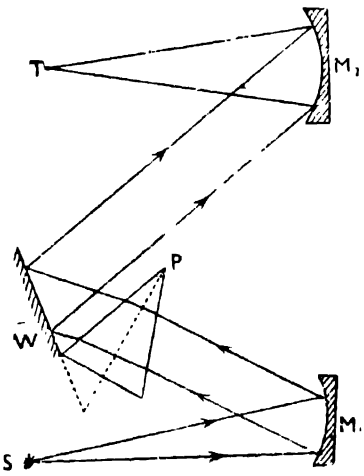


Fig. 253

pile, M_1 the collimating concave mirror, M_2 the focussing concave mirror, P the prism and W the Wadsworth mirror.

In the grating type of the radio-spectrometer either a wire grating, an Echelette grating, a concave grating or a laminar grating may be suitably used according to the range. By this arrangement the wavelengths of the spectral lines can be absolutely determined from a previous determination of the grating space.

The infra-red rays properly sorted out are focussed on a small hole at the centre of a plane metallic mirror M_1 (Fig. 254). This hole is at the focus of the concave mirror C_1M_1. The subsequent paths of the rays are shown in the figure. G is the grating, C_1M_2 and C_1M_3 are two concave mirrors and M_2 is another plane mirror with a central hole like M_1. The beam is finally focussed on to the receiving thermopile T, or some other detector. All the mirrors are fixed. G can only be rotated on a graduated circular scale. From this rotation the diffraction angles corresponding to the lines of different orders can be obtained and hence their wavelengths can be calculated. Besides these methods the interferometer method is also adopted for the determination of the wavelengths of the infra-red radiations.

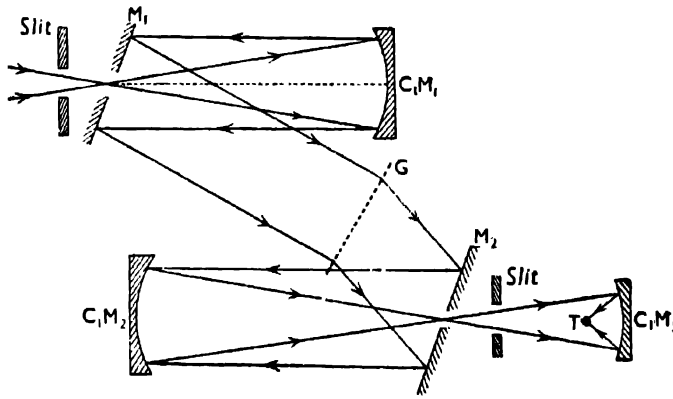


Fig. 254

178. Colours of bodies :—(a) It has been verified by experiments that coloured bodies, whether opaque or transparent, have got no colour of their own. Their colours are determined by (*i*) the nature of the *incident light* ; (*ii*) the proportion of it *absorbed* by them and (*iii*) the sensation of the colours produced in the eye by the colours not absorbed.

Incident Light.—Sunlight is white because all the different constituents of white light are present in it in the necessary proportions, but all the so-called artificial white lights are not really white. They more or less lack some constituents of white light. For example, the

light from an electric lamp contains much more red-orange and less blue-violet ; that from a gas lamp is reddish-yellow and deficient in the blue constituent. So a blue suit looks darker in artificial light. Light from an electric arc is almost as white as daylight. Hence the colours of bodies may be much changed, if the incident light is itself coloured.

(i) **Opaque Bodies**—Of the light incident on an opaque body a portion is reflected at the surface, some may penetrate a little distance into it and then return in part, and some may be completely absorbed. The colour of an opaque body then depends upon the *nature of the incident light* and also the light *absorbed* by it. The body appears coloured with the constituents reflected by it. Thus in white light a red flower appears *red* because it absorbs all the constituents except the red which is reflected by it. A body appears *white* as it reflects all the constituents of white light absorbing nothing, and a body is *black* when it absorbs all the constituents reflecting nothing. Thus the colour of the reflected light is not due to something *added* to the incident light but due to something usually *subtracted* from it.

By passing an object along the *different parts of a spectrum*, the theory of colours may be verified. A *white flower* appears white in white light but it will appear *red* in red light, *green* in green light and so on. A *red flower* will appear bright red in the red part of the spectrum but black in other parts as it absorbs all the colours except the red which it reflects. Seldom we get a body having a pure colour ; so the reflected colour is not always a pure colour, but may be a mixture of its adjacent colour ; hence a body when held in the spectrum may appear bright in one portion, but not totally black in the adjacent portions, as it may reflect these colours also to some extent.

(ii) **Transparent Bodies**—When white light is incident on a transparent body, it absorbs some constituents and transmits the rest to which its colour is due. A piece of red glass appears red because it absorbs practically all the colours except the red which it transmits. Again, if the object be held in a light whose colour is other than that of the body, it will appear black. So a piece of red sealing wax will appear red through red glass, but a blue or green object will appear black because the red light coming through the red glass is absorbed by the blue or the green object and the observer receives no light.

The colours of many coloured glasses are not, however, *pure* ; yellow glass transmits yellow, and green and orange as well, and blue glass allows indigo and green besides blue. So the combination of these two glasses will allow the light common to both of these, *i.e.* green.

Even a good transparent body like water, glass, etc. absorbs some light, which may not be noticeable in thin layers, but the effect is marked in thicker layers. Ordinary deep water looks greenish, but when the depth is very great it may appear black.

(b) Colours of Powders.—The colours of many substances in the powdered state look lighter because the incident light is repeatedly reflected from many particles in different layers and so is unable to penetrate far below to be absorbed. If, however, the powder is very fine, practically no absorption will take place and the powdered mass will appear white due to the diffused light reflected in all directions.

179. Colours and their Mixtures :

(a) Primary and Complementary Colours : Mixture of Spectral Colours —It was shown by Newton that all the seven spectral colours into which white light was split up could not be further analysed, *i.e.* those colours were **pure.** But there are three spectral colours **red, green** and **blue**, by mixing which in right proportions *practically* all colours can be obtained. They are, therefore, called the **primary colours.** Any two spectral colours, which together give white light, are called **complementary colours.** Thus bluish-green and red, yellow and blue, greenish-yellow and violet, are complementary.

If *red* light and *green* light are mixed, the resulting colour is *yellow*, which the eye cannot distinguish from a certain *spectrum yellow* ; but a spectroscope will at once show the difference, for the spectrum yellow seen through a spectroscope will remain yellow, whilst the yellow formed by the mixture, when examined by a spectroscope will be separated into its red and green components.

It is clear from this that wavelengths determine colour but colour does not necessarily determine wavelength.

(b) Colours of Paints or Pigments and their Mixtures.—The mixing of coloured **paints** or **pigments** is not the same as that of mixing of two coloured lights or spectral colours. The colours of pigments depend upon the particular colour or colours each particle absorbs. For example, yellow rays of the spectrum mixed with blue rays produce white light, but the mixture of yellow and blue *pigments* appears green. This is because the yellow particles of the pigment absorb all except yellow and its adjacent colours green and orange whereas the blue particles of the pigment absorb all except blue, indigo and green. Hence the mixture reflects the green rays alone which are absorbed by none of the pigments.

To sum up, we may say that in the case of mixing spectral colours we have the effects of superposition or **addition**, while in the case of mixing of pigments the effect is of absorption or **subtraction** of colours.

Colour Vision :—It has been experimentally found that by suitably mixing red, green and blue components of the spectral colours, it is possible to reproduce any other colour of the spectrum as also most of the different colours that the eye can perceive.

Different theories of colour vision have been put forward of which the *Young-Helmholtz* theory is supposed to be the best. According to this theory the colour vision is due to three different types of receptors in the retinal layer which are responsive to the red, green and blue colours of the spectrum. These receptors are all cones, the rods playing no part in the colour vision. The sensitivity of three types of nerves (*i.e.* conical receptors) in the various parts of the spectrum is shown in Fig. 255. If the light of a particular colour (here yellowish green) corresponding to the wavelength at *o* be considered, this produces responses in the three types of receptor cones as given by *or*, *og* and *ob*. Due to the mingling of these responses, the sensation of that particular colour is produced. Excepting in the extreme red and extreme violet regions where the red and violet-sensitive receptors are only active, the most of the regions of the spectrum are however recognised by the simultaneous responses in the three receptors.

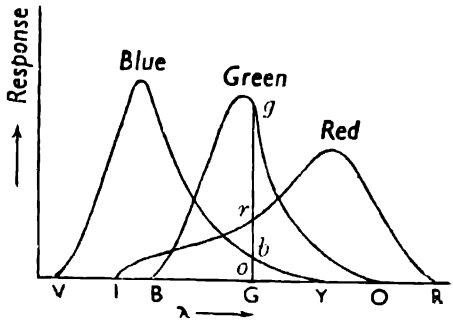


Fig. 255

This theory explains a large number of facts including the defect of vision known as *colour blindness*. In the retinal nerves of a colour-blind man a particular set of receptors are missing and as such he is blind to those colours that are at least partly due to these receptors. Recently it has been found that the above theory is inadequate and that as many as eight types of receptors are required to explain the observed facts.

180. Retinal Fatigue :—When we look *intently* on a bright object for some time and then suddenly turn our eyes on to a white surface, we do not see the object in its actual colours, but the shape of the object is seen in dark outline. This phenomenon is called **retinal fatigue**. This is due to the fact that the nerves of the retina being excited by strong light become insensitive for some time to less powerful source. The colours of the temporary illusive image—called the *after image*—which is seen due to the retinal fatigue, are complementary to the actual colours of the object. Thus when we gaze intently on a red patch painted on white screen for some time and then suddenly look at a white surface we see a greenish-blue patch, as greenish-blue is complementary to red. Similarly, when we gaze at a blue patch painted on a red background, it will appear temporarily to be red on a blue background after suddenly turning our eyes on to a white screen.

181. Importance of spectroscopy in Science and Industry :— In various branches of science spectroscopy has long been used as a powerful tool. In industries also it is now coming into increasing use. When the spectrum emitted by a source is studied, the lines or bands in the spectrum gives us information regarding the composition of the source. Thus if in the spectrum of a mercury vapour lamp we find that in addition to the characteristic mercury lines there also some lines of cadmium, then it may be inferred that the lamp contains some vapour of cadmium in addition to the mercury vapour.

It has already been mentioned that some of the terrestrial elements have also been detected in the atmosphere of the sun from the study of the Fraunhoffer's lines. Similarly if the light coming from a star or a planet be spectroscopically examined, some information regarding their atmospheres can always be obtained. The atmospheres of some of the planets have been studied in this way. The atmosphere of Venus is found to contain a high percentage of carbon dioxide gas, that of Mars a very small percentage of oxygen and that of Jupiter is found to be composed mainly of ammonia and methane. In 1868, a new yellow line was detected in the solar spectrum during an eclipse, whose position did not agree with that of any of the lines of the then known elements on earth. Thirty years later, however, an element was isolated, the yellow emission line of which corresponded to the above yellow line of the solar spectrum. This element has been named *helium* as it owes its discovery to the sun (Greek-Helios). Elements like Caesium, Rubidium, Indium, etc. also have been discovered from the spectroscopic observations. The study of the spectra of stars reveal that the general constitution of the stars are similar to that of the sun. The spectra of the nebulae show that matter is still in the gaseous state in them and pressure of the gases there is sufficiently low.

For the detection of traces of impurities in metals the spectroscopic analysis is supposed to be the most dependable method. The spectroscopic analysis serves as one of the most confirmatory methods for testing the impurities present in minute quantities when other chemical methods fail. The arc spectrum of the metal impregnated with the traces of impurities is first taken and the lines obtained therein are examined wherefrom the lines characteristic of the impurities can be easily detected. In metallurgy this analysis is essential because samples of iron, steel and other metal castings require a definite proportions of certain impurities, which can be easily tested from the relative intensities of the corresponding spectral lines. Brass, so commonly used for casting and construction of small machines and machine parts, acquire a tendency to crack if it contains bismuth beyond a certain percentage. The percentage of bismuth can be ascertained from ordinary chemical

analysis. But in industry, the quicker and more sensitive method of spectrochemical analysis is adopted wherein from comparative study of the intensities of the lines the percentage of Bi present in the sample can be accurately determined.

In detective departments also the method of spectrum analysis may be extensively used. In western countries the method has already been applied successfully in many investigations. The dust particles in the trousers of a suspect tells about his past movements and the nature of these dust particles can be studied correctly by the spectroscopic method. The jemmy of a suspected burglar may reveal the presence of the same spectral lines as found in a broken lock in the burgled house. Various other examples of such possibilities of detecting the crimes by spectrum analysis can be cited. An intelligent officer in the detective department can himself think out the newer possible avenues for the science of spectroscopy to solve the various problems of his department.

PHYSICAL OPTICS

CHAPTER I || WAVE THEORY OF LIGHT

1. Theories of Light :—Various theories have so far been put forward by different scientists to explain the nature of light. A short review of some of these theories are given in the following paragraphs.

(a) **Newton's Corpuscular Theory** :—The first scientific attempt to explain the nature of light was made by the great English scientist and philosopher Sir Isaac Newton (1642—1727). He put forward the corpuscular or emission theory of light. According to this theory a source of light emits a very large number of tiny particles or corpuscles which fly off in space like a hail of shots in all directions with a tremendous speed in straight lines in a homogeneous medium. When these corpuscles strike the eye of an observer, the sensation of light is felt. These corpuscles are so small that they can readily pass through the interstices of particles of some matter. They are very little affected by gravity owing to their high speed and negligible mass. The sensation of the various colours is attributed, according to this theory, to be due to the impact of corpuscles of various sizes on the eye. The phenomena of reflection and refraction were explained by Newton as due to the forces of repulsion and attraction respectively which the corpuscles experience near the surface of the medium on which the light is incident. In order to explain reflection Newton supposed that the light corpuscles, as they approach the reflecting surface, experience a force of repulsion acting normally to that surface. This force of repulsion goes on increasing till the velocity of a light corpuscle is zero. The motion of the corpuscle is then reversed along the dotted path (Fig. 1) with the same velocity as that of the incident corpuscle. The phenomenon of diffraction, *i.e.* the slight bending of light rays round an obstacle, could not be explained by the proposed corpuscular theory of Newton. Young's observations on the interference of light could neither be explained by this theory. Moreover, Newton's explanation of the laws of refraction leads to the absurd result that light travels with a higher speed in a denser medium as will be evident from the following simple considerations.

When a beam of particles enters a denser medium from vacuum, it will bend towards the normal due to a change of velocity suffered at the interface. According to Snell's law, if i and r be the angles

of incidence and refraction respectively, $\frac{\sin i}{\sin r} = \text{a constant} = \mu$,

where μ is the refractive index of the medium. This abrupt change in direction and of velocity at the boundary of the two media was accounted for by assuming the existence of attractive forces which act through distances of the order of the magnitude of the atomic dimensions. If the original velocity of a particle be resolved into two components, one parallel to the interface and the other perpendicular to it, it is only the latter component (*i.e.* the normal component) that will be changed due to the above short-range forces. This is so because the number of particles of the denser medium in the surface layer is negligibly small compared to those in the normal direction. This change in the normal component produces a change in the direction of the incident beam as it crosses the boundary.

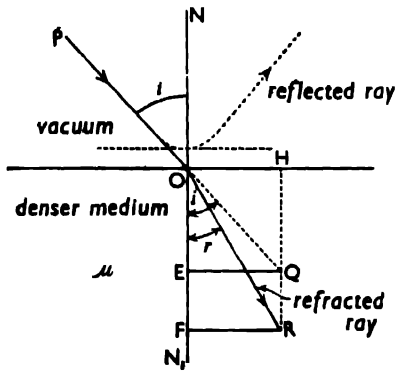


Fig. 1

Let PO (Fig. 1) represent the incident beam of light corpuscles incident at the point O of the boundary between the two media. Let OQ be the continuation of PO drawn such that its length is proportional to c, the velocity of light in vacuum. OE and OH represent respectively the component of OQ normal and parallel to the boundary surface. Due to the attractive force exerted by the denser medium, the component OE changes to the value OF while the other component OH remains unaltered. So the resultant direction of the beam in the denser medium is represented by OR which is the resultant of OF and OH. OR also represents in magnitude the effective velocity of light v_m in this medium. OR being the refracted beam, the angle of refraction is $r = \angle FOR$.

$$\mu = \frac{\sin i}{\sin r} = \frac{EQ}{OQ} \Big/ \frac{FR}{OR} = \frac{OR}{OQ} = \frac{v_m}{c}, \quad (\because EQ = FR).$$

As μ here is greater than unity, v_m is greater than c, *i.e.* light travels with a higher speed in a denser medium.

But the experiments of Foucault on the determination of the velocity of light in different media showed that the velocity of light is smaller in a denser medium. Moreover, the assumptions of a repulsive force in the case of reflection and of an attractive

force in the case of refraction entail difficulties particularly in the cases where both reflection and refraction simultaneously occur. Newton tried to explain away the difficulties by assuming that the corpuscles sometimes reach the surface in a 'fit of easy reflection' and sometimes in a 'fit of easy transmission'. But the explanation seems to be anything but convincing. The above mentioned failures of the corpuscular theory gave a death-blow to it. The theory was, as a consequence, rejected in favour of another theory, known as the wave theory, proposed by Huygens.

(b) Wave theory.—Huygens in 1678 suggested that light is propagated as a wave motion in a subtle, imponderable elastic medium called *ether*, which permeates all matter and space. The supposition of such a medium was necessary as no wave can propagate without a medium. When these waves strike our eyes we feel the sensation of vision. He supposed that the light waves are longitudinal and the phenomena of reflection, refraction, interference and diffraction were successfully explained. But another phenomenon, known as polarisation of light, could not be explained. This difficulty was overcome by Fresnel by supposing that the light wave is transverse. Fresnel also explained the rectilinear propagation of light with the help of the idea of secondary waves or wavelets. We shall develop this idea in detail later on. Though the wave theory, as proposed by Huygens and subsequently modified by Fresnel, Thomas Young and others, could explain a large number of phenomena exhibited by light, it has got a great drawback. It necessitated the adoption of a hypothetical medium called ether which has an extraordinary property of an elastic solid of abnormally low density.

(c) Electromagnetic theory.—Before the discovery of Faraday effect regarding the rotation of the plane of polarisation by the application of a strong magnetic field, light was supposed to have no connection with electricity and magnetism. In 1873, Clark Maxwell working on the hypothesis of displacement current in dielectrics arrived at a totally new conclusion that a electromagnetic disturbance should propagate in space with a speed given by, $v = \frac{c}{\sqrt{k\mu}}$, where k is the specific inductive capacity of the medium, μ = permeability of the medium, and c = ratio of e. m. u. to the e. s. u. of electric charge. The value of $\sqrt{k\mu} = 1$ in vacuum, since in vacuum both k and μ are equal to unity. Thus $v = c = 3 \times 10^{10}$ cm./sec. which is equal to the velocity of light in vacuum. This strange coincidence led Maxwell to put forward his famous electromagnetic theory of light. According to this theory, the vibrations of waves are replaced by the variations of intensities, one electric and the other magnetic, two acting mutually

perpendicular to each other, and also perpendicular to the direction of the energy propagation. Hertz, in 1888, actually demonstrated the truth of Maxwell's theory by producing the electromagnetic waves in the laboratory. The electromagnetic theory could explain the phenomena of reflection, refraction, diffraction, dispersion, etc. quite satisfactorily.

(d) Quantum theory of light:—Einstein first put forward this startling conception of light quanta during the first decade of the 20th century. His conception may be regarded as an extension of the energy quantum first conceived in 1900 by Max Planck while studying the black body radiation. Einstein's conception may be regarded more or less as a revival of the Corpuscular theory. According to this theory, radiation is to consist of indivisible "Radiation Quanta" or photons each quantum or photon carrying energy $h\nu$, where h is a constant known as Planck's constant, and ν is the frequency of radiation. This quantum character of light has got a brilliant success in a class of phenomena involving the interactions between matter and radiation. It can explain the photo-electric effect, Compton Eflect, Raman Effect, etc. But it fails to explain the phenomena of interference, diffraction, etc. Thus in order to explain one class of phenomena we consider light as waves and to explain another class of phenomena we consider light as quanta. Attempts are being made to develop a satisfactory theory which will unify these two concepts into one general co-ordinating principle.

2. Origin of the Wave Theory :—Huygens in 1678 suggested that light is propagated as waves in an elastic medium called ether, which pervades all matter and space. The introduction of such a subtle and imponderable medium was necessary because no wave motion can take place without a medium. However, the theory was not accepted for a long time as it could not explain the bending of light round an obstacle and also the rectilinear propagation of light. The theory was later on developed by Thomas Young and Fresnel to such an extent that it could explain most of the phenomena exhibited by light such as interference, diffraction, etc. It is necessary to understand the production and propagation of waves before other optical phenomena are explained.

A wave is a disturbance, periodic both in space and time, and is characterised by the phenomenon of undulation, *i.e.* up and down motion spreading out of the source in all directions through the medium. The energy is transferred from one part of the medium to another while the individual particles of the medium only oscillate about their mean positions of equilibrium. There is no bodily movement of any part of the medium but the profile of the waves moves continuously. The most common type of waves is observed on the surface of water. When a stone is thrown on the surface of water, the waves spread out from the point of disturbance.

If there is a piece of cork anywhere on still water, the cork goes on oscillating up and down by the action of the wave. This is a very common example of wave motion in a medium. There are in general two types of waves, viz. (i) **longitudinal** and (ii) **transverse.** When the particles of the medium execute a motion parallel to the direction of propagation of the the waves, the waves are said to be **longitudinal** and when the particles of the medium execute a motion perpendicular to the direction of propagation of the waves, the waves are said to be **transverse**. Sound waves are common examples of longitudinal wave motion. Longitudinal elastic waves, may be generated in all media, *i.e.* solid, liquid and gas. Whereas, transverse waves are possible only in solids. The transverse waves are controlled by the shearing elastic forces in the medium. Only in solids, the shearing elastic forces are possible. The vibration of the sonometer string is a common example of transverse vibration. The waves on the surface of water are not, however, elastic waves. The smaller ones are called the *Capillary waves* or the *ripples* and are controlled mainly by surface tension, while the larger ones, called the *gravity waves*, are controlled by gravity. The motion of the particles is not strictly transverse in these cases. In the case of optical phenomena the vibrations are transverse. The ether particles, as the light passes through them, vibrate in a direction perpendicular to the direction of the wave motion. The medium ether may be considered as an elastic solid in reference to light which is supposed to be a kind of elastic waves. Since the velocity c of a wave propagated through a medium of elastic modulus E and of density D is given by $c=\sqrt{(E/D)}$, where for light waves $c=3\times10^{10}$ cms. per sec., the hypothetical ether through which light propagates must have abnormal elasticity and very low density. Lord Kelvin supposed ether to be a jelly-like substance whose modulus of rigidity is 4500 dynes per cm.^{2} and density 5×10^{-18} gm. per c.c. But such a material medium with the above quoted abnormal values of density and elasticity is unknown to our experience. So ether may be termed as an extraordinary or hypothetical medium. Later views regarding the propagation of light waves have discarded the ether hypothesis as gratuitous and have suggested that no medium is at all necessary for the propagation of light waves.

3. **Wave motion** :—Before the explanation of various optical phenomena is given on the basis of the wave theory of light, it is necessary to understand what a wave motion is. A wave motion is produced in a medium due to the periodic motion or vibration of its particles. There being some sort of elastic connection between the adjacent particles in the medium, the vibration of a single particle is not confined to itself, but is transferred from layer to layer. The instantaneous form of this disturbance as a whole

is called the wave, *e.g.* the wave on the surface of water. The simplest form of wave motion is one in which the particles perform Simple Harmonic Motion (S.H.M.). To understand Simple Harmonic Motion consider a particle P moving with a uniform velocity v along a circle of radius a. Let N be the foot of the perpendicular drawn from it on one of the diameters, say YY', of the circle (Fig. 2).

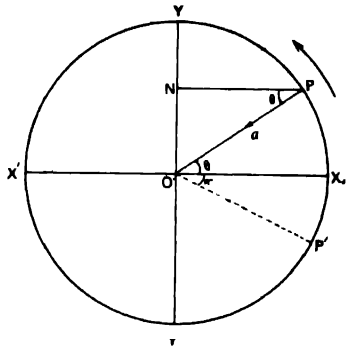


Fig. 2

As the generating particle P starts moving from X to Y, the foot of the perpendicular N moves from O to Y. In this way as the point P moves in the circle, called the **circle of reference**, the point N executes a to-and-fro motion along the diameter YY'. The motion of the foot of the perpendicular along YY' will be simple harmonic. Let the revolving particle go from X to P in time t, describing an angle θ at the centre of the circle. θ will be equal to ωt, where ω is the angular velocity and is equal to $\frac{2\pi}{T}$, and T is the time period of oscillation. The displacement of the foot of the perpendicular is given by $ON = y$, where

$$y = a \sin \theta$$
$$= a \sin \omega t \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

This is the equation of motion of N starting from O. The maximum displacement of N on either side of its mean position O is called its **amplitude** of vibration.

The velocity of the particle executing S.H.M. is given by

$$v = \frac{dy}{dt} = a\omega \cos \omega t, \text{ and}$$

acceleration, $f = \frac{dv}{dt} = -a\omega^2 \sin \omega t = -\omega^2 y$.

Thus the acceleration of the particle executing S. M. H. is proportional to its displacement y from its standard position, O. This satisfies one of the conditions necessary for S.H.M. The negative sign of the acceleration indicates that the acceleration is always directed towards the mean position of equilibrium. Thus the foot of the perpendicular drawn from the generating particle on the reference circle on any diameter satisfies the conditions of simple harmonic motion. Instead of measuring the starting time from the instant $t = 0$, as above (*i.e.* when P is at X or N is at O), we can measure the time from the instant when P is at P', say, so that OP' makes an angle α with OX, then the displacement along $Y'OY$ at time t is

$y = a \sin(\omega t - \alpha)$. If the point P' be above X, then the displacement will be obviously given by $y = a \sin(\omega t + \alpha)$ ... (2)

The phase of a vibrating particle at any instant is the quantity which determines its state of motion, *i.e.* the direction of motion as well as the displacement of the particle at that instant.

The *phase* is measured by the angle swept out by the radius OP [Fig. (1)] on the circle of reference, since the particle last left its mean position. So in equation (1), ωt is the phase angle and in equation (2), $(\omega t + \alpha)$ is the phase angle. The phase angle of the particle at $t = 0$, or at the instant of start is called the **epoch**. From equation (2) we have, epoch $= \alpha$.

If, instead of a single particle executing simple harmonic motion, we have a number of such particles elastically connected to one another, the vibration of one will be communicated to the next one and so on. Particles having the same displacement from the mean position of rest and moving in the same direction are said to be vibrating in the same phase. On the other hand, if any two particles are in their extreme positions in the opposite directions on either side of the mean position, they are said to be vibrating in opposite phases. The minimum distance between any two particles in the medium, executing vibrations in the same phase, or differing in phase by 2π, is known as the *wavelength* of the wave.

So long we have considered only the equation of motion of a single particle executing S. H. M. Next let us consider the equation of wave motion, *i.e.* equation of vibration, of all the particles in the medium as the waves travel through it with a velocity c. Let us suppose that $y = a \sin \omega t$ is the equation of vibration of a particle in the medium. Then another particle at a distance x from the former one will also vibrate simple harmonically but its phase of vibration will be different from the previous one. We have already said that a path difference of λ corresponds to a phase difference of 2π. Therefore, a path difference of x will correspond to a phase difference of $\frac{2\pi}{\lambda} x$. Thus the phase of the second particle will be given by $\left(\omega t - \frac{2\pi}{\lambda} x\right)$ and its equation of motion will be given by,

$$y = a \sin\left(\omega t - \frac{2\pi}{\lambda}.x\right)$$

$$= a \sin\left(\frac{2\pi}{T}.t - \frac{2\pi}{\lambda}.x\right)$$

$$= a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

$$= a \sin \frac{2\pi}{\lambda}(ct - x)$$

since the velocity $c = n\lambda = \lambda/T$, where n is the frequency of vibration.

The equation of motion of any particle in the medium can be obtained by giving the particular value to x in eqn. (3). So this equation represents the general equation of a wave motion.

Amplitude :—The maximum displacement of a particle executing S.H.M., in either direction from its mean position of rest, is called the amplitude. In eqns. (1) and (2), 'a' is the amplitude of vibration.

Wavelength :—The minimum distance between any two particles in a medium executing vibrations in the same phase or differing in phase by 2π, is known as the wavelength,

Frequency :—The number of vibrations executed by a particle per second is called its frequency. If T is the time period of vibration *i.e.* the time taken for one complete vibration, it is evident that

$$n = \frac{1}{T}.$$

The frequency or the wavelength determines the colour and a few other fundamental properties of the light waves.

4. Illumination at a Point :—The *illumination* (or *the intensity of illumination*) at a point is measured by the quantity of light energy passing through unit area surrounding that point per second, when this area is placed perpendicularly to the direction of propagation of light.

During the propagation of light waves, ether particles in the medium execute simple harmonic motions and the equation of vibration of any particle in the medium is given by

$y = a \sin \frac{2\pi}{\lambda}(ct - x)$; where c is velocity of the light wave.

The velocity of any oscillating particle is given by,

$$v = \frac{dy}{dt} = \frac{2\pi ac}{\lambda} \cos \frac{2\pi}{\lambda}(ct - x).$$

and its acceleration,

$$f = \frac{dv}{dt} = -\frac{4\pi^2 c^2 a}{\lambda^2} \cdot \sin \frac{2\pi}{\lambda}(ct - x) \; ;$$

$$\text{or,} \quad f = -\frac{4\pi^2 c^2}{\lambda^2} \cdot y \qquad \ldots \qquad \ldots \qquad \ldots \quad (1)$$

If m be the mass of the particle, its kinetic energy will be given by,

$$\tfrac{1}{2} mv^2 = \tfrac{1}{2} m \times \frac{4\pi^2 a^2 c^2}{\lambda^2} \cos^2 \frac{2\pi}{\lambda}(ct - x).$$

The mean K. E. during a complete time period is given by

$$\frac{1}{T}\int_0^T \frac{1}{2} mv^2.dt = \frac{m\pi^2a^2c^2}{\lambda^2 T}\int_0^T 2\cos^2\frac{2\pi}{\lambda}(ct-x).dt.$$

$$= \frac{m\pi^2a^2c^2}{\lambda^2} \quad \ldots \quad \ldots \quad (2)$$

Again the potential energy of the particle is

$$-\int_0^y m.f.dy = \frac{4m\pi^2c^2}{\lambda^2}\int_0^y y.dy = \frac{2m\pi^2c^2y^2}{\lambda^2}$$

$$= \frac{2m\pi^2c^2a^2}{\lambda^2}\sin^2\frac{2\pi}{\lambda}(ct-x).$$

The mean potential energy during a complete time period is

$$\frac{1}{T}\int_0^T \frac{2m\pi^2c^2a^2}{\lambda^2}\sin^2\frac{2\pi}{\lambda}(ct-x)\,dt = \frac{m\pi^2c^2a^2}{\lambda^2} \quad \ldots \quad \ldots \quad (3)$$

The mean total energy is the sum of the expressions (2) and (3).

Hence, total energy, $E = \frac{2m\pi^2c^2a^2}{\lambda^2}$.

The intensity of illumination or simply the illumination is equal to the product of the energy per unit volume (*i.e.* energy density) of the medium and the velocity. Hence, illumination, $I = E_0.c$, where E_0 is the energy per unit volume and is equal to $\frac{2\rho_0\pi^2c^2a^2}{\lambda^2}$, and ρ_0 is the mass of the vibrating particles per unit volume, *i.e.*, the density of the medium.

$$\therefore \quad I = \frac{2\rho_0\pi^2c^2a^2}{\lambda^2}.\ c = \frac{2\rho_0\pi^2c^3}{\lambda^2}a^2.$$

Hence, $I \propto a^2$; or, $I \propto$ (amplitude)^2.

Thus the illumination or the intensity of illumination is proportional to the square of the amplitude.

5. Wavefront and Ray :—A wavefront is defined as the envelope or trace drawn through all the points on a wave which are exactly in the same condition as regards displacement and direction of motion, *i.e.* in the same phase. Thus, a surface drawn along the crests of a water wave is a wavefront and also a surface drawn along the troughs would be another wavefront.

In a homogeneous transparent medium a wave generated at a point travels out in all directions from the point with the same velocity. At any instant of time, the wave motion lies upon the *surface of a sphere*, whose centre is the generating point and radius is equal to the product of the velocity and time. Such waves set up in a homogeneous medium due to the vibration of a point or a small sphere in it are called *spherical waves*. On this sphere the

particles are all in the same phase of motion. This equi-phase surface is the wavefront at the time. At a very long distance from the source of disturbance, the spherical surface, over a limited region, may be treated as *plane*. So the wavefront may be taken as plane, if the source of disturbance is at a very long distance. A linear source (*e.g.* a narrow slit) will give rise to *cylindrical* wavefront. A normal drawn to the wavefront at any point indicates the direction of propagation of the wave and is known as a *ray*.

6. Huygens' Principle :—According to wave theory, light is propagated in a transparent isotropic medium with a definite velocity in all directions. Huygens' principle gives up a geometrical picture of this mode of propagation. According to Huygens each point of a source of light acts as a centre of disturbance, which spreads out in the form of waves in all directions at a constant speed. So at a later instant the effect of the initial disturbance will be found on the surface of a sphere surrounding the point source. This surface indicates the wavefront at the instant under consideration. In order to explain the mode of propagation of the waves in the medium. Huygens suggests that each point on this wavefront in its turn acts as a *secondary source* sending out disturbances in all directions with a speed identical with that of the original disturbance. These *secondary wavelets* of disturbance will reach the surface of a smaller sphere at a subsequently later instant. The secondary wavelets starting from all the secondary sources of the original wavefront will in a similar way reach the surfaces of so many identical spheres at that particular instant. The envelope of these small spherical surfaces is itself a spherical surface which represents the new position of the wavefront at the instant considered. Each point of this new wavefront again acts as a secondary source and the above process will continue. So as the wave advances the wavefront also moves parallel to itself. Huygens' principle may be illustrated in the following way with the help of a diagram (Fig. 3).

If S (Fig. 3) be a point source of light, the disturbance produced at S according to the wave theory spreads out uniformly in an isotropic transparent medium with equal velocity in all directions. So the wavefront at any instant will be the surface of a sphere drawn with S as the centre and ct as the radius where t is the time and c the velocity of propagation. Let AB represent the trace of such a wavefront at a particular instant on which

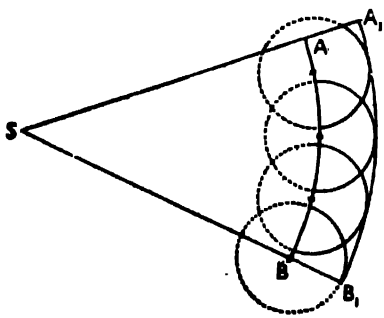


Fig. 3

every point is in the same state of disturbance. At any later instant of time, similarly let the disturbance lie on the trace A_1B_1. Evidently the wave has travelled from AB to A_1B_1 in the interim period. Huygens considers the mechanism of such propagation as follows : Each particle of the medium in the wavefront AB becomes a new centre of disturbance from which spherical waves, called **secondary wavelets**, spread out with the same velocity as in the case of the wave travelling from S to AB provided, of course, the medium remains the-same. The envelope of these secondary wavelets, *i.e.* the spherical surface whose trace A_1B_1 touches all these spheres, is the new wavefront at the end of the period envisaged. The wave travels outwards from A_1B_1 to succeeding fronts as it did from AB to A_1B_1. Evidently the wavefront at any point moves in the direction of the normal to the wavefront at that point.

We have assumed above that the secondary waves are propagated in the forward direction only. As each point of a wavefront is considered as an independent source of disturbance, it is quite likely that it should also emit spherical waves in the backward direction. But in a perfectly homogeneous medium no such backward propagation actually takes place. Rigorous theoretical considerations as well as experiment show that no such back wave is possible.

7. Deduction of the Laws of Reflection from the Wave Theory :—(*Reflection of a plane wavefront*)—Let AB (Fig. 4) be the section of the incident plane wavefront perpendicular to the plane of the paper. MM' is the section of the reflecting surface on the plane of the paper. PBB' and QA are perpendicular to the wavefront AB and hence they are incident rays. The angle i between the incident ray and normal is the angle of incidence.

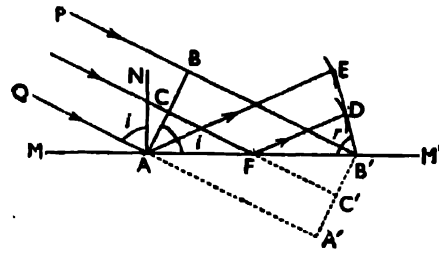


Fig. 4

Since QA is perpendicular to AB and AN is perpendicular to AB', $\angle BAB'$ is also equal to i. Thus the angle of incidence is the angle which the incident wavefront makes with the surface. If c is the velocity of light, the time taken by light to reach from B to B' is evidently $\frac{BB'}{c}$. If there were no reflecting surface, the wavefront after a time interval $\frac{BB'}{c}$, would have occupied the position $B'A'$. Evidently $BB' = AA'$. Due to the presence of the reflecting surface, the secondary wavelets, according to Huygens' principle, would lie somewhere on the surface of a sphere with A as centre and BB' as radius.

The section of the sphere by the plane of the paper is the arc E of a circle. Draw $B'E$ tangent to E, then the plane passing through $B'E$ and perpendicular to the plane of the paper is the reflected wavefront, provided it can be shown that another wavelet from any point C of the incident wavefront also after the same time interval $\frac{BB'}{c}$, occupies a point D of the reflected wavefront.

F is a point on the reflecting surface where the wavelets from point C of the incident wavefront touches the reflecting surface. Draw FD perpendicular to the reflected wavefront $B'E$. Produce CF. It will meet $B'A'$ normally at C'.

From the similar triangles, $AB'E$ and $FB'D$,

we have, $\frac{AE}{FD} = \frac{AB'}{FB'}$ ... ... (1)

Again from the similar triangles, $AA'B'$ and $FC'B'$,

$$\frac{AA'}{FC'} = \frac{AB'}{FB'} \quad \text{... ... ...} \quad (2)$$

From equations (1) and (2), we get,

$$\frac{AE}{FD} = \frac{AA'}{FC'}.$$

But $AE = AA'$. $\therefore$ $FD = FC'$.

Thus the waves from F, which would have gone to C' had there been no reflecting surface, will now go to D in the presence of the reflecting surface. Similarly it can be shown that any wavelet from the incident wavefront AB, will after reflection lie on $B'E$. So $B'E$ is the reflected wavefront.

Now in the triangle ABB' and $AB'E$, $BB' = AE$, AB' is common and $\angle ABB' = \angle AEB' = 90°$.

$\therefore$ The angle of incident $i = \angle BAB' = \angle EB'A$.

But AB' being perp. to AN and EB' being perp. to AE,

$$\angle EB'A = \angle EAN = \angle r.$$

$\therefore$ Angle of incidence i = angle of reflection r.

It is thus clear that a plane wavefront is reflected back from a plane reflecting surface as a plane wavefront. Since AB, $B'E$, MM' are sections of the planes perpendicular to the plane of the paper, the normals to these planes are in the same plane.

Therefore, the incident ray and the reflected ray which are normals to the incident and reflected wavefronts must lie in the same plane as the normal to MM' at the point of incidence.

8. Deduction of the Laws of Refraction :—(*Refraction of a plane wavefront*). Let AC (Fig. 5) be the trace of a plane surface perpendicular to the plane of the paper, separating two media having refractive indices μ_1 and μ_2, where μ_2 is greater than μ_1. Let AIB be the incident plane wavefront, the incident rays being the normals, like PA and QB, to this wavefront.

Due to the presence of the lower medium having a different value of refractive index (μ_2), the light waves from A will go to the lower medium upto E, by the time (t) the wave from B reaches the point C on the separating surface. Therefore, $t = \frac{BC}{v_2} = \frac{AE}{v_1}$ where v_1 and v_2 are velocities of light waves in the upper and the lower media respectively. If we draw a sphere with AE as radius and A as centre, then the section of it by the plane of paper is the arc of the circle E. If then we draw a tangent from C to the arc E, a plane passing through CE and perpendicular to the plane of the paper is the refracted wavefront after a time interval t, provided it can be shown that any other disturbance from the incident wavefront AB will also lie on the refracted wavefront CE after the same time interval.

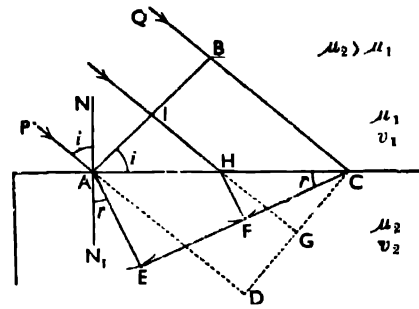


Fig. 5

In the absence of the second refracting medium the incident wavefront AIB after a time t will occupy the position DGC as shown in the figure. Let H be a point on the separating surface AC where the wavelets from the point I of the incident wavefront meets AC. Draw a perpendicular HF on EC from H.

From the similar triangles AEC and HFC, we have,

$$\frac{AC}{HC} = \frac{AE}{HF} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

Again from the similar triangles ADC and HGC, we have,

$$\frac{AD}{HG} = \frac{AC}{HC} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

From eqns. (1) and (2), we have,

$$\frac{AE}{HF} = \frac{AD}{HG};$$

or, $\frac{v_2 t}{HF} = \frac{v_1 t}{HG}$, ($\because AD = BC = v_1 t$ and $AE = v_2 t$) ;

or, $\frac{HF}{v_2} = \frac{HG}{v_1}$.

Thus the time taken by light to go from H to G in the absence of the second medium is the same as it will take to go to F from H in the second medium. Therefore in time t, as the disturbance from B reaches C, the disturbance from I will reach F. As HF is drawn perpendicular to CE, the wavelets from H will also touch CE at F. Thus CE is the section of the refracted wavefront in the second medium in the plane of the paper.

From simple geometry, $\angle ACE = \angle EAN_1 = \angle r$, the angle of refraction and $\angle BAC = \angle PAN = \angle i$, the angle of incidence.

$\therefore$ From the figure, $\sin i = \frac{BC}{AC}$, and $\sin r = \frac{AE}{AC}$

$\therefore$ $\frac{\sin i}{\sin r} = \frac{BC}{AE} = \frac{v_1 t}{v_2 t} = \frac{v_1}{v_2}$ = a constant, for this particular pair of media. This is Snell's law. The constant $\frac{v_1}{v_2}$ *i.e.* the ratio of the velocity of light in the first medium to the velocity of light in the second medium is known as the refractive index of the second medium with respect to the first medium and is denoted as μ_2. This is a constant for a particular colour of light and for a particular pair of media.

Following the same argument as in the previous case it follows that the incident ray, the normal at the point of incidence, and the refracted ray lie in one and the same plane.

9. Total internal Reflection :—In Fig. 5 if AE is greater than AC, no tangent can be drawn from C to the sphere whose centre is A. Obviously the limiting case for which a refracted wavefront can be drawn in the lower medium of refractive index μ_2 is that when $AE = AC$.

We know, $\frac{BC}{AC} = \sin i$, where i is the angle of incidence.

$\therefore$ In the limiting case, $BC = AC \sin i = AE \sin i_c$.

But $BC = v_1 t$ and $AE = v_2 t$,

$\therefore$ $v_1 t = v_2 t \sin i_c$;

or, $v_1 = v_2 \sin i_c$;

or, $\frac{v_1}{v_2} = \sin i_c$.

Here since $\sin r = AE/AC = 1$, $\angle r = 90°$.

Hence the refracted ray grazes along the surface of separation between the two media. As i_c is ordinarily less than 90°, v_1/v_2 is less than unity,

$\therefore$ $v_2 > v_1$. If this is to be satisfied, for the limiting condition of no refraction, the wavefront must then strike the surface

of a rarer medium from a denser medium. (*i.e.* μ_1 should be greater than μ_2). This gives us one of the conditions for total internal reflection.

Thus we see that if the angle of incidence of a wavefront while passing from a denser to a rarer medium is greater than i_c, given by $\sin i_c = \frac{v_1}{v_2} = \frac{1}{\mu}$ where μ is the refractive index of the denser medium with respect to the rare one, no light will pass on to the rarer medium because for the angle of incidence i_c the refracting angle $\angle r$ has already reached its maximum possible value of 90°. For a greater angle of incidence the wavefront, instead of entering the rarer medium, will be turned back in the denser one. This angle i_c is known as the *critical angle* for the pair of media considered and for the colour of light used.

10. Reflection of a plane wavefront at a spherical surface :— Let AB be the section of a plane wavefront incident on a convex spherical reflecting surface whose section by the plane of the paper is shown by XCY (Fig, 6). Let O be the centre of curvature of the spherical reflecting surface. C being the pole of mirror. OC is the principal axis. Suppose that by the time the disturbance from A goes to X and that from B goes to Y of the mirror, the disturbance from C will be reflected normally and reach the point D. $\therefore$ $AX = BY = CD$. Hence XDY is the reflected wavefront.

Let ρ be the radius of curvature of the reflected wavefront, XDY. If the aperture of the mirror is small, we have $EC = \frac{XE^2}{2R}$ and $ED = \frac{XE^2}{2\rho}$, where R = radius of curvature of the spherical mirror $= OC$.

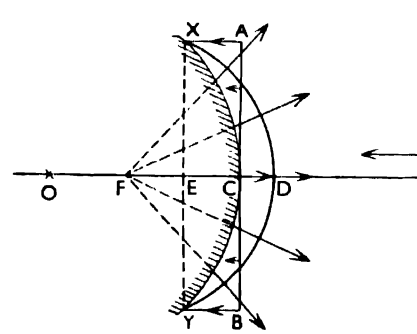


Fig. 6

Now since $2EC = ED$, we have $\rho = \frac{R}{2}$. Thus the radius of curvature of the reflected wavefront is half the radius of curvature of the convex surface. In other words, a plane wavefront after reflection from a convex spherical surface is converted into a spherical wavefront whose radius of curvature is half that of the reflecting surface. Hence, the parallel rays of light incident on a convex spherical surface will appear to diverge from a point called the **focus**, whose distance from the pole of the mirror is $\frac{R}{2}$.

11. Reflection of a spherical wave at a plane surface :— Let O be a point source of light at a distance OC from a plane

reflecting surface XY (Fig. 7). Let ACB be the section of the spherical wavefront in the plane of the paper, meeting the reflecting surface at C. In the absence of the reflecting surface the wavefront would have occupied a position XEY after certain time, as shown in the figure. But due to the presence of the reflecting surface, the disturbance from the point C, instead of going to E, is reflected back and reaches the point D such that $DC = CE$. During the same interval the disturbances at A and B reach the points X and Y respectively. So, XDY becomes the reflected wavefront, which is also spherical in nature having its centre of curvature at I. Therefore, the incident light rays after reflection appear to diverge from the point I.

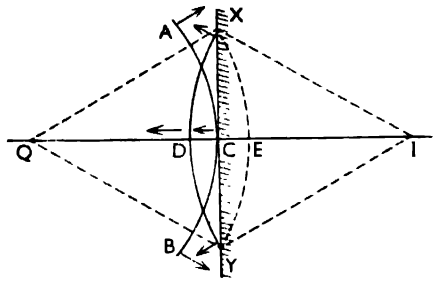


Fig. 7

Since XEY is the arc of a circle having its centre at O, we have,

$$OE = \frac{XC^2}{2CE}.$$

Also since I is the centre of the arc of a circle XDY, we have,

$$ID = \frac{XC^2}{2DC}.$$

Since, $DC = CE$, we have, $OE = ID$.

$$\therefore \quad OC + CE = DC + CI;$$

or, $OC = CI$.

This means that the image I is situated as far behind the mirror as the object O is in front of it.

12. Reflection of a spherical wave at a spherical surface :— Let O be a point source of light situated on the axis of a concave spherical reflecting surface XAY. Let XAY be of small aperture, and let R be its radius of curvature where R is equal to AC. A and C are the pole and centre of curvature of the mirror respectively. XBY is the spherical wavefront incident on the mirror. By the time the disturbance from B reaches the mirror at A, the disturbances from X and Y of the incident wavefront are reflected and reach the points E and F respectively such that $XE = YF = AB$. EAF thus constitute the reflected wavefront, which is also spherical in nature having I as its centre. That light rays emanating from a point O will after reflection at the spherical surface, converge to the point I. Let $AO = u$, $AI = v$ and $AC = R$.

Since the aperture of the mirror is small compared to its radius of curvature, we have,

$$AC = \frac{XC^2}{2R},\ AD = \frac{XC^2}{2v},\ [\text{Since } XC \simeq ED],\ BC = \frac{XC^2}{2u}.$$

Now,

$$AD = AC + CD$$
$$= AC + AB$$
[since $CD \simeq XE = AB$]
$$= AC + AC - BC$$
$$= 2AC - BC.$$

$$\therefore \quad \frac{XC^2}{2v} = 2\frac{XC^2}{2R} - \frac{XC^2}{2u};$$

or, $$\frac{1}{2u} + \frac{1}{2v} = \frac{1}{R};$$

or, $$\frac{1}{u} + \frac{1}{v} = \frac{2}{R}.$$

From the definition of focal length, we know that when $u = \infty$, $v = f$.

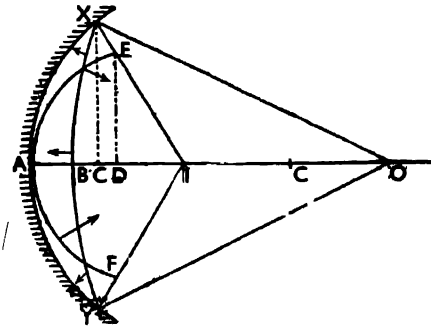


Fig. 8

$$\therefore \quad \frac{1}{f} = \frac{2}{R}. \quad \therefore \quad \frac{1}{u} + \frac{1}{v} = \frac{2}{R} = \frac{1}{f}.$$

This equation gives the relation between u, v and f of a concave spherical reflector.

The case of reflection of a spherical wave at a convex spherical reflecting surface can be dealt with in an exactly similar manner.

13. Refraction of a spherical wavefront at a spherical surface :—Let XAY (Fig. 9) be a concave spherical surface separating two media having refractive indices μ_1 and μ_2, μ_1 being smaller than μ_2. C is the centre of curvature of the spherical surface, XA_2Y represents the position of the incident wavefront at a certain time t, had there been no intervening medium of refractive index μ_2, while XA_1Y represents the actual position of the wavefront at that instant of time when the medium of refractive index μ_2 is present. O is the centre of curvature of the arc XA_2Y which

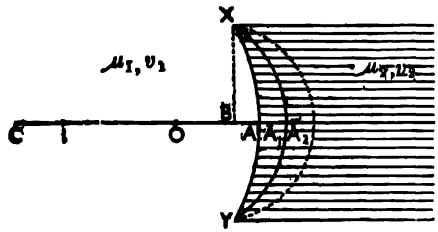


Fig. 9

serves as the source of disturbance. The wavefront XA_1Y which is also spherical appears to diverge from the point I. Hence I is the image point corresponding to the object point O.

Now, $\frac{AA_2}{AA_1}=\frac{v_1}{v_2}=\frac{\mu_2}{\mu_1}$, where v_1 and v_2 are the velocities of the light wave in the two media.

$$\therefore \quad \mu_1.AA_2=\mu_2\ AA_1\ ;$$

$$\text{or,}\quad \mu_1(BA_2-BA)=\mu_2(BA_1-BA).$$

Again from geometry we have,

$$\mu_1\frac{XB^2}{2OA_2}-\mu_1\frac{XB^2}{2CA}=\mu_2\frac{XB^2}{2IA_1}-\mu_2\frac{XB^2}{2CA}\ ;$$

$$\text{or,}\quad \frac{\mu_2-\mu_1}{CA}=\frac{\mu_2}{IA_1}-\frac{\mu_1}{OA_2}.$$

Supposing the aperture of the mirror to be small, we have, $IA_1\approx IA=v$,

$$OA_2\approx OA=u.$$

and let $CA=r=$ radius of curvature of the spherical surface.

$\therefore$ We have,

$$\frac{\mu_2-\mu_1}{r}=\frac{\mu_2}{v}-\frac{\mu_1}{u}$$

In a similar manner as above an exactly similar relation can be deduced for the case when the separating surface is convex spherical.

14. Refraction through a lens—by wave theory :—Let O be a monochromatic point source of light situated on the axis of a convex lens LE' (Fig. 10). Let the radii of curvature of the two surfaces of the lens be r_1 and r_2, and the refractive index of the material of the lens be μ. Let XAY be the incident spherical wavefront touching the lens at A. OA is therefore the object distance u. By the time the disturbance from the regions X and Y of the wavefront reaches the points L and L' of the lens, the disturbances from the point A will traverse a distance AM through the lens such that $\mu.AM=XL=XL'$. The position of the wavefront at that instant within the lens is given by LML'. Again, by the time the disturbance from the point M of the refracted wavefront within the lens reaches the point C on the other surface of the lens, the disturbances from the points L and L' will reach the points X' and Y' respectively, such that $LX'=L'Y'=$

μMC, $X'CY'$ is therefore the transmitted wavefront. It is also spherical in nature with the point I as its centre of curvature.

Thus as a result of refraction through the lens, the disturbances in the transmitted spherical wavefront will converge to the point I, which is the image of O. Hence $CI = v$, the image distance. Let us drop XB, LN and $X'D$, perpendiculars from points X, L and X' on the axis of the lens.

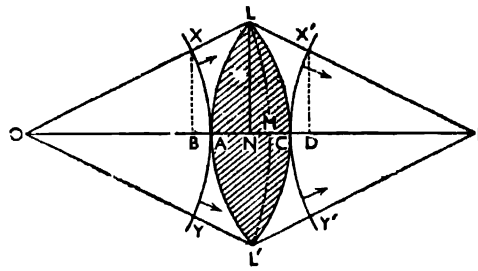


Fig. 10

Now, $AC = AM + MC = \frac{LX}{\mu} + \frac{LX'}{\mu}$.

$\therefore \quad \mu AC = LX + LX' \quad \ldots \quad \ldots \quad \ldots \quad (1)$

The same result also follows from Fermat's principle because the optical path between O and I is the same along any ray considered. If the aperture of the lens is small, we may write,

$$BD = XL + LX' \quad \ldots \quad \ldots \quad (2)$$

Hence, from equations (1) and (2),

$$BD = \mu . AC ;$$

or, $BA + AC + CD = \mu(AN + NC)$;

or, $BA + CD = \mu(AN + NC) - (AN + NC)$;

or, $BA + CD = (\mu - 1)(AN + NC) \quad \ldots \quad \ldots \quad (3)$

Since, the aperture of the lens is small, we may write,

$$XB = LN = X'D = a \text{ (say).}$$

$\therefore \quad BA = \frac{XB^2}{2.OA} = \frac{a^2}{2u}$;

$CD = \frac{X'D^2}{2.CI} = \frac{a^2}{2v}$;

$AN = \frac{LN^2}{2r_1} = \frac{a^2}{2r_1}$, (where r_1 is the radius of curvature of the lens surface LAL').

$NC = \frac{LN^2}{2r_2} = \frac{a^2}{2r_2}$, (where r_2 is the radius of curvature of the lens surface LCL').

Putting these values of BA, CD, AN and NC in equation (3) we have,

$$\frac{a^2}{2u} + \frac{a^2}{2v} = (\mu - 1)\left(\frac{a^2}{2r_1} + \frac{a^2}{2r_2}\right) ;$$

or, $$\frac{1}{u} + \frac{1}{v} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right).$$

This is the relation between u and v of a convex lens forming a real image.

If we adopt the sign convention, that distances measured from the optical centre of the lens *against* the direction of light ray is positive and *along* the direction of light ray is negative, we get,

$$u, +ve\ ;\ v, -ve\ ;\ r_1, -ve\ ;\ \text{and}\ r_2, +ve.$$

Therefore, the above equation becomes,

$$\frac{1}{v}-\frac{1}{u}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right).$$

The focal length f of a lens is defined as $v=f$, when $u=\infty$.

$$\therefore\quad \frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right);$$

or, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$. Thus, we arrive at the usual lens equation.

In the case of a concave lens also a similar relation can be deduced from the wave theory.

Example

A convex lens of focal length 20 cms. and made of glass ($\mu=1{\cdot}50$) *is immersed in a liquid* ($\mu=1{\cdot}33$). *Find the change in the focal length of the lens.*
(*P.U. Suppl. 1954*)

Ans. Here the refractive index of glass with respect to air $=1{\cdot}50$ and that of the liquid with respect to air $=1{\cdot}33$.

$\therefore$ The refractive index of glass with respect to the liquid $=\mu'=\frac{1{\cdot}50}{1{\cdot}33}=1{\cdot}128$.

The focal length of the lens in air is $f_1=20$ cms. Let f_2 be the focal length of the lens in the liquid.

Then, $\frac{1}{f_1}=(1{\cdot}5-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$; and $\frac{1}{f_2}=(1{\cdot}128-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$.

$\therefore\ \frac{f_2}{f_1}=\frac{0{\cdot}5}{0{\cdot}128}$. Hence $f_2=20\times\frac{0{\cdot}5}{0{\cdot}128}=78{\cdot}125$ cms.

So the change in the focal length $=78{\cdot}125-20=58{\cdot}125$ cm.

***15. Fermat's Principle and Wave Theory** :—According to Fermat's principle when light passes from P to Q, we have,

$$\delta\int_P^Q \mu.ds=0.$$

where, $\mu = \frac{c}{v}$, c being the velocity of light in vacuum, and v that in the medium through which light passes.

$$\delta \int_P^Q \frac{c}{v} ds = 0.$$

The quantity v which appears in the above expression is the velocity with which the wavefront advances, *i.e.* it is the velocity of the wavefront according to wave theory.

The principle has got a close resemblance with the Maupertius's principle of least action in dynamics. The above principle may be written in the form,

$$\delta \int_P^Q v.ds = 0, \text{ where } v \text{ is the velocity of the particle.}$$

But Fermat's principle is written as,

$$\delta \int_P^Q \frac{c}{v}.ds = 0 \text{ ; or, } \delta \int_P^Q \frac{ds}{v} = 0. \quad \ldots \quad \ldots \quad (1)$$

($\because$ c is a constant)

So in the latter case, v appears in the denominator. However, the analogy between these two principles of mechanics and optics is only apparent. Hamilton tried to discover the physical basis of it. Later, de Broglie of France and Schroedinger of Germany successfully worked out the physical basis of it. According to their view a kind of wave motion is supposed to be associated with every material particle, and the velocity of a wave is inversely proportional to the velocity of the particle. In terms of this matter wave, the discrepancy between the mechanical principle and Fermat's principle at once vanishes.

Since the rays are nothing but the wave-normals, the equation of a wave surface can be directly deduced from Fermat's principle as laid down in equation (1) above.

From (1), it is evident that $\int_P^Q \frac{ds}{v}$ is a constant, whose actual value depends on the co-ordinates $x_1y_1z_1$ and $x_2y_2z_2$ of P and Q respectively. We may write this value as $V(x_1y_1z_1, x_2y_2z_2)$. The value of $\int_P^Q \frac{ds}{v}$ may also be determined by the time interval from P to Q,

i.e., $\int_P^Q \frac{ds}{v} = \int_P^Q dt = t_2 - t_1$.

Hence, $V(x_1y_1z_1, x_2y_2z_2) = t_2 - t_1$

From this purely geometrical deduction we can now easily pass on to the wave theory. Let P be a fixed point which is chosen as the source of light and as the origin of the co-ordinate system. Then x_1, y_1 and z_1 vanish. Now replacing x_2, y_2, z_2 by x, y, z and putting $t_1 = 0$ and $t_2 = t$, we have,

$$V(xyz) = t \quad \ldots \quad \ldots \quad (2)$$

Equation (2) represents the locus of the terminating points of the rays starting from the source at P and satisfies Fermat's principle. At the same time this equation may as well represent a wave surface at an instant t. Thus the wave theory and the geometrical optics are found to be blended together in the optical principle of Fermat.

Questions (Chapter I)

1. Write a short note on the theories of light. (C. U. 1950)

2. Compare and contrast the explanation of refraction on the corpuscular theory and on the wave theory.

3. State Huygens' principle and show how it can be used to explain the laws of reflection of light. (Bom. U. 1943 ; C. U. 1947)

4. Explain Huygens' principle and deduce the laws of refraction from the wave theory. What is the significance of refractive index according to the wave theory ? (C. U. 1957 ; *cf* C. U. 1955)

5. Clearly explain Huygens' principle and deduce the laws of refraction from it. What is meant by refractive index and how is it related to the velocities of light in different media ? (Agra U. 1955 ; Banaras U. 1946 ; Utk. U. 1949)

6. Derive a formula for refraction at a spherical surface according to the wave theory of light. (P. U. Suppl. 1952)

9. What is the velocity of light in vacuum when that in air is 3×10^{10} cm./sec. and the absolute refractive index of air $= 1{\cdot}00029$? (C. U. 1944)
[*Ans.* $3{\cdot}00087 \times 10^{10}$ cm./sec.]

8. State the fundamental postulates of the undulatory theory of light. Account for the formation of images by refraction through lenses on the wave theory, and hence prove the relation, (Agra U. 1931, '42)

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

9. Derive the lens formula for a thin lens on the wave theory of light. (P. U. 1950 ; Agra U. 1945)

10. Derive the refraction formula for a thin lens on the wave theory. (P. U. 1950)

11. Deduce the laws of refraction from the wave theory of light and explain the meaning of the refractive index according to the theory. (Luck. U. 1946)

12. State the principle of wave propagation. On its basis prove the laws of refraction and explain total internal reflection. (Raj. U. 1953)

13. Apply the principle of wave theory to account for the formation of a real image by a convex lens and with the help of your explanation deduce the formula for the conjugate foci. (P. U. 1946)

14. Derive a formula for reflection from a spherical mirror using the wave theory of light.

15. Apply the principle of Huygens to establish the formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

CHAPTER II

INTERFERENCE : INTERFEROMETERS

16. Introduction :—Interference of light was observed by Newton as early as 1676 in the phenomena of colours of thin films and Newton's rings. Newton, however, could not explain the above phenomena of light adequately with his corpuscular theory. Grimaldi, a contemporary of Newton, in course of his researches to investigate the true nature of light observed that two pencils of light rays when superposed tended to extinguish each other in cartain cases. He allowed sunlight to fall on a screen in a dark room coming from two small holes. Corresponding to each hole, a circular spot surrounded by faint rings was found. At a particular position of the screen the spots partially overlapped. The intensity in the overlapping region was found to be less than in other portions. This apparent lowering of intensity is due to the superposition of two individual diffraction patterns due to the circular apertures. So it was not a true case of interference as will be seen later on.

17. Young's Experiment—Thomas Young (1773—1829), an English scientist, conclusively established for the first time, the wave nature of light by his classical experiment described below. A fine pencil of light rays is admitted through a narrow slit S'. The light coming out of the first slit is then divided into two parts by passing it through two other slits S_1 and S_2. Finally, the two pencils are received on a screen AB, which is, therefore, simultaneously illuminated by the light coming from both S_1 and S_2.

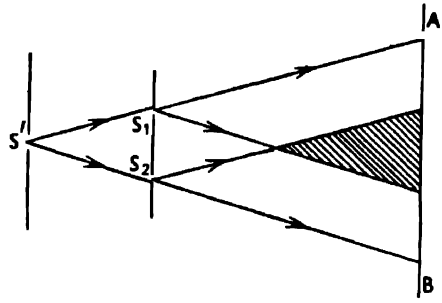


Fig. 11

In the shaded region (Fig. 11) the light from the two sources S_1 and S_2 overlap. Young observed a pattern of alternate dark and bright bands in this overlapping region. Young also observed some characteristics of the dark and bright bands so obtained. (1) As the separation between the two slits was gradually diminished the spacing between the bands increased and as the separation between S_1 and S_2 increased, the bands narrowed down and ultimately disappeared. (2) A uniform illumination on the screen was obtained if either of the slits S_1 or S_2 was closed. (3) If light

were allowed to enter directly through S_1 and S_2 and the slit S' is removed the bands disappeared resulting in a general illumination on the screen.

From his observations Young concluded that the above phenomena is due to the *interference* of two light waves. He obtained a physical explanation of the above phenomenon from the *principle of superposition* of two waves.

18. Superposition of waves.—Two or more sets of waves may be made to cross one another without any one producing any change in the other. At the region of crossing they act simultaneously and interfere with one another. The interference is the result of the superposition of waves in that region and the resultant displacement in the region will be the algebraic sum of the individual instantaneous displacements which would have been produce if each of the wave-trains existed alone.

Let us consider the case of superposition of two wave-trains of equal amplitude and wavelength, but differing in phase, moving in a homogenous medium. The instantaneous displacements of the wave-trains at the region of superposition may be written as,

$$y_1 = a \sin \frac{2\pi}{\lambda}(ct - x).$$

$$y_2 = a \sin \frac{2\pi}{\lambda}[ct - (x + \delta)].$$

where δ is the path difference between the two superposing waves.

The resultant displacement in that region will be the algebraic sum of the individual displacements and will be given by,

$$\begin{aligned} y &= y_1 + y_2 \\ &= a \sin \frac{2\pi}{\lambda}(ct - x) + a \sin \frac{2\pi}{\lambda}[ct - (x + \delta)] \\ &= 2a \cos \frac{\pi\delta}{\lambda} \cdot \sin \frac{2\pi}{\lambda}\left[ct - \left(x + \frac{\delta}{2}\right)\right]. \end{aligned}$$

Therefore, the resultant vibration is also sinusoidal in nature having the same period, but a different amplitude. The amplitude is given by $2a \cos \frac{\pi\delta}{\lambda}$. This shows that the amplitude varies with δ. For $\delta = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, (2n+1)\frac{\lambda}{2}$, $2a \cos \frac{\pi\delta}{\lambda} = 0$. Hence the resultant amplitude is zero for such values of δ. Again for $\delta = 0, \lambda, 2\lambda, 3\lambda, \ldots, n\lambda$, $2a \cos \frac{\pi\delta}{\lambda}$ is maximum, and is equal to $2a$. Thus,

the above result may be interpreted in the following way. When two continuous wave-trains of the same wavelength and amplitude are superimposed the resultant effect on the medium is that, at certain points for which the path differences of the two interfering waves are $\frac{\lambda}{2}$, $\frac{3\lambda}{2}$.....etc. the medium remains stationary. The effect that is produced at these points is called **destructive interference.** While at other points, for which $\delta = 0, \lambda, 2\lambda$, etc., the medium is violently disturbed producing what is known as the **constructive interference** or **reinforcement.** Let a monochromatic light of wavelength λ from two sources S_1 and S_2 superpose on a particle of the medium at P (Fig. 12).

The disturbance at the point P will be dependent on two factors (*i*) the path difference of the point P from the sources S_1 and S_2, and (*ii*) the phase difference of the two waves at the time of leaving the respective sources. If the two sources are derived from the same source in such a manner that the phase difference at the start remains constant for all times, *i. e.* if there is any change in phase of the one there will be an identical phase change in the other, then the disturbance at the point P will depend on the first factor alone, *i. e.* the path difference. The different values of path difference will determine whether the point P will be violently disturbed or will remain undisturbed. Thus a given point for which the path difference from the sources is constant will remain either permanently bright or dark. Since the resultant amplitude is $2a \cos \frac{\pi\delta}{\lambda}$, the intensity will be given by $4a^2\cos^2 \frac{\pi\delta}{\lambda}$. So the intensity distribution in the superposed pattern will follow the *cosine square law*.

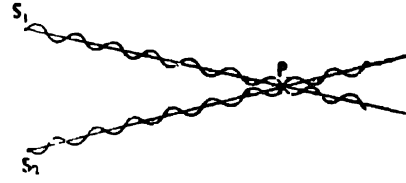


Fig. 12

19. Conditions for sustained Interference :—In order to obtain a phenomenon of continuous or sustained interference of light following conditions are to be satisfied.

(*i*) The two sources should continuously emit waves of the same wavelength.

(*ii*) The phase relation between the two sets of wave coming from the two sources must remain the same for all times, *i.e.* if there is any change of phase in one wave, there must be an identical change of phase in the other. In other words, the sources of light waves must be **coherent.**

(*iii*) The amplitudes of the interfering waves must be equal or very nearly so.

(*iv*) In order to obtain an observable interference pattern the distance between the two coherent sources must be as small as possible.

(*v*) The two light waves must be in the same state of polarisation.

(*vi*) For complete destructive interference so that the resultant amplitude and hence the intensity may be zero, the phase difference between the two waves from the two sources must be π, or path difference $\frac{\lambda}{2}$, $\frac{3\lambda}{2}$, etc. For complete reinforcement, the phase difference between the two waves from the two sources must be 0 or 2π, or the path difference 0, λ, 2λ.....etc.

Light waves from two separate candles or from two separate portions of the same candle cannot produce any sustained interference phenomenon on meeting at a point. This is because an agitation or vibration of a particle from a source of light which is responsible for the production of the light waves is subject to abrupt change of phase many times per second due to its collision with other particles. Since, in a source, there are thousands of luminous particles vibrating in all possible phases any constant phase relation between any two particles is impossible. The phase of each particle so rapidly changes with time that as a result there is a rapid alternation of brightness and darkness at any point and as such the eye is unable to recognise the interference phenomenon that may be produced by two incoherent sources of light. Hence we get a general illumination from light coming from two different candles.

By interference of light we get alternate darkness and brightness. In the regions of darkness, the intensity of light falls to zero. Hence light energy, which is proportional to the intensity, also appears to have been completely annihilated. But the complete annihilation of energy goes against the principle of conservation of energy. This apparent contradiction disappears if we consider the intensity and energy in the region of brightness. In the region of the bright fringes the amplitude of the resultant disturbance is $2a$. Hence the intensity is proportional to $4a^2$. Each of the original waves is of intensity proportional to the square of its amplitude, *i.e.* proportional to a^2. So when they superpose at a point the expected intensity (and hence energy) should be proportional to $2a^2$. But in the dark regions this $2a^2$ is absent whereas in the bright regions this intensity (or energy) becomes proportional to $4a^2$. So it is obvious that there takes place a redistribution of energy, the total energy of the dark regions being transferred on to the bright regions making the intensity there double that of the expected value. Thus *the interference phenomenon does not involve any destruction of light energy.*

20. Corresponding points.—We have already said that interference cannot occur between any two points of an extended source. But we can have continuous or sustained interference between the waves produced by a vibrating source or its image, or two images of the same source. For in that case, if there is any abrupt change in phase in one source, there will be a corresponding change of phase in its image, and as such the phase difference will remain constant. In practice, we take a narrow slit backed by a luminous source and its image as the interfering sources or two images of the same slit acting as the two sources producing interference. In such cases we have a large number of luminous points in the slit, and continuous interference will occur only between such points in the slit and their virtual images which produce coherent waves. Such points on the source and its images are called **corresponding points.**

21. Methods of obtaining Interference Effects.—All interference effects can be broadly classified in two groups :

(1) Methods involving *division of wavefront*, *i. e.* methods requiring a point or line source, from which light waves spread out as spherical or cylindrical waves. Any two points on such a spherical or cylindrical wavefront are taken as the interfering sources, which are evidently coherent. Fresnel's biprism and bimirror, Lloyd's mirror, Billet's split lens, Rayleigh interferometer, diffraction gratings, etc. are examples of this class.

(2) Methods involving *division of amplitude*, *i.e.* methods requiring an extended source. Here the interference takes place between a ray and its reflected or refracted component or between two reflected or refracted components of the same ray. Some of the examples of this class are the interference effects in thin films, Jamin interferometer, Michelson's interferometer, Fabry Perot interferometer, Lummer-Gehrcke plate and Newton's rings.

We shall first of all discuss a few cases of interference phenomena belonging to the first group (*i.e.* the method of division of wavefront) and then deal with the different types of interference patterns that are formed as a result of the division of amplitude.

22. Explanation and Theory of Young's Experiment :—In Young's experiment, as we have already described, if a monocromatic beam illuminates the narrow slit and if we consider a section of this slit in the plane of the paper it will practically be a point at S (Fig. 13). From this point spherical waves originate and finally fall on the two narrow adjacent slits, whose sections in the plane of the paper are the points S_1 and S_2. Evidently S_1 and S_2, lie on the same wavefront. S_1 and S_2, according to Huygens' principle, act as secondary sources and emit fresh spherical wavelets. So the phenomenon that will be observed in the case occur as a result of division of wavefront. In the space beyond S_1 and S_2 the

waves becomes superposed as shown in Fig. 13. Two sets of wavefront starting from S_1 and S_2 are shown in the figure. The thick-line arcs represent the wavefronts at distances $\lambda/2$, $3\lambda/2$, $5\lambda/2$ etc. from the sources whereas the discontinuous arcs represent the wavefronts at distances λ, 2λ, 3λ etc. from the sources. At the positions where two troughs or two crests intersect, the waves evidently meet in the same phase and there will be reinforcement. These points

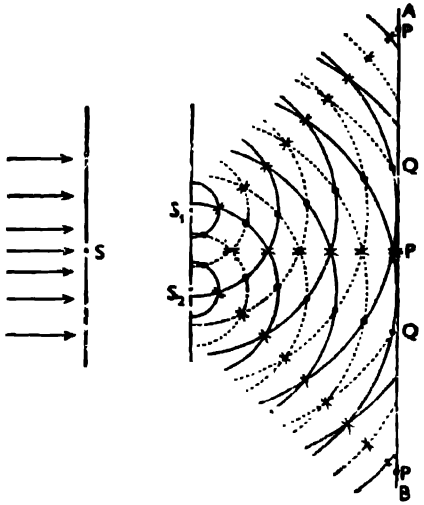


Fig. 13

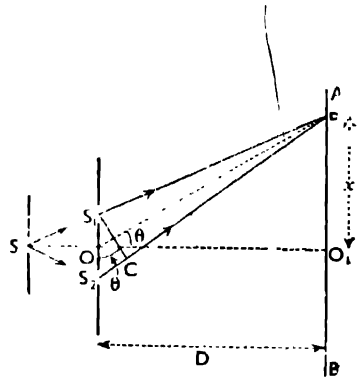


Fig. 14

of reinforcement are designated by cross (×) marks in the figure. Again at points shown by the black dots in the figure, the two waves meet in the opposite phases and as such there will be complete darkness (*i.e.* destructive interference) at those points. So if *A* screen is placed at the position *AB*, bright fringes will be obtained at the points *PPP* and dark fringes will be formed at *QQ*. Though shown on a magnified scale, it must be remembered that the actual fringes are formed only near the central region of the screen and not at regions much away from S_1 and S_2.

To find out whether as a point *P* on the screen, there will be formation of a dark or a bright fringe, let us consider Fig 14. Let *D* be the distance of the screen from the plane containing S_1 and S_2. Let the distance between S_1 and S_2 be *a*. Let *P* be at a distance x from O_1 the corresponding point of *O* the screen. Evidently two wave-trains S_1P and S_2P will produce interference at *P*. The intensity at *P* will depend on the path difference between these wave-trains which is given by

$$\delta = S_2P - S_1P \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

$\therefore$ $\delta = S_2C = S_1S_2 \sin\theta = a \sin\theta$ when θ is the angle S_2S_1C.

Remembering that P is a point close to O_1, this angle θ is equal to angle POO_1.

$$\therefore \quad \delta = a \sin POO_1 = a\frac{x}{PO} = a\frac{x}{D} \text{ (approximately).}$$

According to the theory of interference, the resultant amplitude of the waves reaching P is given by $2a \cos\frac{\pi\delta}{\lambda}$ where δ is given by $\frac{ax}{D}$, λ being the wavelength of the light used.

So when $\frac{ax}{D} = n\lambda$, (n being an integer), or when $x = \frac{nD\lambda}{a}$, the amplitude and hence the intensity at P will be maximum and a *bright fringe* will be observed at that position.

Again if $\frac{ax}{D} = (n+\frac{1}{2})\lambda$;

or, $x = (n+\frac{1}{2})\frac{D\lambda}{a}$, the intensity at P will be zero and a dark fringe will appear at that position. So alternate bright and dark fringe will be formed on the screen, depending on the position of the point of observation. These fringes are the interference fringes. Since these fringes are obtained at any position of the screen beyond $S_1 S_2$, they are *non-localised fringes.* The distance between any two consecutive dark fringes or any two consecutive bright fringes are always equal. Hence the *interference fringes are equi-spaced fringes.* This will be evident from the following simple calculations.

If x_n and x_{n+1} be the positions of the nth and $(n+1)$th bright fringes, then the distance between them is,

$$x_{n+1} - x_n = (n+1)\frac{\lambda D}{a} - \frac{n\lambda D}{a} = \frac{\lambda D}{a}.$$

Hence for a monochromatic radiation this distance is constant. For dark fringes also the same statement holds good. If a composite beam of light (*e.g.* white light) is used, the fringe widths will be varying with λ.

We know that the condition of formation of bright fringes is given by the relation $S_2P - S_1P = n\lambda$ and that of dark fringes by the relation $S_2P - S_1P = (n+\frac{1}{2})\lambda$. If n has a definite value, $S_2P - S_1P$ is constant in both the cases. So the loci of P for different values of n will be so many hyperbolas, with S_1 and S_2 as their foci. If these hyperbolas be rotated about the line S_1S_2, we get hyperboloids of revolution. *These hyperboloidal surfaces represent the alternate dark and bright fringes in space.* The sections of these hyperboloids on the screen will be so many hyperbolic fringes.

In the central region practically they will be straight lines. If the slits S_1 and S_2 be of small lengths the corresponding fringes, which will be also short, would appear to be practically straight over a wider region. When slits are long the hyperbolic nature of the fringes becomes prominent even at a short distance from the central region.

23. Billet's Split Lens :—Billet devised a convenient method of studying the interference fringes. Here a convex lens is cut into two halves L_1 and L_2 which are placed close together as shown in Fig. 15. The separating distance between L_1 and L_2 can be adjusted by a micrometer screw. The space between L_1 and L_2 is covered up by a piece of black paper so that no light can pass through it. A slit illuminated by a strong monochromatic source casts two real images at S_1 and S_2. These two images serve as the two interfering sources of light. In the overlapping region PQ of the screen AB interference fringes will be formed.

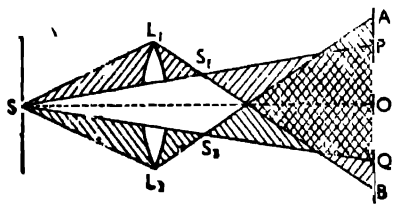


Fig. 15

24. Fresnel's Bi-mirror :—Here two mirrors M_1 and M_2 silvered on the front surface are inclined at an angle very nearly 180°. The arrangement is shown in Fig. 16 Light from a narrow slit S_1 strongly illuminated source is received by M_1 and M_2. The reflected rays from M_1 and M_2 appear to come from the virtual images S_1 and S_2 of the slit respectively. So S_1 and S_2 serve as the virtual interfering sources and the reflected rays will produce interference fringes on the screen placed at AB. The fringes will be formed in the region near O_1 where OO_1 is an axis symmetrical with respect to S_1 and S_2.

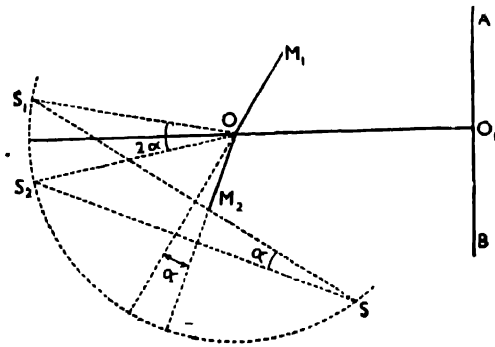


Fig. 16

Since S_1 and S_2 are images of S formed by two inclined mirrors, the points S, S_1 and S_2 lie on a circle with O as the centre. If α be the acute angle between the mirrors, then

$\alpha = \angle S_1SS_2$ from simple geometry. Hence, $\angle S_1OS_2 = 2\alpha$ and $S_1S_2 = 2r\alpha$, where r is the radius OS of the circle.

From the general theory of the formation of fringes as given in the case of Young's experiment (Art. 22) we know that the nth bright fringe is formed at a distance x from O_1 when $x = \frac{n\lambda D}{a}$. Here $a = S_1S_2 = 2r\alpha$ and $D = (r + b)$ where b is the distance OO_1.

$$\therefore \quad x = \frac{(r+b)n\lambda}{2r\alpha}.$$

25. Fresnel's Biprism :—To obtain two coherent sources Fresnel used a biprism ABC (Fig. 17). The angle B of the biprism is highly obtuse, *i.e.* about 178°. Necessarily the refracting angles A and C are very small So the biprism ABC may be considered to be made up of two thin acute-angled prisms with their bases placed together. S is the source of light, usually a fine slit illuminated by a sodium flame. Light from the source is allowed to be divided into two pencils by refraction and these are then allowed to recombine. The two virtual images, S_1 and S_2 of the source S serve as two coherent sources which will produce interference in the overlapping region $BMDENB$. So a screen placed anywhere in this region or an eye-piece placed in this region would receive the

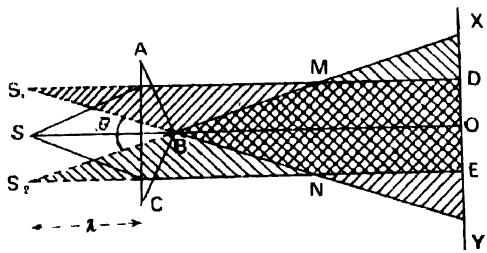


Fig. 17

fringes, which are non-localised. XY is the screen on which DOE is the region where two beams overlap. So interference fringes will be observed in this region. O is the central point which is equidistant from S_1 and S_2 and the waves from them reach O at the same time and hence in the same phase. Therefore, O would be the position of a bright band, known as the *central bright band.* As we proceed both ways from O, we alternately get dark and bright bands due to the change of path difference. When the path difference is $n\lambda$, we get the nth bright band and when the path difference is $(2n+1)\frac{\lambda}{2}$, we get the nth dark band. These fringes

may be conveniently seen with the help of an eye-piece fitted with a cross-wire. The nature of the fringes is shown in Plate II (*b*).

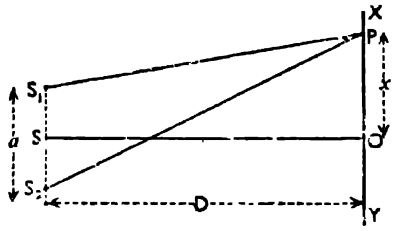


Fig. 18

To find an expression for the width of a fringe let us proceed as follows :

In Fig. 18, where XY represents the screen and S_1, S_2 the two virtual sources, bisect S_1S_2 at S and draw SO perpendicular to S_1S_2 to meet the screen at O. Let $S_1S_2 = a = 2d$ and $SO = D$. It is obvious from the diagram that O is equidistant from S_1, S_2 and as such the central bright band will be situated at O. Next let us consider the effect at P where $PO = x$. Join S_1P and S_2P. The path difference between the rays reaching P from S_1 and S_2 is $S_2P - S_1P$.

Now, $S_2P^2 = D^2 + (x+d)^2$

$S_1P^2 = D^2 + (x-d)^2$

$S_2P^2 - S_1P^2 = (x+d)^2 - (x-d)^2$

$= 4dx = 2ax.$

$\therefore \quad S_2P - S_1P = \dfrac{2ax}{S_2P + S_1P}.$

Now $S_2P + S_1P \approx 2D$

$\therefore \quad S_2P - S_1P = \dfrac{ax}{D}$ (approximately).

(This calculation may also be done by the same method as has been followed in Art. 22.)

If we suppose that the nth bright band is situated at P, then the path difference $S_2P - S_1P$ will be equal to $n\lambda$.

$\therefore$ We get,

$$\frac{ax_n}{D} = n\lambda,$$

where x_n denote the distance of the nth bright band from the centre.

Similarly, for the $(n+1)$th bright band, we have,

$$\frac{ax_{n+1}}{D} = (n+1)\lambda.$$

From these two equations,

$$\frac{a(x_{n+1}-x_n)}{D}=\lambda\ ;$$

$$\text{or,}\quad x_{n+1}-x_n=\frac{D\lambda}{a}=p,\ \text{(say)}.$$

This equation gives the distance between any two consecutive bright bands. This equation also provides us with a convenient and direct method of determining the wavelength of light, $(x_{n+1}-x_n)$, can be easily measured with a micrometer eye-piece fitted with a cross-wire. Therefore if D, and a be known λ can be easily determined.

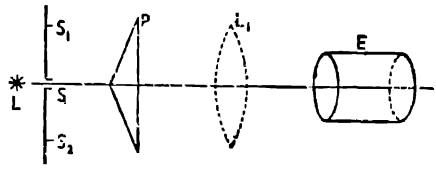


Fig. 19

Experiment :—The experiment can be conducted on a suitable optical bench along the bed of which uprights carrying the slit, the biprism and the eye-piece fitted with a micrometer screw and cross-wire can slide. The slit is illuminated with a monochromatic light (Fig. 19). By adjustments, the slit S, the edge of the biprism P and one of the cross-wires of the eye-piece E are made perfectly vertical and all in the same vertical plane and at the same height from the bench. The fringes are then seen through the eye-piece. The eye-piece is now moved along the bench and the biprism and the cross-wires are adjusted in such a way that there is no relative shift between the cross-wire and a fringe as the eye-piece is brought nearer to or away from the slit.

The cross-wire is then made coincident with a bright band and the micrometer scale reading noted. It is then shifted at right angles to the bench and is made to coincide with another bright band. The reading of the micrometer scale is again noted. From the difference between these two readings and the number of intervening bands crossed, we get the distance between the consecutive bright bands.

D can be easily measured from the difference of the readings of the optical bench corresponding to the eye-piece and the slit. However, the necessary corrections for the index error are to be made.

Next a convex lense L_1 of suitable focal length is placed on another upright between the eye-piece and the biprism. The distance between the slit and the eye-piece should be at least four times the focal length of the lens. There are two positions of the lens for which real images of S_1 and S_2 will be seen in the eye-piece kept at the same place. If a_1 and a_2 be the distances between the real images of S_1 and S_2 for the two positions of the lens, then $a=\sqrt{a_1a_2}$.

Thus D, a and p, the distance between any two consecutive bright bands, can be easily determined. From these values, the wavelenght λ of the light used can be calculated.

The distance between the two virtual sources S_1 and S_2 can also be measured from a knowledge of the refracting angle α of the biprism. Since each of the constituent prisms of the biprism is thin, the deviation produced in a ray is given by, $\delta = (\mu - 1)\alpha$, where μ is the refractive index of the prism material. The two virtual images of S are deviated by double this amount. As shown in Fig. 17, $\angle S_1 B S_2 = \theta = 2\delta$.

$$\therefore \quad \theta = 2(\mu - 1)\alpha.$$

Now since $S_1S_2 (= a)$ is small, $\frac{a}{l} = \theta = 2(\mu - 1)\alpha$, where l is the distance between the slit and the biprism.

or, $a = 2l(\mu - 1)\alpha$

Hence from this relation knowing l from the optical bench and μ and α from auxiliary spectrometric measurements, a can be found out.

White Light Fringes :—We have already seen that the distance of the nth bright fringe from the centre is given by,

$$x_n = \frac{Dn\lambda}{a}.$$

For the central band, $x_n = 0$, for all the wavelengths of white light. Thus the central band will be white for white light. At some position off from the centre, where the condition of darkness is satisfied for a particular λ, the condition of brightness may be simultaneously satisfied for some other λ. So with white light the central region will be white and on either side of it we shall get coloured bands over a comparatively small region beyond which there will be a general illumination due to overlapping of the differently coloured components.

Examples

1. Two straight narrow slits 0·3 mm. apart are illuminated by a monochromatic source of wavelength $5{\cdot}9 \times 10^{-5}$ cm. Fringes are obtained at a distance of 30 cm. from the slit. Find the width of the fringes.

Ans. Here a (the distance between S_1 and S_2) = 0·3 mm. = 0·03 cm. ;

$\lambda = 5{\cdot}9 \times 10^{-5}$ cm. ;

$D = 30$ cm.

$$\therefore \text{ Width of the fringes} = \frac{D\lambda}{a} = \frac{30 \times 5{\cdot}9 \times 10^{-5}}{0{\cdot}03} = 0{\cdot}059 \text{ cm.}$$

2. Interference fringes are produced by a Fresnel's biprism in the focal plane of a reading microscope which is 100 cm. from the slit. A lens placed between the biprism and the microscope gives two images of the slit in two positions. If the distance between the two virtual slits be 4·05 mm. and 2·90 mm. in the two

positions, calculate the width of the interference bands formed with sodium light of wavelength 5893 Å. U

Ans. From the given data, we have,

$a_1 = 4{\cdot}05$ mm. $= 0{\cdot}405$ cm. ; $a_2 = 2{\cdot}90$ mm. $= {\cdot}290$ cm.

$\therefore\ a = \sqrt{a_1 a_2} = \sqrt{{\cdot}405 \times {\cdot}290} = {\cdot}3428$ cm.

$\lambda = 5893$ Å $U = 5893 \times 10^{-8}$ cm. ; $D = 100$ cm.

$$\text{Fringe width} = \frac{D\lambda}{a} = \frac{100 \times 5893 \times 10^{-8}}{{\cdot}3428} = 0{\cdot}0172 \text{ cm.}$$

3. In an experiment with Fresnel's biprism sodium light is used and bands 0·0196 cm. in width are observed at a distance of 100 cm. from the slit. A convex lens is then put between the observer and the prism so as to give an image of the sources at a distance of 100 cms. from the slit The distance apart of the images is found to be 0·70 cm. the lens being 30 cm. from the slit. Calculate the wavelength of sodium light. *(P. U. 1951)*

Ans. Here the images of virtual sources are formed by a lens placed at a distance of 30 cm. from the slit and the image is seen at 100 cm. from the slit.

$\therefore\ u = 30$ cm and $v = (100 - 30) = 70$ cm.

$$\therefore\ \text{Magnification} = \frac{70}{30} = \frac{7}{3} = \frac{\text{size of the image}}{\text{size of the object}}.$$

The size of the image is 0·70 cm. (given).

$\therefore$ The actual distance between the virtual sources (*i.e.* size of the object) $= 0{\cdot}70 \times \frac{3}{7} = 0{\cdot}30$ cm. $= a$.

$D = 100$ cm. and band width $= 0{\cdot}0196$ cm.

$$0{\cdot}0196 = \frac{100 \times \lambda}{0{\cdot}30}\ ;\quad \text{or,}\quad \lambda = \frac{0{\cdot}0196 \times 0{\cdot}30}{100} = 5880 \times 10^{-8} \text{ cm.}$$

4. The inclined faces of a biprism of refractive index 1·50 makes an angle 2° with the base. A slit illuminated by a monochromatic light is placed at a distance of 10 cm. from the biprism. If the distance between the two dark fringes observed at a distance of one metre from the biprism is 0·18 mm. find the wavelength of the light used. *(P. U. 1949)*

Ans. We know, $a = 2l(\mu - 1)\alpha$. Here $l = 10$ cm., $\mu = 1{\cdot}50$.

$\alpha = 2° = \frac{\pi}{90}$ radians and $D = 100 + 10 = 110$ cm.

$$\text{Band width} = \frac{D\lambda}{a} = 0{\cdot}018 \text{ cm. (given).}$$

$$\therefore\ \lambda = \frac{0{\cdot}018 \times 2 \times 10 \times 0{\cdot}5 \times \pi}{110 \times 90} = 5714 \times 10^{-8} \text{ cm.}$$

5. Parallel white light is incident on two very narrow parallel slits which are 1 mm. apart. A lens of 1 metre focal length is used to focus the interference fringes on a screen. If a small hole is made in this screen on one side of the central white fringe at a distance of 3 mm. from it, and the light is examined by a spectroscope, what wavelengths between 4000 Å and 8000 Å will be missing ? *(C.U.H. 1952)*

Ans. The missing λ's evidently here refer to dark bands, the condition for which is (by Art. 22), $x = (n + \frac{1}{2})\frac{D\lambda}{a}$, when x is the position of the band with reference to the central white fringe, a = separation of slits, D = distance of screen, n = order number,

Here $x = {\cdot}3$ cm., $D = 100$ cm., $a = {\cdot}1$ cm.

$$\lambda = \frac{2 \times {\cdot}3 \times {\cdot}1}{100 \times (2n + 1)} = \frac{60{\cdot}000}{(2n+1)} \times 10^{-8} \text{ cm.} = \frac{60{,}000}{(2n+1)} \text{ Å}$$

when $n=0$, 1, 2, 3, 8 and higher integers, the corresponding λ's fall beyond the given range. But,

when $n=4$, $\lambda = 60{,}000/\ 9 = 6666{\cdot}6A$;

when $n=5$, $\lambda = 60{,}000/11 = 5454{\cdot}5A$;

when $n=6$, $\lambda = 60{,}000/13 = 4615{\cdot}4A$; and

when $n=7$, $\lambda = 60{,}000/15 = 4000A$.

So missing wavelength are, $4000A$, $4615{\cdot}4A$, $5454{\cdot}5A$ and $6666{\cdot}6A$, whithin the given range.

26. Lloyd's single Mirror :—This simple and convenient method of producing interference fringes was first devised by Dr. Lloyd in 1834. In this case interference fringes are formed by two pencils of light, one proceeding directly from the source and the other being reflected from a mirror MM.

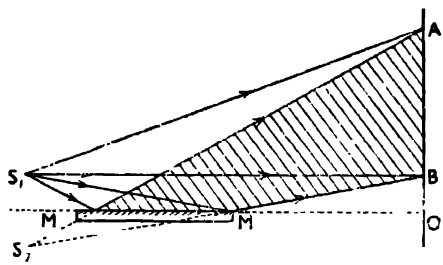


Fig. 20

Rays from a narrow slit, parallel to a glass plate blackened at the back, are incident on the latter at a grazing angle. Here, the source S_1 and its virtual image S_2 act as two coherent sources. The reflected light (*i.e.*, the light coming from the virtual image S_2) meets the direct light and produce interference fringes on the screen ABO (Fig. 20). The interference between the two wave-trains occurs in the region of overlapping as is shown by the shaded region in the figure.

The distribution of intensity and the phase relationship are quite simple, as in the case of biprism. With white light, there being no dispersion of the virtual sources obtained on reflection, the separation $2d$ between the two interfering sources will be same for all wavelengths. The fringes can be received on a screen or can be observed by an eye-piece. By setting up the mirror on an optical bench, and by measuring the separation between any two bright bands the wavelength of a monochromatic light can be determined with the help of the formula,

$$\lambda = \frac{ap}{D}$$ where p is the band width.

With monochromatic light, when the screen is placed in contact with the end of the mirror, the first fringe close to the mirror is dark, instead of bright as we would expect. As a result, in this case we obtain a minimum at a place where we expected a maximum from our normal considerations. This means that one of the two beams has undergone a phase change of π on reflection. Since the direct beam cannot change its phase, it is obvious that the reflected one has undergone the phase change at reflection. If the

slit is backed by white light, and if a thin film of mica is introduced in the path of the direct light, the central fringe will be shifted upward. In this case, the central **dark fringe** with coloured fringes on either side of it will be clearly visible.

27. Displacement of Fringes due to Interposition of Thin films :— Let S_1 and S_2 be two coherent monochromatic sources which are producing interference fringes on the screen PP' (Fig. 21). P is the position of the central bright band. If a thin strip of mica, say, of uniform thickness t is introduced in the path of one of the light rays, the path of that ray is retarded as light travels slower in that medium. Hence the point, at which the two rays arrive in the same time, *i.e.* the position of the central fringe will be displaced. Let P be the position of the central fringe when the mica strip is not introduced. After the introduction of the mica strip of thickness t and of refractive index μ, the central fringe is displaced to P'. Let C and C' be the velocities of light in air and in the medium respectively. Hence the time taken by light to travel the path S_1P' will be equal to the time taken by light in travelling over S_2P'.

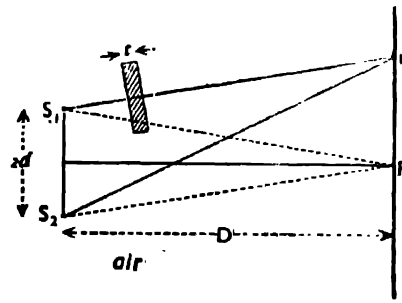


Fig. 21

Therefore, $\frac{S_2P'}{C} = \frac{S_1P'-t}{C} + \frac{t}{C'}$;

or, $S_2P' = S_1P' - t + \frac{C}{C'} \cdot t$; or, $S_2P' = S_1P' - t + \mu t$;

or, $S_2P' - S_1P' = (\mu - 1)t$ = path difference between the rays.

If the displacement PP' of the central fringe be equal to x, we have, path difference $= \frac{a.x}{D}$, where $a = S_1S_2$, and D = distance of the screen from the sources.

$$\therefore \quad \frac{ax}{D} = (\mu - 1)t.$$

Thus the central fringe is shifted through a distance x given by,

$$x = (\mu - 1)t.\frac{D}{a} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (1)$$

The distance p, between any two consecutive bright bands is given by,

$$p = \frac{D}{a}\lambda.$$

$$\therefore \quad \frac{D}{a} = \frac{p}{\lambda}.$$

Therefore, the shift of the central bright fringe can be expressed as,

$$x = (\mu - 1)t \frac{D}{\lambda}.$$

Therefore, by measuring x, the thickness t of the film can be determined, if μ, the refractive index of the film is known. Also, μ can be determined by knowing the thickness and the fringe displacement. This method was applied by Fresnel and Arago to study the variation of refractive index of glass due to a small rise in temperature. A very high order of accuracy in the measurement of μ or t is claimed in this method.

28. Change of Phase due to Reflection (Stokes' treatment) :— The change of phase suffered by a ray due to reflection from the surface of a denser medium, as has been observed in Lloyd's mirror experiment wherein in the expected positions of maxima we get distinct minima, has been explained in a simple way by Stokes.

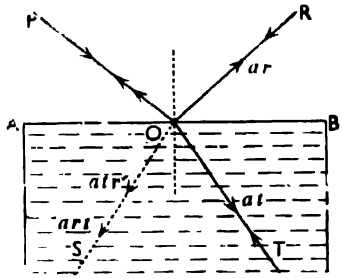


Fig. 22

Let AB represent the separating surface of two media of which the one below AB (Fig. 22) is denser. A ray of light PO is incident on AB. Suppose a to be the amplitude of the incident light wave. This light is partly reflected along OR and partly transmitted along OT in the denser medium. If r and t be the reflection and transmission coefficients respectively, the amplitude of OR is $a.r$ and that of OT is $a.t$. Supposing there is no absorption in the medium, we have, $r + t = 1$. Now if the reflected and the transmitted rays are reversed, they should recombine along OP to yield the original amplitude a.

If OR is reversed, the part of amplitude reflected along OP is $ar.r = a.r^2$, while that transmitted along OS is $a.r.t$. Again if OT is reversed, the part reflected along OS is atr' and that transmitted along OP is $at.t'$ where r' is the reflection coefficient in the denser medium and t' the transmission coefficient in the rarer medium. Since the two amplitudes along OP combine together to give the original amplitude a, we have $att' + ar^2 = a$.

So the total amplitude along OS is zero ; or, $art + atr' = 0$.

$$\therefore \quad tt' + r^2 = 1 \text{ and } r + r' = 0.$$

Hence $tt' = (1 - r^2)$, and $r' = -r$.

The last relation is significant because here the negative sign indicates that one of the rays has a positive displacment while the other has a negative displacement. So the two rays, one reflected from a denser medium on to a rarer medium, and the other reflected from a rarer medium on to a denser medium, differ in phase by π. Actually a phase change of π has been noticed in the former case (reflection from a denser on to a rarer medium) in Lloyd's mirror experiment, Newton's rings etc. But no such phase reversal is found to occur in the latter case of reflection from a rarer on to a denser medium.

29. Achromatic Fringes.—When a source of white light is used the interference fringes produced are found to be coloured and to extend over a small region near the central bright band. At the central point of the fringe system, as all the differently coloured components meet in the same phase, the resulting band will be white. But as the width of a fringe depends on the wavelength of the light, at all other positions the width of a fringe is different for different colours. So the dark or bright fringes due to the different component wavelengths of white light will be partially overlapped by the simultaneous presence of fringes of some other wavelengths at the same position. The coloured fringes will thus be tinged with other colours too. Even these coloured fringes will be only a few in number on both sides of the central fringe. This is so becaue at a certain distance from the central fringe, all the wavelengths will overlap in such a way as to produce a resultant impression of white light there to a naked eye. But if the fringe width can be made identical for all colours, achromatic fringes may be obtained even with white light. We know that the fringe width is given by $\frac{D\lambda}{a}$ where D is the distance between

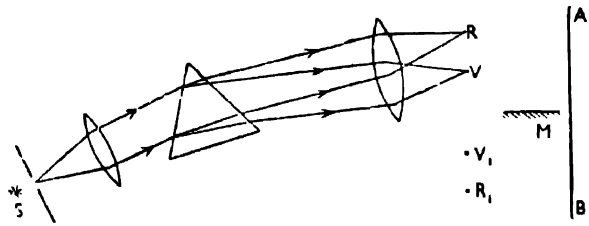


Fig. 23

the sources and the screen and a is the distance between the interfering sources. So for a particular value of D, if $\frac{\lambda}{a}$ is made constant for all colours, the band width becomes the same for all

colours and the bands obtained are achromatic. The arrangement for getting such bands is shown in Fig. 23 wherein a pure spectrum is obtained either by a prism spectroscope or a grating spectroscope. R and V represent the red and violet bands of the spectrum. A Lloyd's mirror M is so placed in front of RV that the corresponding images R_1 and V_1 in the mirror satisfy the relation, $RR_1/VV_1 = \lambda_r/\lambda_v$, where λ_r and λ_v refer to the wavelengths of red and violet light respectively. Hence $\frac{\lambda_r}{a_r} = \frac{\lambda_v}{a_v}$ or, $\frac{\lambda}{a}$ is constant in this case. So the fringes are of equal widths. This is approximately true also in the cases of the intermediate colours. In a grating spectrum the dispersion being accurately proportional to the wavelength, the condition of perfect equality of width is attained for all colours whereas for a prism spectrum it is only approximately attained. Once the above condition is satisfied in a Lloyd's mirror, the fringes obtained will be achromatic due to perfect superposition. Here alternate dark and white fringes will be obtained on a screen at AB. They can be conveniently observed also by means of an eyepiece placed in front of M.

Example

Fringes are produced by monochromatic light of wavelength $5{\cdot}45 \times 10^{-5}$ cm. A thin plate of glass of refractive index 1·50 is then placed normally in the path of one of the interfering beams and the central bright band of the fringe system is found to move into the position previously occupied by the third bright band from the centre. Calculate the thickness of the glass plate. *(P. U. 1944)*

Ans. We know that the shift of the central bright fringe is given by, $x = (\mu - 1)\, t\, \frac{p}{\lambda}$, where p is the fringe width.

Here, $x = 3p$; $\mu = 1{\cdot}50$; $\lambda = 5{\cdot}45 \times 10^{-5}$ cm.

Here, $3p = (1{\cdot}50 - 1)\, t\, \frac{p}{5{\cdot}45 \times 10^{-5}}$;

or, $t = \frac{3 \times 5{\cdot}45 \times 10^{-5}}{0{\cdot}50} = 3{\cdot}27 \times 10^{-4}$ cm.

30. Interference Involving Multiple Reflections : Colours of Thin Films :—We shall now study a few cases of interference belonging to the second group, *i.e.* the interference by the division of amplitude.

The most common practical example of this type of interference is found in the brilliant colours seen when a drop of oil is spread on the surface of water. In white light colours are exhibited in all thin transparent films *e.g.* colours of soap films and of tempered steel, flashing colours of the wings of certain insects, colours of lead skimmings, etc.

Light coming from the same source is reflected from the upper and lower surfaces of the film AB (Fig. 24) many times. Thus we get pencils of light, reflected under different conditions, and all coming from a single ray. These pencils are in a position to interfere and to produce the colour effect. Another set of trasmitted light waves will be obtained which are also in a position to interfere. A multiple beam of light is evidently obtained by successive reflections, but for a thin film only the first few pencils are of importance, as the pencils suffering a large number of reflections or transmissions are of low intensities rapidly diminishing to zero. Fig. 24 shows the formation of multiple beam by reflection as well as transmission from a single incident beam SO. Either beam can be collected by a lens or an unaccommodated eye. In both the cases, due to interference, colour effects will be exhibited.

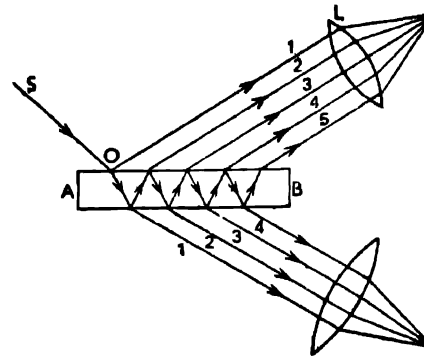


Fig. 24

Let us now calculate the path difference between any two successive reflected rays.

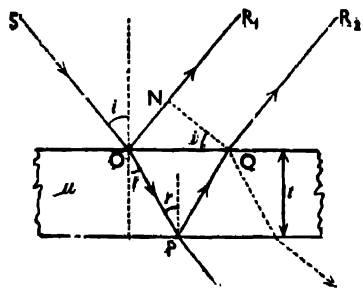


Fig. 25

Consider a thin parallel-sided film of thickness t shown in Fig. 25 and refractive index μ situated in air. Let a single wave-train SO of wavelength λ be incident on the film at O. A part of it will be reflected in the direction OR_1 and the other part will be transmitted in the direction OP and incident on the lower surface of the film. Now the ray OP will again be divided into two parts one reflected in the direction PQ and again transmitted in air along QR_2, and the other part transmitted in the downwards direction. Let us confine our attention to the reflected portion only. The rays OR_1 and QR_2 are derived from the same point of the source, and will have *approximately** same intensity,

* Actually, however, the intensity of the first reflected beam is found to be equal to the sum of the intensities of other parallel beams coming out after successive internal reflections. But these latter beams are of gradually diminishing intensities.

They are in a position to interfere provided they are brought together. Since the film is parallel-sided, OR_1 and QR_2 will be parallel to each other. Drop a perpendicular QN from Q upon OR_1. Then the plane passing through QN and perpendicular to the plane of the paper is a new wavefront. Light waves moving in the direction QR_2 and NR_1 have got an initial phase difference at this plane. This phase difference is evidently due to the difference between the path of light OPQ in the medium of the film and the path OM in air.

Considering the optical paths,* the path difference,

$$\delta = \mu(OPQ) - ON = \mu(OP + PQ) - ON$$
$$= 2\mu.OP - ON, \quad (\text{since } OP = PQ)$$
$$= 2\mu.OP - OQ \sin i$$
$$= \frac{2\mu t}{\cos r} - 2.t \tan r \sin i.$$
$$(\because \quad OP \cos r = t \text{ and } OQ = t \tan r)$$
$$= \frac{2\mu t}{\cos r} - 2t \sin r. \frac{\sin i}{\cos r}$$
$$= 2\mu t \left(\frac{1}{\cos r} - \frac{\sin^2 r}{\cos r}\right), \quad \left(\because \quad \frac{\sin i}{\sin r} = \mu\right)$$
$$= \frac{2\mu t}{\cos r}(1 - \sin^2 r) = 2\mu t \cos r.$$

It is to be remembered that the rays under consideration have undergone reflections under opposite conditions ; the first, while passing from air to the film its reflected from the denser medium of the film and the second while passing from the film to the air is reflected from the rarer medium. Observations suggest that the first ray having external reflection undergoes a phase change of π, and the second ray having internal reflection does not. As such a phase difference of π or a path difference of $\frac{\lambda}{2}$ is introduced. Hence we have path difference,

$$\delta = 2\mu t \cos r \pm \frac{\lambda}{2} ;$$

---

* Optical path is also known as equivalent air path. If t is the time taken by light to traverse a distance x' in the medium of refractive index μ, then in the same time t the distance x, that light would traverse in air, is known as its equivalent air path. Let c and c' be the velocities in air and medium respectively, then we have,

$$x' = c't, \text{ and } x = c.t :$$

or, $\frac{x}{x'} = \frac{c}{c'} = \mu$; or, $x = \mu \times x'$;

or, path in air $= \mu \times$ path in medium.

or, phase difference,

$$\Delta = \frac{2\pi}{\lambda}(2\mu t \cos r) \pm \pi.$$

It is evident that the interference effect may be due to either of the two variables t and r. If one of the factors is constant, the effect is entirely due to the variation of the other factor. For a thin film, if observation is made at a constant value of r, say *almost normally*, then phase difference,

$$\Delta = \frac{2\pi}{\lambda} \cdot 2\mu t \pm \pi$$

depends only on the different values of the thickness t.

Such fringes, for which the value of r is constant and the fringe formation depends upon the thickness only, are known as **fringes or curves of equal thickness**. Here the variation from maximum to minimum intensity in pattern is mainly due to variation of thickness. For a particular fringe (*i.e.* fringe of a particular order) t has constant value. Hence this fringe system has been named as the fringes of equal width or thickness.

Again if the thickness of the film remains constant and fringes are obtained by varying r, each fringes would correspond to a particular value of r or a fixed value of inclination. Such fringes are known as **fringes or curves of equal inclination.**

A bright fringe is determined by the condition,

$$\delta = 2\mu t \cos r \pm \frac{\lambda}{2} = n\lambda ;$$

$$\text{or, } 2\mu t \cos r = (n \pm \tfrac{1}{2})\lambda.$$

Similarly a dark fringe is determined by the condition,

$$\delta = 2\mu t \cos r \pm \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2} ;$$

$$\text{or, } 2\mu t \cos r = n\lambda.$$

If the incident wave-train of light is white, instead of monochromatic, then for a particular thickness of the film, some of the colours will be destroyed, and all other colours will be reinforced in that part of the film. As a result the film will appear coloured. If the wave-trains are parallel and come from a monochromatic source and the film has got a uniform thickness t, t and r are constant for the whole film. As a result the film will either appear completely dark or bright according as the particular values of t, μ, r etc., satisfy the condition of darkness or brightness. If again the thickness t of the film increases or decreases gradually

from one end to the other, then due to this change of thickness the conditions of brightness or darkness are alternately satisfied. So we shall get bright and dark bands with monochromatic light depending on the thickness of the film. For thin films of varying thickness, the colour effect with white light will be magnificient due to the presence of varieties of colour in different parts of the films.

A similar interference phenomenon is seen to occur with the rays transmitted through the film and emerging on the other side of it. In the case of transmitted beams, there are only internal reflections (*i.e.* reflection while passing from denser to rarer medium). As this causes no change in phase, the phase difference of these rays will be purely determined by their path difference $2\mu t \cos r$. So for transmitted system, we have the following conditions of brightness or darkness.

$$2\mu t \cos r = n\lambda \rightarrow \text{for brightness,}$$

$$2\mu t \cos r = (2n+1)\lambda/2 \rightarrow \text{for darkness.}$$

These conditions are just opposite to those obtained with reflected light. As a result the reflected and transmitted lights are complementary to each other. This means that the position which corresponds to the maxima of the reflected system will be the position of minima of a transmitted system and vice versa. The films exhibiting the above colour effects are only a few wavelengths thick and the faces of the film are not strictly parallel. They actually form a wedge of a very small angle of the order of a few minutes. As the film faces are not parallel the coloration or fringes produced are neither formed at infinity. The effect is definitely a localised one as will be evident from the treatment of interference in a wedge shaped film. If the film be very thin, say, of the order of a fraction of a wavelength, no interference effect is exhibited and the film appears to be black.

Though in the above discussions we have only considered the cases where t and r change, in a more general way all the possible variables in the expression $2\mu t \cos r$ are to be taken into account. If r and t remain constant, even then due to the variation of μ with λ, there will be a colour effect with incident white light. We know that if t is kept constant and r changes, the path difference also changes. So in a film different colours will be seen with white light when viewed from different angles. But as in natural films there are variations of thickness in different parts, they form wedges and the *fringes are generally of the constant thickness type when viewed from a particular position.*

Note : The colours of films are seen with the help of an extended source of white light, *e.g.* light reflected from the sky. So apparently

the condition of coherence of the interfering sources appears to have been violated. But if we observed the phenomenon rather more analytically, this apparent anomaly disappears. Let F be a thin oil film on a pool of water seen by light reflected from the sky. The point A of the film is seen by light coming from the point S of the sky, while A_1 is seen by light from S_1 (Fig. 26). In general each point is seen by light coming from a particular point of the sky. Here as the interference takes place by amplitude division, we have point to point coherence. To scan the whole film the eye has to be moved about. But for a particular position of the eye focussed on a given point of the film light comes from a definite point of the sky. So though an extended source is needed to see the whole of the film, the fundamental condition of the interfering waves issuing from a single point of the source is at the same time satisfied.

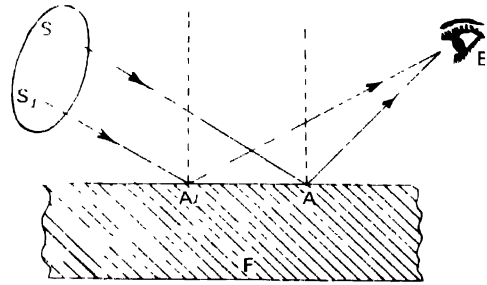


Fig. 26

Example

White light fringes are produced by an air film 10^{-4} cm. thick and the fringes with reflected light are examined by a spectroscope; find the number of dark bands seen in the spectrum between the wavelength range 4000 A to 7000 A, when light is incident at an angle 30°. *(A. U. 1945)*

Ans The condition of dark-band formation is $2\mu t \cos r = n\lambda$ as given in the article above. (Here $\mu = 1$).

So when $\lambda = 4000A$, $n_1 = \dfrac{2\mu t \cos r}{4\times 10^{-5}} = \dfrac{2\times 10^{-3}\times \cdot 8660}{4\times 10^{-5}}$

$$= \frac{86\cdot 6}{2} = 43.$$

Again when $\lambda = 7000A$.

$$n_2 = \frac{2\times 10^{-3}\times \cdot 8660}{7\times 10^{-5}} = 24.$$

Now both the 43*rd* and the 24*th* bands being within the given range, the total number of dark bands in the range is $43 - 23 = 20$.

***31. Interference in a Wedge-shaped film : Location of Fringes :**—We have already stated that in thin films the thickness

varies from point to point. So actually wedges of very small angles are formed in the films. Let ABC represent such a wedge of angle θ as shown in Fig. 27. If a parallel beam of monochromatic light falls on the face AB normally, the components reflected from the upper and the lower surfaces will interfere to produce alternately dark and bright bands. If the angle of the wedge is very small, the beam incident normally to the face AB may very well be considered normal to the face BC too. Neglecting the wedge angle which is very small, the path difference between the two interfering beams referred to above will be $2\mu t + \frac{\lambda}{2}$ at the point P where the thickness of the wedge is t.

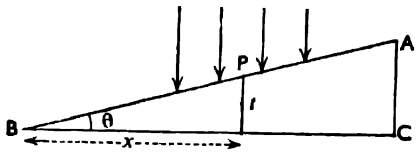


Fig. 27

So at this position of the film there will be a dark fringe when

$$2\mu t + \frac{\lambda}{2} = (n + \tfrac{1}{2})\lambda, \quad \text{or,} \quad (2n+1)\frac{\lambda}{2};$$

$$\text{or,} \quad \text{when } 2\mu t = n\lambda.$$

But from Fig. 27, $\theta = \frac{t}{x}$, or, $t = \theta . x$.

$\therefore$ The nth dark fringe appears at P when $2\mu\theta x = n\lambda$ ... (1)

Similarly at a distance x_1 from B, if $(n+m)$th dark fringe is formed, we have,

$$2\mu\theta x_1 = (n+m)\lambda \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

$$\therefore \quad 2\mu(x_1 - x)\theta = m\lambda \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

Thus if we can count the number of fringes in a space $(x_1 - x)$ with a travelling microscope, we can measure any one of the quantities μ, θ and λ if the remaining two are known.

The fringes formed in a wedge-shaped film are always formed at a short distance from the surface of the film and as such they are localised. But in the case of a parallel faced film since the interfering beams are parallel, the fringes are formed at infinity. To find out the exact location of the fringes in a wedge-shaped film, let us consider a wedge of a small angle θ (Fig. 28), PO is a ray incident at O on the wedge. The externally and the internally reflected component rays OQ and ORO_1Q intersect at Q which is evidently the location where these two rays would interfere. So the fringe system here will be localised and can be seen by a naked eye or a microscope focussed at Q. The fringes formed are straight parallel fringes parallel to the edge of the wedge when they are formed near this edge.

Let i be the angle of incidence of the ray PO and $(i-di)$ be the angle subtended by the O_1Q with the normal to the surface at O_1. The angle between the two interfering rays is evidently di. Let $(r-dr)$ be the angle subtended by RO_1 with the normal at O_1. From the figure this angle is $r-2\theta$. Hence $dr=2\theta$.

From Snell's law,

$$\sin i=\mu \sin r,$$

where μ is the refractive index of the wedge material.

Differentiating we have,

$$\cos i.di=\mu \cos r.dr ;$$

or, $$di=\mu\frac{\cos r}{\cos i}dr$$

$$=2\theta\mu\frac{\cos r}{\cos i}.$$

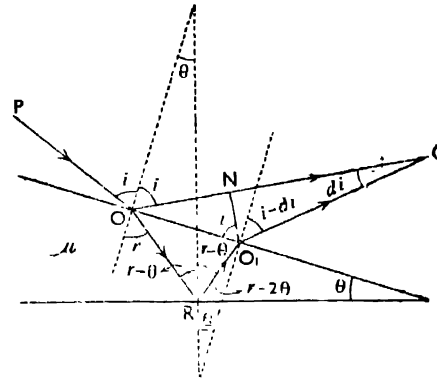


Fig. 28

Since di is a very small angle, $O_1Q.di=O_1N$.

But $O_1N=OO_1\cos i$ and $OO_1=2t \tan r$ (approximately), since θ is very small.

$$O_1Q=\frac{O_1N}{di}=\frac{2t.\tan r\cos i}{dt}=\frac{2t.\tan r.\cos^2 i}{2\mu\theta.\cos r}$$

$$=\frac{t}{\mu\theta}.\frac{\sin r\cos^2 i}{\cos^2 r}.$$

This distance O_1Q gives the location of a fringe formed in a wedge at a position where the thickness of the wedge is t. If the incidence is normal, i and r are both very small. Hence O_1Q (practically the same as OQ) will reduce to $\frac{t}{\mu\theta}.\sin r$ which is a very small quantity. So the fringes will be formed very close to the wedge. If the film be parallel-faced, the wedge angle θ is zero. In that case O_1Q becomes infinity and the fringes are formed at infinity.

Note : If the wedge-angle is not neglected the path difference between the two interfering beams OQ and ORO_1Q will be $2\mu t\cos(r-\theta)$. This is evident from Fig. 28 wherein O and O_1 are close points and t is the mean thickness at the position (in between O and O_1). Hence proceeding exactly in the same way as followed in Art. 30, the path difference between the interfering beams comes out as $2\mu t\cos(r-\theta)$. For normal incidence and for small wedge angle, $\cos(r-\theta)\simeq 1$, and this path difference becomes $2\mu t$.

Example

Two thin glass plates are separated by a thin wedge of air and produce a series of frings with the monochromatic light of $\lambda = 6500$ Å. *How thick is the air film where the 22nd dark fringe is observed?*

Ans. The thickness of air film equal to $\lambda/2$ will produce one dark fringe. So to produce 22 dark fringes, the thickness of air space there must be,

$$22 \times \frac{6500 \times 10^{-8}}{2} = 715 \times 10^{-6} \text{ cm.}$$

32. Fringes of Equal Thickness and those of Equal Inclination :— If a parallel beam of light is incident on a film of varying thickness, the interference fringes are formed due to variation of t only. The fringe of any order follows a path of constant width. Such fringes are known as the *fringes (or curves) of equal thickness.* Newton's ring-fringes, fringes formed in wedge-shaped films, *etc.* are examples of such fringes. These fringes are always *localised.*

When a beam of light from an extended source falls on a parallel-sided film, the varying path difference will be caused by the variation of r, and we shall get concentric rings or curves bulging in or out. Such fringes are known as *fringes of equal inclination* or **Haidinger's fringes.** In order to observe the fringes, the eye must be focussed for infinity. Haidinger fringes may be observed with a piece of mica of uniform thickness, by allowing diffused light to be reflected from the mica foil. The fringes will be circular. The foot of the perpendicular dropped from the eye on the surface of the mica-piece will be the centre of the fringe system.

The nature of the fringes of constant inclination or constant width, may be analytically dealt with in the following simple way.

We know that a fringe is the locus of the points for which the relation, $2\mu t \cos r = n\lambda$, holds good. Differentiating this relation, we have,

$2\mu (\cos r\, dt - t \sin r.\, dr) = \lambda\, dn$ ($\because$ λ and μ are constant for an incident monochromatic radiation).

$$\therefore \cos r\, .\, dt - t \sin r\, dr = \frac{\lambda}{2\mu} dn\ ;$$

$$\text{or, } \frac{dt}{t} - \frac{\sin r\, .\, dr}{\cos r} = \frac{\lambda}{2\mu t \cos r} .dn = \frac{dn}{n} \quad \ldots \quad \ldots \quad (1)$$

In a wedge-shaped film r is kept constant as in a normally incident parallel beam. $\therefore$ $dr = 0$.

Hence from (1) by integration, we get,

$$\log t = \log nk\ ;\ \text{or, } t = nk \text{ (where } k \text{ is a constant).}$$

So t is constant for a particular order of the fringe, which is consequently known as a **fringe of equal width.**

Again in a parallel-faced film, t is constant and hence $dt = 0$. Hence from (1) we get by integration,

$$\log (\cos r) = \log Cn\ ;\ \text{or. } \cos r = Cn, \text{ where } C \text{ is a constant.}$$

So r is constant for a particular order of the fringe which is therefore called a **fringe of equal inclination.**

33. Newton's Rings :—One very good example of fringes of equal thickness is afforded by what is known as the Newton's rings. The rings are observed when a convex lens of large focal length is placed on the plane surface of an optically flat glass plate. A thin film of air is formed between the lens and the glass plate. The point where the lens touches the glass plate, the thickness of the air film is zero. The thickness of the air film increases all round this point as we move away from the centre. The air film thus enclosed is wedge-shaped, and the loci of all points having the same thickness of air are circles. When white light is made to fall normally on such a film, a series of concentric coloured rings are visible all round the point of contact, both in the reflected and in the transmitted rays. If monochromatic light is used, dark and bright rings are obtained.

Experimental arrangement :—Light rays from an extended source *S*, rendered parallel by a lens *L* (Fig. 29) fall on a glass plate *G* inclined at an angle of 45° to the vertical. A portion of light reflected from the glass plate *G* falls on a plano-convex lens *L* placed on an optically flat glass plate *P*. The reflected light is received by a long focus microscope *M* placed vertically above *G*. Newton's rings are observed in the field of view of the microscope.

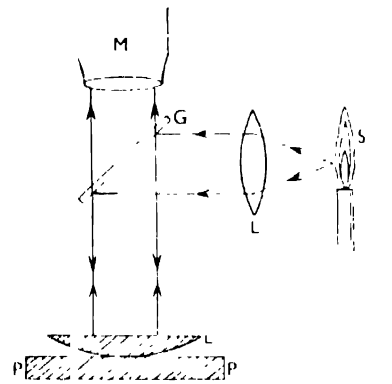


Fig. 29

Let us consider a wave-train *AB* of monochromatic light of wavelength λ (Fig. 30) incident on plano-convex or a double convex lens placed on a plane glass plate *PP*. A part of it is internally reflected at *B* and proceeds towards *BF*. It does not undergo any phase change in this reflection. The other part goes along *BC* and suffers reflection on the glass plate and proceeds along *CDE*. This ray undergoes a phase change of π on reflection. These two wave-trains *BF* and *CDE* are derived from the same wave-train *AB* and as such they are in a position to interfere at the point of their intersection. As the air film is wedge-shaped

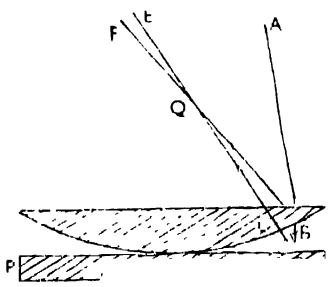


Fig. 30

the fringe system becomes localised. Q indicates the position where BF and CDE will interfere, the distance of Q being very close to the air film (shown magnified in the figure).

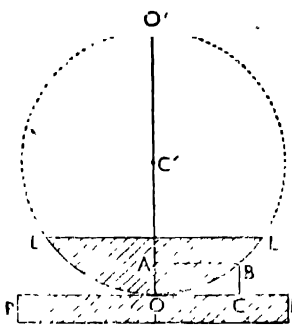


Fig. 31

Let LOL (Fig. 31) be the convex surface of a lens in contact with the plane surface PP. O is the point of contact. Let C be a point on PP at a distance ρ from the point of contact. $CB = t$ is the thickness of the air film at C. Then the path difference between the interfering rays at this position due to the film alone is given by $2\mu t \cos r$.

Let C' be the centre of the spherical surface LOL, having radius of curvature equal to R. The thickness of the air film $CB(=t)$ is given by $AB^2 = OA.AO'$,

where OO' is the diameter of the spherical surface.

or, $OC^2 = CB.AO'$.

Since $OC = AB = \rho$,

$$\rho^2 = CB.(2R - OA)$$
$$= 2R.CB - CB^2$$
$$= 2R.CB \text{ approx.,}$$

since the thickness of the air film $CB(=OA)$ is very small compared to the radius of curvature of the lens.

$$\therefore \quad CB = \frac{\rho^2}{2R}; \quad \text{or,} \quad t = \frac{\rho^2}{2R}.$$

Thus with the thickness of the air film at C, destructive interference will occur between the two wave-trains under consideration if the condition, $2\mu t \cos r + \frac{\lambda}{2} = \left(n + \frac{1}{2}\right)\lambda$, or, $2\mu t \cos r = n\lambda$ is satisfied, the path difference $\frac{\lambda}{2}$ being introduced as a result of the extra phase change π suffered on account of reflection at the surface of the plate PP. [As the air film formed between the lens and glass plate is really wedge-shaped, the path difference should be actually $2\mu t \cos (r - \theta)$. (vide Art. 31, Note). As θ is very small and the incidence is normal this ultimately reduces to $2\mu t$]. For air film, $\mu = 1$ and for normal incidence $\cos r = 1$. Therefore, the condition for destructive interference at the position of C reduces to $2t = n\lambda$.

But $t = \frac{\rho^2}{2R}$. Hence $\frac{\rho^2}{R} = n\lambda$;

or, $\rho^2 = n\lambda R$ ... ... (1)

This equation gives the radius of the nth dark ring round O. We shall indicate the radius of the nth dark ring by ρ_n. Therefore, $\rho_n^2 = n\lambda R$ for air film. For any other film of refractive index μ, (μ being less than the refractive index of the material of the lens or glass plate) $\rho_n^2 = \frac{n\lambda R}{\mu}$.

At the point of contact, O, the thickness of the film is zero. But still a phase difference of π is there between the two wave-trains suffering reflections under two different conditions. Hence the central spot of the ring system will be dark as shown in Plate II(a). The radii of the successive dark rings are given by putting $n = 1, 2, 3, \ldots$ etc. in relation (1).

Thus $\rho_1^2 = \lambda R$; or, $\rho_1 = \sqrt{\lambda R}$.

$\rho_2^2 = 2\lambda R$; or, $\rho_2 = \sqrt{2\lambda R}$

$\rho_3^2 = 3\lambda R$; or, $\rho_3 = \sqrt{3\lambda R}$.

Hence $\rho_1 : \rho_2 : \rho_3 \quad = \sqrt{1} : \sqrt{2} : \sqrt{3}. \quad \ldots$

Thus the radii of the successive dark rings are proportional to the square roots of the natural numbers. Similarly, the radius of the nth bright ring is given by,

$$\rho_n'^2 = (2n+1)\frac{\lambda}{2} \cdot R,$$

and the radii of the successive bright rings are given by,

$$\rho'^2_1 = \frac{\lambda}{2} R, \quad \rho'^2_2 = 3\frac{\lambda R}{2}, \quad \rho'^2_3 = 5\frac{\lambda R}{2}.$$

Hence, $\rho'_1 : \rho'_2 : \rho'_3 \quad \ldots \quad = \sqrt{1} : \sqrt{3} : \sqrt{5}$ and so on. Thus the radii of the successive bright rings are proportional to the square root of the odd natural numbers.

Newton's rings provide us with a convenient method of determining the wavelength of a monochromatic light. The diameter of the nth dark ring is given by, $d_n^2 = 4\rho_n^2$,

or, $d_n^2 = 4n\lambda R$,

and the diameter of the $(n+m)$th ring is given by

$$d^2_{n+m} = 4(m+n)\lambda R,$$

$$\therefore \quad d^2_{n+m} - d^2_n = 4Rm\lambda,$$

$$\therefore \quad \lambda = \frac{d^2_{m+n} - d_n^2}{4Rm}.$$

Thus, with the help of the experimental arrangements shown in Fig. 29 the diameter of the rings can be measured by means of the travelling microscope and the value of λ calculated, provided the radius of curvature of the lens surface is known. The radius of curvature may be determined with the help of an accurate spherometer. But the best thing would be to determine the radius of

curvature of the lens surface with the help of light of another known wavelength using the above formula. It is, however, not necessary to know the actual order of the rings in this experiment.

Newton's rings may be obtained with transmitted light also. In that case the ring system will be complimentary to the reflected one. The sum of the two intensities shall be equal to that of the incident light. The central spot will be bright in the transmitted system as there is no phase difference between transmitted rays at the centre of the system. The reflected system shows a better contrast between the dark and bright rings as the background illumination can be conveniently reduced to a minimum, whereas it is not possible in the case of the transmitted system which has always got a general bright background

34. Necessity of an Extended Source :—In studying Newton's rings, an extended source of light, such as a large flame is necessary. With a point source, light is reflected from a single point of the film to the eye placed at a particular point. Depending on the thickness of the film at that point and the particular angle of reflection, the point under consideration will appear bright or dark. Thus to get a system of broad fringes light must reach the eye from a wide area of the film. This is only possible when the actual source is broad, *i.e.* an assemblage of a large number of point sources. It is to be noted that coherent rays coming from a *single point of the source only produce interference effect at a particular point* of the film. (See, Note in Art. 30). The phase difference depends only on the thickness of the film at that point. So if any other ray coming from another point of the broad source is incident at that particular point of the film, there will be the same phase difference between the reflected components and as such they will produce the same effect. So the fringes will be more distinct and the joint effect will be very bright. The mutual incoherence between the different points of the extended source is immaterial here.

35. Refractive Index of a Liquid by Newton's Ring method :—The arrangement for studying Newton's rings may be used for finding the refractive index of a liquid. A small quantity of the liquid is placed between the glass plate and the lens and the diameters of the nth and $(n+m)$th rings are measured by the usual method. If light of wavelength λ be used, the radius of the nth ring as proved ealier is given by,

$\rho_n^2 = \frac{n\lambda R}{\mu}$ where μ is the refractive index of the liquid.

Hence the corresponding diameter is $d_n^2 = \frac{4n\lambda R}{\mu}$.

Similarly the diameter of the $(n+m)$th ring is $d^2_{n+m} = \dfrac{4(n+m)\lambda R}{\mu}$.

Hence $$\mu = \frac{4m\lambda R}{d^2_{n+m} - d_n^2} \quad \cdots \quad (1)$$

So if m, λ and R are known μ can be found by actually measuring $(d^2_{n+m} - d_n^2)$. If R is not known, then measuring diameters D of the nth and the $(n+m)$th fringe with an air film in between the lens and plate, we get

$$1 = \frac{4m\lambda R}{D^2_{n+m} - D_n^2} \quad \cdots \quad (2)$$

Then dividing (1) by (2), we get,

$$\mu = \frac{D^2_{n+m} - D_n^2}{d^2_{n+m} - d_n^2} \quad \cdots \quad (3)$$

Hence from relation (3), μ of the liquid can be found.

36. The Testing of Planeness of a Surface :—The surface to be tested, if transparent, should be held at a very small distance over the clear surface of mercury contained in a trough so that a small air film is included in between. With a monochromatic source this would give interference fringes. If all these fringes are exactly parallel (either straight or slightly curved), the surface is optically plane. If there is irregularity anywhere, the surface at that position is not optically plane. If the surface to be tested is not transparent but reflecting, the planeness can be tested by placing a lens of large focal length on it and observing the regularity of the Newton's rings formed in the case.

Examples

1. A wedge-shaped film of air is illuminated by a monochromatic beam of light of $\lambda = 4655$ Å. The distance between two consecutive fringes is 0·12 cm. when viewed vertically. Calculate the wedge angle.

Ans. From Art. 31, relation (3), $\theta = m\lambda/2\mu(x_1 - x)$ For two consecutive fringes, $m = 1$ and $(x_1 - x) = 0{\cdot}12$ cm. Here $\mu = 1$.

Hence, $\theta = \lambda/2(x_1 - x)$ radians $= \dfrac{4655 \times 10^{-8}}{0{\cdot}24} \times \dfrac{180}{\pi}$ degrees

$= (4655 \times 10^{-8} \times 180 \times 60 \times 60)/(0{\cdot}24 \times 3{\cdot}14)$ sec $= 40$ seconds.

2. A parallel beam of sodium light of wavelength 5890×10^{-8} cm. is incident on a thin glass plate of refractive index 1·5 such that the angle of refraction into the plate is 60°. Calculate the smallest thickness of the plate which will make it dark by reflection.

Ans. Let t be the smallest thickness. If μ and r be respectively the refractive index and the angle of refraction, the path difference is $2\mu t \cos r$.

For darkness, $2\mu t \cos r = n\lambda$ and for the smallest thickness $n = 1$.

$$\therefore \quad 2\mu t \cos r = \lambda ;$$

$$\text{or,} \quad t = \frac{\lambda}{2\mu \cos r} .$$

$$\therefore \quad t = \frac{5890 \times 10^{-8}}{2 \times 1{\cdot}5 \times \cos 60^\circ} = \frac{5890 \times 10^{-8}}{3 \times 0{\cdot}5} = 3926 \times 10^{-8} \text{ cm.}$$

3. The diameter of the 10th dark ring in a Newton's ring experiment is 0·50 cm. in the reflected system Calculate the thickness of the air film at the position and also the radius of curvature of the lens. Given $\lambda = 5{\cdot}9 \times 10^{-5}$ cm.

Ans. For dark fringes, $2\mu t \cos r = n\lambda$

Here $r=0$, $\mu=1$, $n=10$, $\lambda=5{\cdot}9\times10^{-5}$.

$\therefore$ $2t=10\times5{\cdot}9\times10^{-5}$.

or, $t=2{\cdot}95\times10^{-4}$ cm.

Again, $R=\frac{\rho^2}{2t}$ where $\rho=\frac{0{\cdot}50}{2}={\cdot}25$ cm. from the given data,

$\therefore$ $R=\frac{(0{\cdot}15)^2}{2\times2{\cdot}95\times10}=105{\cdot}9$ cm.

4. Newton's rings are formed with reflected light of wavelength 5890 A°.U using a plano-convex lens and a plane glass plate with a liquid between them. The diameter of the 3rd bright ring is 2 mm If the radius of curvature of the curved surface of the lens is 90 cm., find the refractive index of the liquid. (Delhi U. 1948)

Ans. For brightness, $d_n^2=\frac{(2n+1)}{\mu}4R\frac{\lambda}{2}$.

Here $\lambda=5890\times10^{-8}$ cm., $d_3=2$ mm. $=\cdot2$ cm., and $R=90$ cm.

$$\therefore\quad \mu=\frac{(6+1)\times2\times90\times5890\times10^{-8}}{(\cdot2)^2}$$

$$=\frac{7\times180\times5890\times10^{-8}}{\cdot04}=1{\cdot}85.$$

***37. Colour Photography** :—The photography of object in their natural colours or colour photography in the original form, provides us with an instructive application of the phenomenon of interference of light.

Since light is propagated in the form of waves it is expected that if light is reflected perpendicularly from a reflecting surface, the reflected and the incident waves will superpose and form stationary waves. As there is a phase change of π at the reflecting surface, nodes will be formed at the surface and also at distances $\frac{\lambda}{2}$, λ, $\frac{3\lambda}{2}$ etc. from it. Between these nodes lie the equidistant antinodes. Wiener, in course of his experiments to determine whether the electric or the magnetic vector of the electro-magnetic waves is photographically effective, demonstrated the existence of such stationary waves in 1890. Lippmann in 1891 introduced colour photography based on the formation of such stationary waves. He placed the photo-plate in a camera-box with its glass side facing the objective. The sensitive layer of photographic emulsion was backed by a layer of mercury. When the plate is exposed to light coming from an object and subsequently developed and fixed, thin parallel layers of silver deposit are formed in the positions of the expected antinodal planes. This is because any displacement in the nodal planes being absent, the photographic plate is not affected in these planes whereas due to the maximum displacement in the antinodal planes the photo-emulsion is most affected in those positions and thin parallel layers of silver are found in those planes after fixation. These layers are separated by thin transparent layers of gelatine—the distance between successive silver layers

being $\frac{\lambda}{2}$ where λ is the wavelength of the incident light. Each of the spectral colours in the object will produce its own silver layers on the plate. In order to reproduce the photo image in the original colours of the object, white light is made incident normally on the glass side of the plate The waves of differet colours will get totally reflected from their corresponding silver layers while layers formed by a particular λ will absorb all wavelengths other than this λ. So the overall effect of the reflections from these differently distributed silver layers will be an exact reproduction of the image in the natural colours of the object itself.

Lippmann's process of colour photography is rather difficult to carry out and has been very little used in actual commercial colour photography.

Commercial methods of Colour Photography and Colour Printing.– We know that all colours in nature can be produced by an appropriate mixture of the three primary colours of the spectrum—namely red, green and blue According to Young's theory of colour vision, the colour perception by the brain is due to the excitation of three kinds of sensitive nerves in our eye by these three primary colours. Impression of different colours is due to the different degrees of excitation of those three types of nerves and also to their intensities of excitation The commercial processes of colour photography are founded on the theory of primary colours and colour vision. Here a proportionate combination of the three primary colours produces the same colour sensation as the original object. In all the processes three colour filters—red, green and blue —are employed.

In one of the commercial processes, three different negatives of the object are taken through three different filters. The red portion of the object appears on the negative taken through the red filter and the green portion on the negative taken through the green filter and so on. The adjacent colours of the filters are to a certain extent always transmitted through them and this effect can further be enhanced by selecting the pigments of the filters such that the ratio effect on the negatives is almost the same as that on the sensitive nerves of the eye. The three negatives so obtained correspond to the three fundametal colour sensations. Corresponding to each of these negatives a positive is produced. Then each positive is combined with its corresponding filter, through which its negative copy was taken. With the help of a triple projection device consisting of three condensers and three lenses, the three positive pictures are simultaneously cast on the screen where they superimpose exactly reproducing the image of

the original object in its natural colours. Many methods of colour cinematography are based on this principle. This process, wherein three pictures in the primary colours are superimposed to produce the original colour of the object, is known as the *additive process* of colour photography. Now-a-days in the additive process it is possible to take the three separate negatives on the same plate. This method was first used by Joly. Two different types of such plates are found in the market. They are known as the Dufay colour plates and Finlay plates. The Lumiere plates and the Agfa and Lignose films are older varieties of Dufay colour plates. Dufay colour plates are ordinary colour sensitive plates on which is printed a very fine transparent pattern of alternate blue and green squares and red lines. While taking the negative the squares and lines act as filters. After development the image in the emulsion is reversed. So it now acts as the positive and gives the image in its natural colours when it is held in white light.

A Paget plate (the older variety of Finlay plates) consists of a glass plate on which small squares of primary colours are so very closely arranged that they are visible only under a microscope. The negative is taken on a panchromatic plate with the above plate as the filter. Next a positive is produced. When this positive is placed in contact with the Paget plate acting as the colour screen, the picture appears in its natural colours in white light.

In commercial tri-colour printing and half-tone work the *subtractive* process is adopted. Here the negatives are taken exactly in the same way as in the additive process through three filters. Then the pictures are directly printed on paper on the top of one another, each in a colour which is complementary to the corresponding filter colour. The resultant picture now appears in its original colours.

38. Interferometers :—An interferometer is an arrangement for obtaining the interference effect with a large path difference between two interfering waves. The method of interferometry was first developed by Michelson in 1892, and is now widely used in science, in the study of the fine structure of spectral lines, and in determining the variation of refractive index of gases and salt solutions with density, temperature, etc. So far as the study of spectral lines is concerned, interferometry dates back as early as 1862, when Fizeau observed that the intensity of the Newton's ring fringes practically disappeared in the region of the 500th fringe but regained their original clarity in the region of the 1000th fringe. From this he concluded about the doublet nature of sodium line. When an interferometer is used to determine the refractive index of a substance, it is also called a *refractometer*.

***39. Rayleigh's Interferometer (or Refractometer) :**—Rayleigh devised this instrument to determine the refractive indices of rare gases. The instrument is as well used to determine the refractive indices of liquids in the form of weak solutions. The plan of the arrangement is shown in Fig 32. Light from a narrow vertical slit source S is made parallel by a lens L, and is passed through the double slit A, B. The two beams then pass through two similar tubes T_1 and T_2 and a compensator arrangement. The emergent rays are then brought to a focus by a telescope T. In the focal

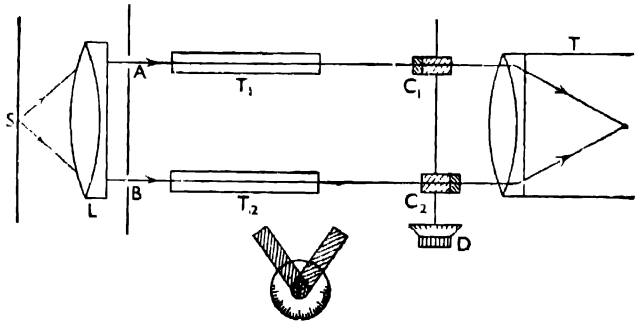


Fig. 32

plane of the telescope interference fringes due to double slit appear. The compensator arrangement consists of two thin glass plates C_1 and C_2 inclined at an angle of 45° to the path of light. The plates can be rotated about a horizontal axis perpendicular to the incident light by means of a micrometer screw D carrying a divided drum. By rotating the drum D a path difference between the two rays coming from the two slits can be set up, or any previous path difference can be compensated. Or, in other words, the fringe system can be given any desired displacement or any previous displacement of the fringe system can be compensated by rotating the drum. The two tubes, T_1 and T_2, are exactly similar. The end pieces of glass are optically flat and have the same thickness. At first, using a monochromatic light the graduations of the drum are calibrated in terms of the fringe shift, *i.e.* the rotation of the drum for the displacement of a single fringe is found.

Next the fringes are produced with white light when there is vacuum in two tubes T_1 and T_2. The central white fringe is made to coincide with the cross-wire of the telescope by rotating the drum D. Now a gas at a suitable pressure is introduced in one of the tubes. A displacement of the central white light

fringe would occur due the introduction of this additional path difference. The drum is again rotated, say, through an angle θ, to bring the central white light fringe on the cross-wire again.

If l is the length of the tube and μ is the refractive index of the gas at that pressure, the path retardation introduced is $l(\mu-1)$.

In n fringe-shifts occur, with monochromatic light of wavelength λ for a rotation of α divisions of the drum, then $\frac{\lambda n}{\alpha}$ is the path retardation due to one division rotation of the drum. So the path retardation for θ divisions rotation of the drum is $\frac{n\lambda}{\alpha}\theta$. Therefore we have,

$$l(\mu-1)=\frac{n\lambda}{\alpha}\theta.$$

From this relation the value of μ can be determined. The refractometer provides us with a very convenient and accurate method of studying the variation of refractive index with temperature and pressure.

It is also used to detect a slight amount of impurity in a gas or for finding the concentration of salt solutions.

Rayleigh's interferometer was improved by Haber and Loewe who substituted for the cross-hairs a second system of fixed interference fringes which served as a reference system. The system of fringes is viewed through a cylindrical eye-piece instead of an ordinary eye-piece. As a result we see two sets of fringes one above the other and just separated by a dark line. Any displacement of one set of fringes due to any path retardation is measured with reference to the fixed set of fringes, and as a result a higher accuracy is possible. Moreover, any distortion of the frame work due to temperature changes or other causes affects both the systems alike and hence introduces no error.

***40. Jamin's Interferometer :**—This interferometer, which is used as a refractometer, consists essentially of two rectangular glass blocks of exactly equal thickness, cut accurately plane from the same material. The plates are silvered at the rear surfaces. Light from an extended source S_1 is made parallel by a convex lens. It then falls on the surface of the glass plate P_1 (Fig. 33). The incident amplitude is divided at A and the component rays are recombined at D. The disturbances traversing the ray paths $ABCD$ and $AEFD$ are obviously coherent and can interfere at D. If the glass plates are absolutely parallel, the optical paths $ABCD$ and $AEFD$ are equal. Thus the rays will interfere constructively at D and hence the field of view of the telescope T will be uniformly

bright. If the glass plates are slightly out of parallel, a path difference between the two beams will be introduced. It is

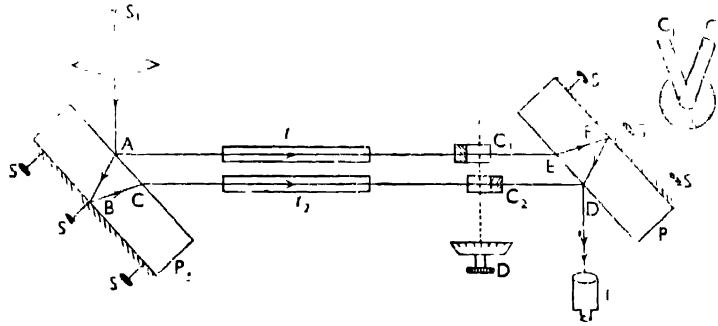


Fig. 33—Jamin's Interferometer

evidently given by $2\mu t\ (\cos r_1 - \cos r_2)$, where r_1 and r_2 are the angles of internal reflection at the two plates. So the fundamental formula is

$$2_\mu t(\cos r_1 - \cos r_2) = n\lambda.$$

For a bright fringe n is an integer.

By means of the back screws S, the plates can be rotated about a vertical, as well as a horizontal axis. If the plate P_1 be kept fixed, and the plate P_2 is given a slight tilt about the horizontal axis, so that the line of intersection of the plates becomes horizontal a number of horizontal bands, very nearly straight appears in the field of view. An exactly similar pair of tubes (t_1 and t_2) is placed in the path of the two beams. A compensator consisting of two similar plates C_1 and C_2 making a small angle with each other and movable about a horizontal axis by a drum D provided with a circular scale, is placed in the path of the two beams. By rotating the drum D, an additional path difference between the two beams may be introduced. The circular scale of the compensator may be previously calibrated by observing the fringe shift of a monochromatic light. If n fringe shifts of light of wavelength λ occur due to rotation of α divisions of the circular scale, $\frac{n\lambda}{\alpha}$ is the path retardation due to a rotation of one division of the circular scale.

To measure the refractive index of a gas the two tubes are first evacuated and the central fringe with white light is brought on the cross-wire of the telescope. Now the gas whose refractive index is to be determined is introduced in the one of the tubes. Due to the introduction of the additional path difference, $(\mu - 1)l$, the central fringe with white light will be shifted. The compensator

is then rotated by θ divisions to bring the central white fringe back to its original position. The path retardation thus introduced by the compensator is $\theta . \frac{n\lambda}{\alpha}$. Obviously this must be equal to $(\mu - 1)l$, where μ is the refractive index of the gas and l is the length of the tube,

$$\therefore \quad (\mu - 1)l = \theta . \frac{n\lambda}{\alpha}.$$

Knowing l, μ can be calculated. This also provides us with an accurate method of studying the variation of refractive index of a gas with temperature and pressure.

The thickness of a thin film, whose refractive index is known can also be determined with this instrument. First, the central fringe with white light is brought on the cross-wire of the telescope Then the thin film of thickness t is introduced into path of one of the interfering rays. A path retardation occurs which is compensated by rotating the compensator by an angle β, say, and bringing the central fringe again on the cross-wire of the telescope. Then $(\mu - 1)t = \frac{n\lambda}{\alpha} . \beta$. Knowing μ, the film thickness t can be found out. This method is suitable for determining the thickness of very thin films.

***41. Michelson's Interferometer** :—In Michelson interferometer, light from an extended source is divided into two beams of equal intensity (*i.e.*, amplitude is divided) by partial reflection at the surface of a glass plate. These beams are then sent in mutually perpendicular directions where they suffer reflections on plane mirrors M and M', whence they are brought together again to form the interference fringes. The arrangement is shown schematically in Fig. 34. Light from an extended source S falls on an optically worked plane glass plate A, half silvered at its rear surface and inclined at 45° to the incident beam. The two

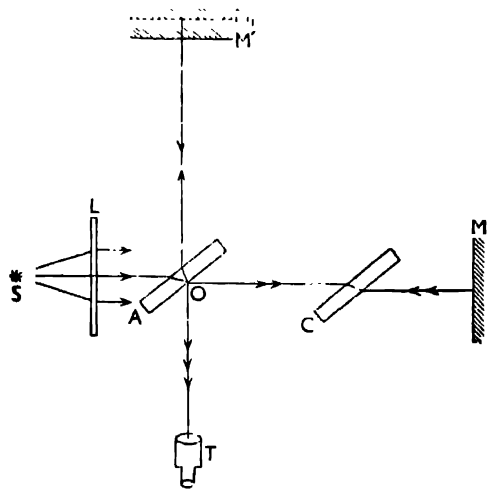


Fig. 34

beams, one produced by reflection at its rear surface and the other by transmission, fall normally on the plane mirrors M and M' and are reflected back. On return they are reunited at the half-silvered surface of A and pass on to the telescope T. Since the beam to M' passes three times in all through the thickness of the glass plate A, whereas, the beam to M passes through the glass plate A only once, an unsilvered compensating plate C, exactly similar and parallel to A is inserted in between A and M. This obviously makes the two separate beams to have equal paths in glass. The mirror M is mounted on a carriage and can be moved parallel to itself along well machined ways or tracks (not shown in the figure).

Two distinct types of fringes can be obtained by this instrument. When the mirrors M and M' are perpendicular to each other and the plate A is at 45° to the direction of incident light and to either of M and M', the image of M in the plate A will be at M_1 and will be parallel to M'. Hence the arrangement is equivalent to a thick film of air enclosed between M_1 and M', and the fringes will be similar to those of a parallel-sided film of air. With an extended monochromatic source, the fringes are the usual concentric fringes of equal inclination formed at infinity and can be viewed with a telescope. In a direction inclined to the normal to M' at an angle θ, the path difference between the superposed disturbances is given by $2t \cos \theta$, where t is the distance between M' and M_1. The rays will reinforce each other to produce maxima for those angles θ, for which the relation $2t \cos \theta = n\lambda$, is satisfied, provided the silver coating on A is thin.* With a monochromatic light circular fringes are visible even for a very large path difference.

If one of the mirrors be tilted so that M_1 and M' are inclined at a small angle, the fringes will then be those of a thin air wedge. With an extended source, straight fringes, parallel to the line of intersection of M_1 and M', and localised in the air wedge will be formed. To get such fringes the film should be very thin *i.e.* t should be very small. As t is increased, the fringes moves towards the thin end of the wedge. Another effect arises due to the variation of path difference with θ, which causes the fringes to get curved towards the thicker part of the wedge.

***Theory of Fringe Shape :—**

Let the mirror M' and the image mirror M_1 of Michelson interferometer be inclined at an angle α [Fig. 34(*a*)]. Let the eye be placed at O and R be the foot of the perpendi-

*If the silver coating is thin the phase change in the internally reflected ray is identical with that in the externally reflected ray at O (both equal to π) as has been shown by Edser. In other cases there will always be an extra phase change π suffered by the ray from M externally reflected at O.

cular drawn from O to the plane of M' ($X-Y$ plane). Let $OR=z$ and let us consider the effect at O due to the rays from P of co-ordinates x, y, lying in the M' plane. If θ be the angle between OP and OR, the path difference between the rays from the front and rear planes reaching O from P is $2t\cos\theta$ where t = plate separation at P.

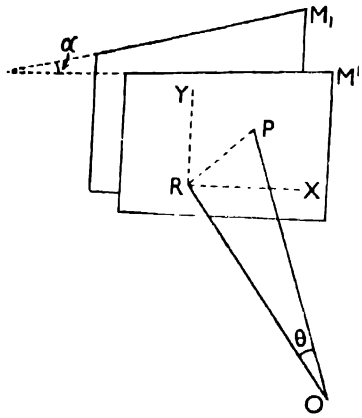


Fig. 34(*a*)

$$\therefore \quad OP=\sqrt{x^2+y^2+z^2} \text{ and } \cos\theta = OR/OP$$
$$= z/\sqrt{x^2+y^2+z^2}.$$

If the film thickness at R be e, that at P is $t=e+x\tan\alpha$.

Hence for a fringe formation, we have,

$$\frac{2(e+x\tan\alpha)z}{\sqrt{x^2+y^2+z^2}}=n\lambda \qquad (1)$$

When the two planes intersect at R, then $e=0$ and eqn. (1) reduces to $n\lambda = \dfrac{2xz\tan\alpha}{\sqrt{x^2+y^2+z^2}}$.

Now if z is very large compared to x and y, this relation reduces to $n\lambda = 2x\tan\alpha = 2x\alpha$ (for small values of α). This is the eqn. of a straight line. Hence the fringes are straight and equispaced separation between successive fringes being $\lambda/2\alpha$. They are localised practically on the plane M'.

If the point O be near the plane M' such that x and y are not negligible compared to z, then the equation represents a hyperbola whose axis coincides with the line of intersection of M' and M_1. Thus the fringes may be curved according to the tilt.

When M' and M_1 are parallel, $\alpha=0$, and eqn. (1) reduces to $n\lambda = 2ez/\sqrt{x^2+y^2+z^2}$.

Squaring and rearranging,

$$x^2+y^2=\left[\frac{4e^2-n^2\lambda^2}{n^2\lambda^2}\right]z^2.$$

If the position of eye be fixed, z is constant and as e is also constant, the above equation represents a circle with centre at R and of radius $[z(4e^2-n^2\lambda^2)^{\frac{1}{2}}/n\lambda]$. They are formed at infinity.

So if two plates are inclined and intersect, straight line fringes are formed particularly near the intersection line. If they are inclined but do not intersect, the fringes are sections of hyperbolae. When the plates are parallel, the fringes are circular and appear at infinity.

Uses of the Instrument.—

(a) To find the wavelength of a monochromatic light.— Let the order of the fringe at the centre of the field of view be n, then

$$2t = n\lambda, \qquad [\text{since } \cos\theta = 1].$$

If one of the mirrors is moved back slowly so as to increase the path difference between two normally reflected waves by λ, we have then

$$2t + \lambda = n\lambda + \lambda,$$

$$\text{or,} \quad 2(t + \lambda/2) = \lambda(n + 1).$$

Thus when t increases by $\lambda/2$, a fringe of the next higher order appears in the field of view. Let one of the mirrors be shifted by a distance x, so that N fringes are found to cross the centre of the field of view, then

$$x = \frac{N\lambda}{2}.$$

N can be very easily counted, and x, the distance through which the mirror is shifted, may be determined from the readings of the micrometer scale attached to the movable mirror. Therefore, λ the wavelength of the radiation can be determined.

(b) Study of the fine structure of Spectral line.—If a source of light is truly monochromatic, then the increase of the path difference will not in any way affect the clearness of the fringes. If, however, a spectral line is composite in nature, the clearness of the fringes is impaired. To measure $d\lambda$, the difference in wavelengths of two near lines, the movable mirror is first adjusted to give a set of distinct fringes. If now the path difference between the two beams is increased by moving the mirror back, the fringes will be most indistinct when the two components are equally intense and one component gains half a wavelength over the other. If the mirror is moved over a distance l, the additional path difference introduced between the two rays is $2l$. Let λ_1 and λ_2 be the wavelengths of the components and n_1 and n_2 be the number of waves of these two wavelengths respectively contained in $2l$.

Then for the first superposition of the dark band of one on the bright band of the other the difference between n_1 and n_2 is $\frac{1}{2}$.

Let λ_1 be known and n_1 be greater than n_2,

Then $\lambda_1 < \lambda_2$.

Now since $n_1 = \frac{2l}{\lambda_1}$ and $n_2 = \frac{2l}{\lambda_2}$,

$$n_1 - n_2 = 2l\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$\frac{2l(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2}$$

$$\frac{2l\, d\lambda}{\lambda_1\lambda_2},$$

or, $d\lambda = \frac{\lambda_1\lambda_2}{4l} = \frac{\lambda^2}{4l}$, since $n_1 - n_2 = \frac{1}{2}$.

Here λ indicates the average wavelength of the composite spectral line.

The distance l through which the mirror is shifted can be determined with great accuracy from the readings of the micrometer screw attached to the movable mirror. The average value of the wavelength (λ) can be determined by the method described in (*a*). Hence $d\lambda$ can be calculated. If the two components are of unequal intensity, the movable mirror is to be so shifted that observations either on two successive distinct fringe systems (of maximum visibility), or on two successive indistinct systems (of minimum visibility) can be made. In this case as $(n_1 - n_2) = 1$, and if L be the shift of the mirror, $d\lambda = \lambda^2/2L$.

(c) Standardisation of the Metre in terms of wavelength of light :—We have seen that in Michelson's interferometer, if either of the mirrors is moved, the fringes move across the field of view. By counting the number of fringes the distance moved by the mirror may be determined in terms of the wavelength of the monochromatic light used. This provides us, at least in principle, with a method of determining the number of wavelengths in the standard metre. The standard metre is the distance between two fine scratches on a bar of platinum-iridium alloy at 0°*C*. kept at the International Bureau of weights and measures near Sev'res, France. In spite of the secondary standards, it was thought desirable to define metre in terms of some non-destructible unit or length, free from any secular change or any chance of accident. The wavelength of light was considered to be most suitable for the purpose. Evidently it became necessary to evaluate the standard metre in terms of wavelength of light. This task was first undertaken by Michelson and Benoit in 1892. Michelson worked with the red line of cadmium, the most homogeneous light sources available. The task of counting a large number of fringes was itself an impossible one at the time of experiment. So Michelson constructed nine intermediate sub-standards, called etalons, each of which consists of a pair of mirrors M_1, M_2

mounted on a rigid base and exactly parallel, as indicated by Fig. 35. The mirror separation of the etalons varied from 10 cm. to 0·39 mm. each successive one being of half the length of the previous one.

At first the smallest etalon was mounted on the interferometer and the number of waves contained in the space between the planes of the two mirrors was determined accurately with the help of a modified arrangement of the Michelson interferometer. Next the second etalon was compared with the first to the fraction of a fringe. In this way each etalon was compared with the next higher one until the 10 cm. one was reached. In the final operation, the longest intermediate standard (10 cm.) was compared with the prototype metre, the former being stepped off ten times with the help of the fringes. Thus the number of wavelengths of red cadmium line in a standard metre was determined. The result obtained by Michelson was,

1 metre = 1,553,163·5 wavelengths

where $\lambda = 6438\ 4722$ Angstroms.

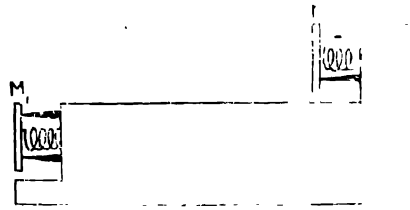


Fig. 35 - Etalon

The cadmium red line is now-a-days the primary standard in spectroscopy. It is now internationally agreed that under the conditions specified by Michelson the cadmium red line has an wavelength of 6438·4696 Å. U. in dry air at 15°*C.* and at a pressure of 760 mm of Hg.

Twyman and Green constructed an interferometer which resembles a Michelson interferometer in many respects but wherein the extended source is replaced by a monochromatic point source. It is a very useful type of interferometer extensively used for testing the imperfections in prisms and lenses.

Examples

1. When the movable mirror of a Michelson interferometer is shifted through 0·0589 mm., a shift of 200 fringes is observed. Calculate the wavelength of the light used in Angstrom Units.

Ans. $2l = n\lambda$; $l = 0{\cdot}00589$ cm.

and $n = 200$.

$$\lambda = \frac{2l}{n} = \frac{2 \times 0{\cdot}00589}{200} = 0\ 0000589 \text{ cm.}$$

$$= 5890 \text{ Å. U.}$$

2. In an experiment with Michelson interferometer for studying close doublets, the readings corresponding to maximum indistinctness were 0·6939 mm. and 0·9884 mm. If the mean wavelength be 5893A, evaluate the difference between the wavelengths of the doublets.

Ans. $\frac{\lambda^2}{2L}$ = ·02945 cm.

Substituting, $d\lambda = \frac{5893 \times 5893 \times 10^{-16}}{5890 \times 10^{-5}} = 6 \times 10^{-8}$ cm. = 6 A U.

***42. Fabry-Perot Interferometer** :—This interferometer consists of two parallel optically plane glass plates E_1 and E_2 (Fig. 36),

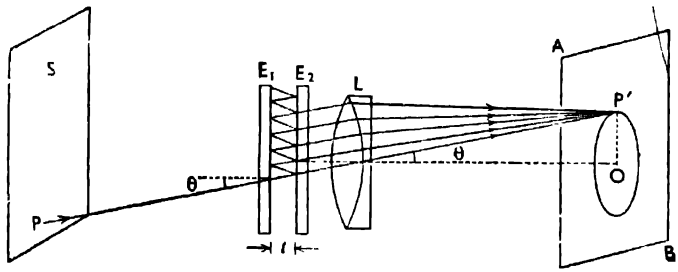


Fig. 36

which are thinly silvered on the inner surfaces. The silvered surfaces are placed facing each other and the separation between them is fairly large (0·1 to 10 cms.). The fringes are produced in the transmitted light after multiple reflections in the air film enclosed between the glass plates. The observations are made at nearly the normal direction and the fringes observed come under the designation of *Haidinger fringes*. They are formed at infinity and are to be observed with the help of lens or telescope.

Light from an extended monochromatic sources S is allowed to fall on the interferometer plates E_1E_2. Any ray incident on the interferometer plate will suffer multiple reflections in the air film of constant thickness t, enclosed between the glass plates and will be broken up into a series of parallel transmitted rays. A lens L is used to bring these transmitted parallel rays to a focus. The lens L may also be an unaccommodated eye lens. A ray from the point P of the extended source, incident at an angle θ, produces a series of parallel rays also at the same angle θ, which are brought together by L at the point P'. The condition for reinforcement or maxima at P' is given by,

$$2\mu t \cos \theta' = n\lambda ;$$

or, $2t \cos \theta = n\lambda$. [when $\mu = 1$ and $\theta' = \theta$, θ' being the angle of refraction]

This condition will be satisfied by all points on the circle passing through P' with centre O, the intersection point of the axis of the lens

with the screen AB. Thus we shall get a bright ring passing through P'. As θ is decreased cos θ will increase until n is greater by 1, 2, 3 ..etc., so that we get higher order bright rings with radius of the rings diminishing. Thus, if an extended source of monochromatic light is viewed through the Fabry-Perot plates, concentric rings will appear in the field of view. If the plates of interferometer are unsilvered, the fringes are broad and their visibility is low. But if the reflecting power is increased by lightly silvering the surfaces, the bright fringes become sharper and more intense while the dark fringes get broadened, as will be evident from the following treatment.

***Intensity of the Haidinger fringes :**

Let a be the amplitude of the incident wave represented either by the real or imaginary part of $ae^{i\omega t}$ where ω is the pulsatance of the wave. Let r and T, be the reflectivity of the bounding surfaces of the plate and the transmissivity of the air-film respectively. Here r is defined as the fraction of the incident light intensity reflected at each surface and T, the fraction transmitted, so that the corresponding fractional amplitudes will be $\sqrt{r}$ and $\sqrt{T}$. As here we are concerned with the rays transmitted from the plate E_2, the amplitudes of these successive emergent rays will be $a\top$, $a\top r$, $a\top r^2$ etc. as evident from Fig. 36(*a*). As the path difference between the successive rays is $2t$ cos θ, the corresponding phase difference, $\delta = (4\pi t \cos\theta)/\lambda$. Let ϕ be the phase difference between the first transmitted ray (1) and the incident ray, then the series of emergent wave disturbances can be represented by,

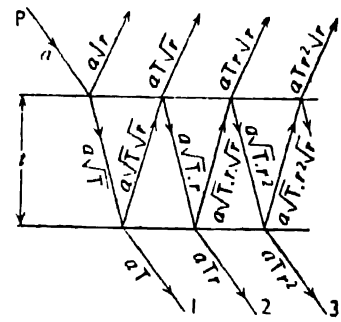


Fig. 36(*a*)

$$a\top e^{i(\omega t-\phi)},\quad a\top re^{i(\omega t-\phi-\delta)},\quad a\top r^2e^{i(\omega t-\phi-2\delta)}$$ and so on.

When these disturbances are brought to a point by a lens, the resultant disturbances there will be, $a\top e^{i(\omega t-\phi)} + a\top re^{i(\omega t-\phi-\delta)} + a\top r^2e^{i(\omega t-\phi-2\delta)} + \cdots$

$$= a\top e^{i(\omega t-\phi)}\left[1 + re^{-i\delta} + r^2e^{-2i\delta} + \cdots\right] = \frac{a\top e^{i(\omega t-\phi)}}{1-re^{-i\delta}}$$

The complex amplitude of the resultant disturbance is thus $a\top e^{-i\phi}/(1-re^{-i\delta}$ Hence the resultant intensity $I = a^2\top^2/(1-re^{-i\delta})(1-re^{i\delta})$

$$= \frac{a^2 \top^2}{1 + r^2 - r(e^{i\delta} + e^{-i\delta})} = \frac{a^2 \top^2}{1 + r^2 - 2r\cos\delta} = \frac{a^2 \top^2}{(1+r)^2 + 4r\sin^2 \delta/2}$$

$$= \frac{a^2 \top^2}{(1-r)^2} \cdot \frac{1}{1 + \frac{4r}{(1-r)^2}\sin^2 \delta/2} \quad \cdots \quad (2)$$

[The intensity of fringe system emerging in the same side as the source can be obtained from the relation $I_0 = I + I_r$, Ir being the required intensity. Here I is the intensity of the transmitted light as given by relation (2) and $I_0 = a^2$.

$$\therefore \quad I_r = a^2 - \frac{a^2 \top^2}{(1-r)^2 + 4r\sin^2 \delta/2} = (a^2 . 4r \quad \sin^2 \quad \delta/2)/\{(1-r)^2 + 4r\sin^2 \delta/2\}]$$

From (2), it is evident that I varies with δ as also with film properties. It is maximum when $\delta = 2\pi n$ and minimum when $\delta = (2n+1)\pi$, n being an integer.

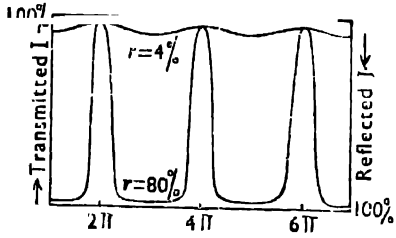


Fig. 36(*b*)

$\therefore$ For maximum, $2t\cos\theta = n\lambda$ and for minimum, $2t\cos\delta = (n + \frac{1}{2})\lambda$.

$$\therefore \quad I_{max.} = \frac{a^2 \top^2}{(1-r)^2} \text{ and } I_{min.} = \frac{a^2 \top^2}{(1+r)^2}$$

$$\therefore \quad I_{min.} / I_{max.} = \left[\frac{1-r}{1+r}\right]^2.$$ For ordinary unsilvered glass surfaces $r = 0{\cdot}02$ and this ratio becomes $0{\cdot}9$ only. So the contrast between maximum and minimum is not at all sharp. If, however, $r = 0{\cdot}8$, $I_{min.}/I_{max.}$ becomes $(\cdot 2/1{\cdot}8)^2 = \frac{1}{81}$. So here the intensity contrast is very pronounced and the fringes are very sharp as shown in Fig. 36(*b*), where intensity curves for $r = \cdot 04$ and $r = \cdot 8$ have been drawn to show the contrast. The visibility of the fringe-system, as defined by Michelson, is given by $V = \frac{I_{max.} - I_{min.}}{I_{max.} + I_{min.}} = \frac{2r}{1+r^2}$.

If r is very high tending to unity, $V \to 1$ also. So with increase in r visibility of the fringe system also increases.

***Study of the Structure of Spectral lines :—**

As the instrument has a high resolving power, it is convenient to use a Fabry-Perot interferometer for the study of the structure of the spectral lines. Let us, for example, consider the study of two close doublets of wavelengths λ_1 and λ_2 (as in sodium D_1 and D_2

lines). The interferometer is set so as to obtain exactly concentric circular fringes with the light which is composed of two doublets only. The movable plate is slowly displaced such that the bright rings of λ_1 and λ_2 are superposed on each other. Now the separation between the plates is further increased gradually till maximum discordance occurs when the maximum of λ_1 coincides with the minimum of λ_2 and vice versa. In this setting, at the centre of the pattern $\cos\theta = 1$ and hence,

$$2t_1 = n_1\lambda_1 = (n_1 + \tfrac{1}{2})\lambda_2 \quad \ldots \quad (1) \text{ [taking } \lambda_1 > \lambda_2\text{]}$$

Further displacing the plates the rings will again be in concordance and then again maximum discordance will follow. In this new setting,

$$2t_2 = n_2\lambda_1 = (n_2 + 1\tfrac{1}{2})\lambda_2 \quad \ldots \quad \ldots \quad (2)$$

From (1) & (2), we have,

$$2(t_2 - t_1) = (n_2 - n_1)\lambda_1 = (n_2 - n_1)\lambda_2 + \lambda_2 \quad \ldots \quad (3)$$

From (3), $(n_2 - n_1)(\lambda_1 - \lambda_2) = \lambda_2$; or, $n_2 - n_1 = \lambda_2/(\lambda_1 - \lambda_2)$

$$\therefore\ 2(t_2 - t_1) = \lambda_1\lambda_2/(\lambda_1 - \lambda_2) \doteq \frac{\lambda_1^2}{\lambda_1 - \lambda_2}$$

$$\therefore\quad \lambda_1 - \lambda_2 = \frac{\lambda_1^2}{2(t_2 - t_1)}.$$

Thus knowing λ_1 and the plate shift $(t_2 - t_1)$ during the above adjustment the wavelength difference between the close doublets can be determined.

***Resolving Power :—**

According to Rayleigh's criterion of resolution, two spectral lines are said to be resolved when the central maximum of one line coincides with the first minimum of the other. If two lines have the same intensity it can be shown that two lines will be resolved when the intensity of the saddle portion (S) of the resultant curve is $\frac{8}{\pi^2}$, or ·81 times the maximum on either side. Neglecting the intensity of the minimum at the position of the adjacent maximum for simplicity of calculations, the intensity at the point of overlap (D) should, according to the above consideration, be 0·405 times the intensity of each of the lines A and B. In the figure [Fig. 36(C)] where intensity I is plotted against the order number n, A and B represent the intensities of lines of wavelengths λ and $\lambda + d\lambda$.

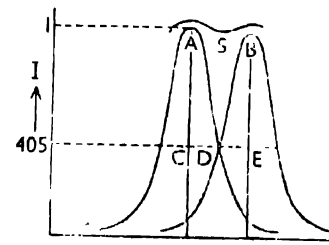


Fig. 36(c)

$$\therefore\ \text{Intensity at } D \text{ is } I_D = I_{max.} \Big/ \left[1 + \frac{4r}{(1-r)^2}\sin^2\delta/2\right]$$

$$= I_{max.}/[1 + F\sin^2\delta/2],$$

where, $F = 4r/(1-r)^2$, and intensity at A is $I_{max.}$

Under the condition of resolution, $I_D = 0{\cdot}405\ I_{max.}$, so that

$$\sin^2 \delta/2 = \frac{1 - {\cdot}405}{{\cdot}405F} = \frac{1{\cdot}469}{F}\ ;\ \text{or, } \delta = 2\sin^{-1}\left(\sqrt{\frac{1{\cdot}469}{F}}\right) = 2\sin^{-1}\left[\frac{1{\cdot}21}{\sqrt{F}}\right]$$

This phase difference δ corresponds to CD in the diagram. Hence the phase difference CE is $2\delta = 4\sin^{-1}\left(\frac{1{\cdot}21}{\sqrt{F}}\right)$, which corresponds to an order change $dn = \frac{2\sin^{-1}(1{\cdot}21/\sqrt{F})}{\pi}$, since unit change of order is equivalent to 2π (phase change). Since at the centre, the order n is given by the relation, $n\lambda = 2\mu t$ ($\because \cos 0 = 1$) $= 2t$, ($\because \mu = 1$)

$\therefore$ Differentiating, $nd\lambda + \lambda dn = 0$; or,

$$\frac{\lambda}{d\lambda} = -\frac{n}{dn}.$$

So the resolving power of the interferometer is,

$$R.P. = \frac{\lambda}{d\lambda} = -\frac{n}{dn} = \frac{-n\pi}{2\sin^{-1}(1{\cdot}21/\sqrt{F})}$$

For large values of r, F is also very large and hence the angle $\sin^{-1}(1{\cdot}21/\sqrt{F})$ can be replaced by its sine.

$$\therefore\quad R.P. = \frac{-n\pi\sqrt{F}}{2\times 1{\cdot}21} = -n\times 1{\cdot}3\sqrt{F}$$

If, however, the intensity of the minimum at the position of the adjacent maximum is taken into account, this expression of the resolving power reduces to $R.P. = -n \times 1{\cdot}5\sqrt{F}$ (approx.).

The $R.P.$ of an instrument is essentially a positive quantity. Here the negative sign occurs because a positive change of λ corresponds to a negative order change.

It can be shown that for an wavelength of 5,000 Angstroms and a mirror separation of 10 mm., the resolving power is 139,600 with a reflecting power of 50 per cent, and the resolving power is 349,200 with a reflecting power of 75 per cent. Thus the resolving power greatly increases with the increase in the reflecting power. This high resolving power of Fabry-Perot interferometer makes it very useful for studying the fine structures of spectral lines.

***43. Lummer-Gehrcke Interferometer** :—Another method of producing Haidinger's fringes was first devised by Lummer and Gehrcke. Here a long plate LL of glass or quartz with perfectly plane and parallel surfaces is used. Light is admitted through a right-angled prism P cemented at one end of LL. Light entering the

prism, suffers total internal reflection and passes into the glass plate. It then travels along the length of plate by suffering multiple reflections within the plate. Two sets of parallel rays at a nearly grazing angle will emerge from the plate. The rays may be collected with the help of a telescope T (shown in Fig. 37), in the focal plane of which

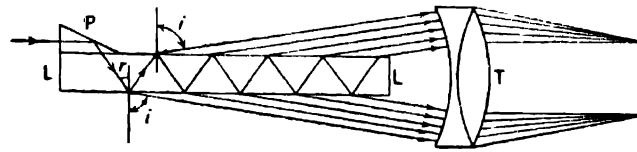


Fig. 37

nearly straight interference fringes are produced. The fringes comprise bright and narrow maxima with wide minima in between. The instrument has got a high revolving power and is generally used to study the fine structures of individual spectral lines.

To study the hyperfine or fine structure of spectral lines, it is required to photograph the fringes formed on opposite sides of Lummer plate simultaneously and to measure the distance between the identical fringes on opposite sides. Let D_p and D_{p+1} represent the diameters of the pth and $(p+1)$th main bright lines of wavelength λ, the order or fringes being counted from the first bright fringe starting from the emergence angle $i = 90°$, on either sides of the plate. Let d_p be the diameter of the pth faint line of wavelength $\lambda + d\lambda$. Then it can be shown that since angle made by the emergent rays with the plate is very very small, the spectral separation of the two lines is

$$d\lambda = \lambda^2[2t\sqrt{\mu^2 - 1}]^{-1} \frac{d_p^2 - D_p^2}{D_{p+1}^2 - D_p^2},$$

approximately. Hence $d\lambda$ can be calculated knowing λ, plate thickness t, its refractive index μ, d_p, D_p and D_{p+1}.

Resolving Power :

Considering Fig. 37, the width of the emergent wavefront entering the telescope objective either on the upper or on the lower side of the plate is $l \cos i$ where l is the effective length of the plate and i is the angle of emergence (angle of refraction on the emergence side). If $l \cos i$ be considered to be the aperture width, then from the theory of diffraction at a single slit, the angle δi between the central maximum and the first minimum of wavelength λ, *i.e.* the half the angular width, is given as, $\delta i = \lambda / l \cos i$ ... ... (1)

If t be the plate thickness and r, the angle of incidence of the rays within the plate, the path difference between successive emergent rays is $2\mu t \cos r$. Hence for a fringe formation,

$$2\mu t \cos r = n\lambda.$$

Squaring and rearranging, $4\mu^2 t^2(1-\sin^2 r)=n^2\lambda^2$

or, $4t^2(\mu^2-\sin^2 i)=n^2\lambda^2 \quad (\because \sin i=\mu \sin r)$ (2)

Differentiating we get,

$$4t^2\left(2\mu\frac{du}{d\lambda}-2\sin i\cos i\frac{di}{d\lambda}\right)=2n^2\lambda \tag{3}$$

Substituting $n^2\lambda^2$ from (2) and rearranging,

$$\frac{di}{d\lambda}=\left(\mu\lambda\frac{d\mu}{d\lambda}-\mu^2+\sin^2 i\right)\Big/\left(\lambda\sin i\cos i\right) \tag{4}$$

$\therefore$ From (1) and (4),

$$\frac{\lambda}{l\cos i}=\frac{\left(\mu\lambda\frac{d\mu}{d\lambda}-\mu^2+\sin^2 i\right)d\lambda}{\lambda\sin i\cos i}$$

$\therefore$ Resolving power $=\frac{\lambda}{d\lambda}=-\frac{l}{\lambda\sin i}\left[\mu^2-\sin^2 i-\mu\lambda\frac{d\mu}{d\lambda}\right]$

At grazing emergence, $\sin i=1$ and as the dispersion term introduces only a small correction, the expression for resolving power may be written as,

$$R.P.=-\frac{l}{\lambda}(\mu^2-1) \tag{5}$$

***44. Michelson's Stellar Interferometer** :—This instrument is useful in determining the diameter of a star as well as the distance between the two stars which are very close together.

The principle may be explained thus : Let us suppose that light from a distant star is incident on a telescope objective, the whole of which is stopped out except that passing through two slits at its two end. The star may be considered as a point object, producing a double slit interference pattern in the focal plane of the telescope. It can be shown that the angular separation between any two consecutive bright fringes is given by $\frac{\lambda}{d}$, where d *is* the slit separation. If now we have two such stars acting as point sources, each will give rise to its own system of fringes whose angular separation is $\frac{\lambda}{d}$. The two systems of fringes will evidently overlap. And by changing the slit separation d it is possible to obtain such a value for which the maxima of one set fall on the minima of the other. As a result the fringes disappear and we get a general illumination in the field of view. In this case the angular separation between the central

maxima of two systems of fringes is evidently $\frac{\lambda}{2d}$, which is also the angular separation of the stars, *i.e.*, the angle subtended by the stars at the telescope. Thus it is possible to measure the angular separation between the stars. For a single star it can be shown that the interference pattern will disappear when the angle subtended by the star at the telescope is $1{\cdot}22\,\frac{\lambda}{d}$. This enables one to determine the diameter of the stars. Since for stars this angle is very small, the value of d should be so great that it is not feasible to have such an adjustable slit in front of telescope objective. Michelson made a special device in which the two slits may be effectively placed beyond the aperture of the telescope objective.

The arrangement is shown in Fig. 38. Light from the distant star (or stars) is received by the mirrors M_4 and M_1, which are set at 45° ; from M_4 and M_1 light passes on to two other mirrors M_2 and M_3, which are fixed also at 45°. Light from M_2 and M_3 then passes straight into the telescope objective as two pencils. In this arrangement M_1 and M_4 serve as two apertures, the distance between which may be varied symmetrically. In the focal plane of the telescope, the interference pattern due to double slit is formed. By increasing the distance between M_1 and M_4 the fringes can be made to disappear. The separation between M_2 and M_3 is however kept constant.

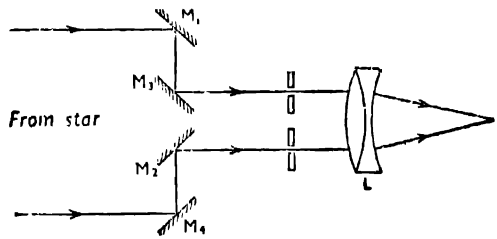


Fig. 38—Stellar Interferometer.

Michelson made his first observation on the bright star Betelgeuse with the 100 inch reflecting telescope at Mount Wilson observatory. He set the mirrors at a distance of 306·5 cm. when the fringes vanished. Taking the effective value of wavelength as 6750 Å, he obtained the angular diameter of the star as 0·047 sec. Using a known distance of the star its linear diameter came out to be 240 million miles.

Questions (Chapter II)

1. Under what conditions are interference effect observed in light? Explain the colour of thin films. (*cf.* Utk. U. 1966) (C. U. 1951 ; *cf.* C. U. '66)

2. (*a*) Describe a method of obtaining achromatic fringes. (Agra U. 1953)
(*b*) Discuss the formation of fringes by Lloyd's single mirror and explain why the central fringe is black. (Raj. U. 1953)

3. Describe an experiment to illustrate the interference of light and explain with necessary theory how from your observations you can determine the wavelength of light. (C. U. 1949)

4. Discuss briefly the phenomenon of interference in relation to the law of conservation of energy. (*cf* Del. U. 1967, '68)

Explain the method of finding the wavelength of monochromatic light by a biprism, giving the adjustments in detail. (C. U. 1952)

5. How would you obtain a beam of monochromatic light and what experiments would you perform in order to determine the wavelength of light? Describe one experiment in details. (C. U. 1947)

6. Discuss the phase change due to reflection of light from the surface of a denser and a rarer medium. (Bom. U. 1953 ; P. U. 1956)
Describe and explain the appearance of Newton's rings in reflected light and deduce an expression for the radius of a ring in terms of the wavelength of light used. (Bom. U. 1953)

7. Discuss the colours of thin transparent films. Give the theory of Newton's rings and show how from their study the wavelength of monochromatic light can be determined (C.U. 1954)

8. Give with necessary theory, Newton's rings method of determining the wavelength of monochromatic light (*cf.* Luck. U. 1951 ; Del. U. 1953 ; C.U. '63)

9. Explain the formation of colours in thin films and show that a film which appears bright in reflected light appears dark in transmitted light.
(Ban U. 1948 ; P.U. 1953, '56 ; Baroda U. 1953)

10. Give the theory of Newton's rings. Show how it can be used for determining the refractive index of water. (*cf.* P. U. 1948, '50 ; Luck. U. 1949)

11. Explain clearly the phenomenon of interference of light. Mention clearly the conditions under which two light beams can produce interference fringes.
(Agra U. 1946, '50 ; Bom. U 1947 ; All. U. 1946 ; *cf.* C. U. '68)

12. How do you account for the colours of thin films? A thin film of air is contained between two parallel faced glass plates, the distance and angle between which can be varied. How would you adjust the plates in order to obtain (*a*) straight (*b*) circular fringes? Describe with the help of diagrams the method of illumination and of viewing the fringes you would use.
(*cf.* Utk. U. '70S) (Gau. U. 1951, '54)

13. (*a*) How can the wavelength of monochromatic light be measured with the help of a Fresnel's biprism. Give the theory of the method and the arrangement of the apparatus. (*cf.* Utk. U. 1966) (Del. U. 1967, '69)
(P. U. Suppl. 1952, Pat. U. 1948 ; Utk. U. Supl. 1954 ; Agra U. 1946)

(*b*) A biprism is placed at a distance of 5 cm. in front of a narrow slit, illuminated by sodium light, and the virtual images of the slit formed by the biprism are ·05 cm. apart. Find the width of fringes formed on a screen 75 cms. in front of the biprism. ($\lambda = 5{\cdot}89 \times 10^{-5}$ cm.) [*Ans.* 0·094 cm.) (C. U. 1968)

14. Explain the effect of introducing a thin plate of glass in the path of one of the interfering beams. Show how this method is used for finding the refractive index and wavelength of light. (Bom U. 1947)

15. Describe how would you use Newton's rings to measure the wavelength of light? Give an outline of the necessary theory. (Bom. U. 1947)

16. Describe an experimental arrangement for producing Newton's rings by means of reflected sodium light and prove that the diameter of the dark rings are proportional to the square root of the natural numbers.
(Bom. U. 1953 ; Pat. U. 1942)

17. Describe the principle and construction of Michelson's interferometer. Also describe some of its applications. (P. U. 1946, '54)

18. Describe Jamin's interferometer. How will you find the refractive index of a gas with its help ?

19. Describe a Michelson interferometer. How would you used it to find the wavelength of (*a*) monochromatic light, (*b*) spectral line ?
(Luck. U 1950 ; Raj. U. 1951)

20. Describe Michelson's interferometer. How can it be used (*a*) to standardise distances, (*b*) to study the spectral structure ?
(Agra U. 1949 ; *cf.* Del. U. '68)

21. Discuss the formation of fringes by a Fabry-Perot interferometer.

22. Write short notes on :—

(a) Rayleigh refractometer ; *(b)* A stellar interferometer ; *(c)* Lummer-Gehrcke plate.

23. In Newton's ring experiment with violet light, the following data are obtained :

Radius of curvature of lower surface of the convex lens = 10 mm. ; radius of the nth dark ring = 4 mm. ; radius of $(n+5)$th dark ring = 6 mm.

Calculate the wavelength of the given light. (C. U. 1946)

[*Ans.* 4000×10^{-8} cm]

24. In a Newton's ring experiment, the diameters of two successive rings formed by light of wavelength 5893×10^{-8} cm. were obtained as 4 mm. and 4·2 mm. Calculate the radius of curvature of the lower surface of the lens.

[*Ans.* 69·6 m.]

25. The faces of a biprism are inclined at 2° with the base of the prism and the slit is 10 cm. from the prism. If λ for the light used be 5000×10^{-8} cm calculate the fringe width at a distance of 1 metre from the prism.

[*Ans.* 0·0172 cm]

26. A drop of oil of $\mu = 1{\cdot}58$ is introduced between a convex lens and a glass plate both of $\mu = 1{\cdot}50$. If the radius of curvature of the lower surface of the lens be 200 cm. and if the light used has wavelength 5890 Å.U., calculate the diameter of the 5th dark ring.

[*Ans.* 0·386 cm.]

27. The diameter of the 3rd and the 23rd bright ring in a Newton's rings experiment are respectively 0·181 cm. and 0·501 cm. If the lower surface of the lens has a radius of curvature of 50 cm. calculate the wavelength of the light used.

[*Ans.* 5456 Å. U.]

28. Monochromatic light of $\lambda = 5893$ Å. U. passes through a narrow slit and then through a biprism whose refracting edges are parallel to the slit. Find the spacing of the fringes formed in a plane 90 cm. from the biprism.

(Distance of the biprism from the slit is 10 cm. ; the refracting angle of the biprism is 1° ; and the refractive index of the glass of the biprism is 1·5)

[*Ans.* 0·338 mm.]

29. Write an essay on colour photography and tri-colour printing.

30. Describe a Michelson's interferometer and give the theory of the fringe formation. How the interferometer can be used to study the doublet structure of sodium *D*-line ?

31. Explain the principle of Fabry-Perot interferometer and discuss how it can be used to resolve the structure of spectra. (C. U. H. 1962)

DIFFRACTION

45. Diffraction of Light : When a beam of light passes through a narrow slit, or is incident on the sharp edge of an obstacle, it spreads out to some extent into the region of the geometrical shadow. This bending of light round an obstacle is the simplest example of *diffraction* of light. This bending is greater for the light of longer wavelengths and less for that of shorter wavelengths.* The phenomenon can be satisfactorily explained by assuming the wave character for light. According to Huygens' principle, each point on the wavefront is a source of secondary disturbance and the envelopes of such secondary waves form the subsequent wavefronts. If we draw the wavefront of a light wave passing through an aperture, then according to Huygens' constructions, the encroachment of light within the geometrical shadow of the aperture may be explained. Since the wavefront at any instant is the locus of all the particles which are in the same phase of vibration, the secondary sources on the wavefront might be termed as coherent sources. So these secondary waves should be able to produce interference effects. The interference of these secondary waves derived from the same wavefront gives rise to the so-called *diffraction fringes.* Diffraction fringes are therefore the effect of the superposition of different secondary waves coming from the different parts of the same wavefront, whereas interference is the effect of superposition of two waves coming from two distinct coherent sources. So in a sense Young's classical experiment on interference is really due to the diffraction effect.

46. Fresnel's treatment of the plane wavefront and its application to the rectilinear propagation of light :—On the basis of mutual interference of the secondary wavelets from the various points of a wavefront, Fresnel gave an explanation of the diffraction of light. The wavefront may be plane, spherical or cylindrical. Let us consider the simplest case of a plane wavefront and calculate its effect at an external point P (Fig. 39).

* It should be remembered in this connection that in sound waves, where wavelength is about 100 cm. for a frequency of 332, the effect is scarcely shown by small obstacles. If the obstacles be nearly a million times greater than that used for light waves, the effect becomes perceivable.

If *ABCD* represents a plane wavefront (Fig. 39) of monochromatic light travelling towards the right, every point on this plane may be supposed to be a source of secondary wavelets. Let us find out the resultant effect of these secondary wavelets at the point *P*. For this purpose, the whole wavefront is divided into a number of zones, called **Fresnel's half period zones** in the following manner. From *P* a perpendicular *PO* is drawn on the wavefront. The point

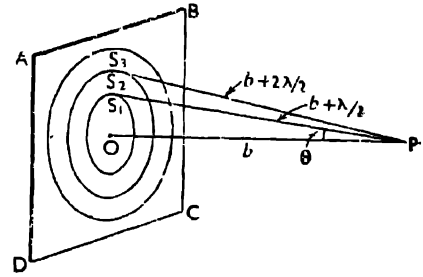


Fig. 39

O is called the *pole* of the wavefront with respect to *P*, Let *PO* be equal to *b*. With *P* as centre and radii $\left(b+\frac{\lambda}{2}\right), \left(b+\frac{2\lambda}{2}\right), \left(b+\frac{3\lambda}{2}\right)$, ...etc. spheres are drawn. The sections of these spheres by the plane of the wavefront *ABCD* will be circles S_1, S_2, S_3.....etc. with *O* as centre. Obviously the first circle will be at a distance $\left(b+\frac{\lambda}{2}\right)$, the second circle at $\left(b+\frac{2\lambda}{2}\right)$, and so on from the point *P*. The area enclosed by the first circle is called the first half period zone, the annular area between the second and first circles is called 2nd half period zone, and so on.

The area of the first half period zone

$$= \pi(OS_1)^2$$
$$= \pi\left\{\left(b+\frac{\lambda}{2}\right)^2 - b^2\right\}$$
$$= \pi b\lambda, \ [\lambda^2, \text{ being very small, is neglected}]$$

The area of the second half period zone

$$= \pi(OS_2)^2 - \pi(OS_1)^2$$
$$= \pi\left[\left(b+\frac{2\lambda}{2}\right) - b^2 - \left(b+\frac{\lambda}{2}\right)^2 + b^2\right]$$
$$= \pi[2b\lambda - b\lambda]$$
$$= \pi b\lambda, \ (\lambda^2 \text{ being neglected}).$$

Thus, the area of all zones are approximately equal The radius of the first zone is $OS_1 = \sqrt{b\lambda}$ and that of the second zone is, $OS_2 = \sqrt{\left(b+\frac{2\lambda}{2}\right)^2 - b^2} = \sqrt{2b\lambda}$. Similarly that of the *n*th zone is $\sqrt{nb\lambda}$ approximately. Thus the outer radii of the zones are proportional to the square root of their zone numbers.

If we consider a spherical wavefront whose radius of curvature is a, then it can be shown that the area of nth zone is given by,

$$A_n = \frac{\pi a}{a+b}\left[b\lambda + \frac{(2n-1)\lambda^2}{4}\right]. \qquad \text{(See Note below)}$$

For a plane wavefront, a is very large compared to b. Therefore,

$$A_n = \pi\left[b\lambda + \frac{(2n-1)\lambda^2}{4}\right] \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

Thus the areas of all zones are not strictly speaking equal to $\pi b\lambda$; they depend upon the zone number. The areas will increase slowly with n.

According to Huygens' principle we may regard every point on the wavefront as sending out secondary disturbances in the same phase. But these secondary disturbances will reach P with different phases, because they travel different distances. Since each zone is on an average $\lambda/2$ further from P, it is evident that the successive zones will send out waves which will reach P in opposite phases. Let d_1, d_2, d_3,.....etc. be the amplitudes at P due to the wavelets coming from the first, second, third etc. half-period zones respectively. The resultant amplitude at P will then be given by,

$$D = d_1 - d_2 + d_3 - \quad \ldots \quad \ldots \quad + (-1)^{n-1} d_n \ldots \quad (2)$$

considering that there are n such zones in the wavefront.

In order to evaluate D in the expression (2), three factors are to be considered : (1) The slight increase of the area of the zones with the zone number ; (2) The inverse variation of the amplitude with the average distance b_n of the zone ; (3) The decrease in amplitude with the obliquity or the inclination θ (Fig. 39) of the zone.

The obliquity factor has been shown to be equal to $(1+\cos\theta)$ by Kirchhoff. Thus the amplitude due to the nth zone may be written as,

$$d_n = \text{const.} \times \frac{A_n}{b_n}(1+\cos\theta) \quad \ldots \quad \ldots \quad (3)$$

The first two factors neutralise each other and the influence of the obliquity factor gradually diminishes the amplitudes as the zones recede from the pole. So the contributions of the distant zones at the point P diminishes with increase of θ. Remembering these facts we can now evaluate D in the following way.

Taking n to be odd, series (2) may be written as

$$D = \frac{d_1}{2} + \left[\frac{d_1}{2} - d_2 + \frac{d_3}{2}\right] + \left[\frac{d_3}{2} - d_4 + \frac{d_5}{2}\right] + \ldots\ldots\ldots + \frac{d_n}{2} \quad \ldots \quad (4)$$

But $\frac{d_1}{2}+\frac{d_2}{2}=d_2$, $\frac{d_3}{2}+\frac{d_5}{2}=d_4$, and so on because the contributions of successive zones are approximately equal and opposite.

$$\therefore \quad D=\frac{d_1}{2}+\frac{d_n}{2}.$$

If n is even, with a similar argument the value of the series becomes

$$D=\frac{d_1}{2}+\frac{d_{n-1}}{2}-d_n$$

If we allow n to become large enough, the obliquity factor causes, d_n and d_{n-1} to become negligible and the resultant amplitude due to whole wavefront is just half that due to the first half period zone alone, *i.e.*

$$D=\frac{d_1}{2}.$$

[Note :—Area of a Half period zone in a spherical wavefront.

Let the divergent rays from a point source reach the spherical wavefront $ABCD$ at an instant. Let P be the point of observation and O the pole of the wavefront where $OP=b$. With P as centre and radii $b+\lambda/2$, $b+2\lambda/2$, $b+3\lambda/2$, etc., spheres are drawn which cut the wavefront in circles and the half period zones, OM_1, OM_2, OM_3, etc. are obtained as shown in Fig. 39(a). To evaluate the area of the nth such zone, let us refer to Fig. 39(b), where S is point source and a, the radius of the spherical wavefront. Let the outer edge of the nth zone (where $PM_n=b+n\lambda/2$) subtend a semi-vertical angle θ_n at S. The solid angle subtended by the whole spherical cap M_nOM_n at S is

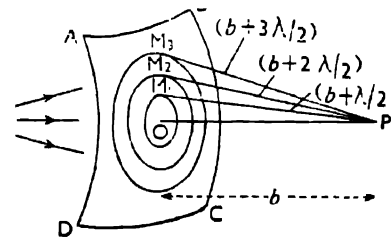


Fig. 39(a)

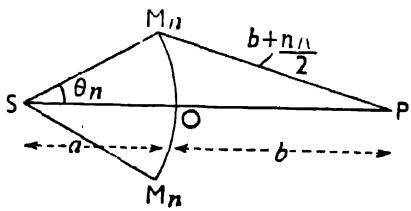


Fig. 39 (b)

$$\omega=\frac{\text{normal area}}{a^2}=\frac{A}{a^2}$$

$=2\pi(1-\cos\theta_n)$ (vide College Physics, vol. II, Magnetism, Art. 38)

Here A represents the total area of all the n zones in the cap M_nOM_n. $\therefore \quad A = 2\pi a^2(1-\cos\theta_n) \quad \ldots \quad \ldots \quad (1)$

Now in $\triangle SM_nP$, $a^2+(a+b)^2-2a(a+b)\cos\theta_n = \left(b+\frac{n\lambda}{2}\right)^2$

$$\therefore \quad \cos\theta_n = \frac{(a+b)^2+a^2-(b+n\lambda/2)^2}{2a(a+b)}$$

From (1), area of n zones

$$= 2\pi a^2 - \frac{\pi a}{a+b}\left[2a^2+2ab-bn\lambda-\frac{n^2\lambda^2}{4}\right]$$

and area of $(n-1)$ zones

$$= 2\pi a^2 - \frac{\pi a}{a+b}\left[2a^2+2ab-b(n-1)\lambda-\frac{(n-1)^2\lambda^2}{4}\right]$$

Hence the area of the nth zone

$$= A_n = \frac{\pi a}{a+b}\left\{b\lambda+\frac{(2n-1)\lambda^2}{4}\right\}$$

Explanation of the Rectilinear propagation of light.—From the above considerations it is clear that light reaching P may be considered to be coming from a portion whose area is half that of the first half period zone. Since the wavelength λ of light is small, the area $\pi b\lambda$ of the first half period of zone is very small. Only half of this area is thus responsible for producing intensity at P. If now an opaque obstacle be placed in the path of the light ray, so that it covers the first half period zone, the resultant amplitude at P will be, $\frac{d_2}{2}$, and the intensity proportional to $\left(\frac{d_2}{2}\right)^2$. If now the size of the obstacle be increased gradually, so as to cover the first two, three and four half period zones, the resultant intensity at P will be proportional to $\left(\frac{d_3}{2}\right)^2$, $\left(\frac{d_4}{2}\right)^2$ and $\left(\frac{d_5}{2}\right)^2$ respectively, As d_1, d_2, d_3, etc. are in descending order of magnitude, the resultant intensity at P will be gradually diminished. When the size of the obstacle is sufficiently large so as to cover a large number of half period zones, the point P will be almost dark. Since a small obstacle is sufficient to cover a considerable number of half period zones, any point beyond that obstacle will remain dark. This explains the approximate rectilinear propagation of light.

Thus Fresnel's modification of Huygens' principle accounts for the rectilinear propagation of light. It shows that the law is only approximate having its limitation due to the short wavelength of light. If the wavelength is large, *e.g.* sound or wireless waves, the bending round the obstacle is very marked even for obstacles of ordinary size, whereas for light waves it is predominant only for obstacles of very small size.

Example

Calculate the radius of the first and the 36th half period zones of a plane wave-front for a point 20 cm. ahead of it taking wavelength as 5000×10^{-8} cm.

Ans. : Radius of the 1st zone $= \sqrt{b\lambda} = \sqrt{20 \times 5000 \times 10^{-8}} = 0.0316$ cm.

Radius of the 36th zone $= \sqrt{36b\lambda} = 0.1896$ cm.

47. Zone Plate :—It is a practical application of the theory of Fresnel's half period zones. According to construction of half period zones, light from any two successive zones reaches the point under consideration in opposite phases. As a result, the intensity is greatly diminished. If then the alternate zones are made opaque, either the odd or the even members will be effective and the waves reaching any point P will be reinforced producing intense illumination. A transparent plate made on this principle is known as a **zone plate.**

Let O be a point source of light in the plane of the paper and AS_n an imaginary plane perpendicular to the plane of the paper. Let us find out the amplitude at a point P on the other side of this plane lying on the line OAP normal to AS_n. Let $OA = u$ and $AP = v$. Let us take points S_1, S_2, S_3 etc. on the plane so that

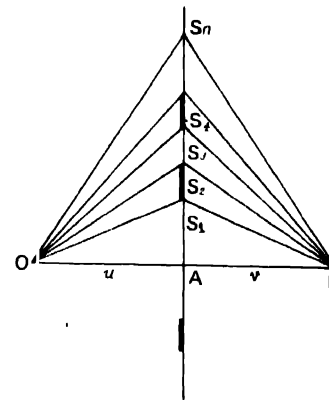


Fig. 40

$$OA + AP + \frac{\lambda}{2} = OS_1 + S_1P;$$

$$OA + AP + \frac{2\lambda}{2} = OS_2 + S_2P;$$

... ...

$$OA + AP + \frac{n\lambda}{2} = OS_n + S_nP.$$

Then, $$OS_n + S_nP = u + v + \frac{n\lambda}{2} \qquad (1)$$

Then area of the circle with radius AS_1 is the first half period zone, and the annular zone between the circles of radii AS_1 and AS_2 is the 2nd half period zone and so on. Let $AS_1 = r_1$ and $AS_2 = r_2, \ldots\ldots, AS_n = r_n$.

Then, $OS_n^2 = u^2 + r_n^2$; or, $$OS_n = u\left[1 + \frac{r_n^2}{2u^2}\right] \text{ approx.} \quad \cdots (2)$$

$$PS_n^2 = v^2 + r_n^2 \text{ ; or, } PS_n = v\left[1 + \frac{r_n^2}{2v^2}\right] \text{ approx.} \quad \cdots (3)$$

Therefore, from equations (1), (2) and (3), we have,

$$\frac{r_n^2}{2u} + \frac{r_n^2}{2v} = \frac{n\lambda}{2};$$

$$\text{or,} \qquad r_n^2 = \frac{uv}{u+v} n\lambda. \qquad \cdots \qquad \cdots \qquad (4)$$

Putting $n = 1, 2, 3$ *etc.*, we get the values or r_1, r_2, r_3 *etc.* Thus the radii of the half period zones are proportional to the square root of the natural numbers (or zone numbers).

Now if on a transparent plate circles are drawn with radii proportional to the square root of the natural numbers, 1, 2, 3,... etc. and the alternate annular zones are blackened, such a plate is called a *zone plate*. In Fig. 41 (*a*) & (*b*) two such zone plates are shown, one with its central zone opaque and the other with its central zone transparent. The zone plate so constructed behaves as *convergent lens*. This may be shown as below.

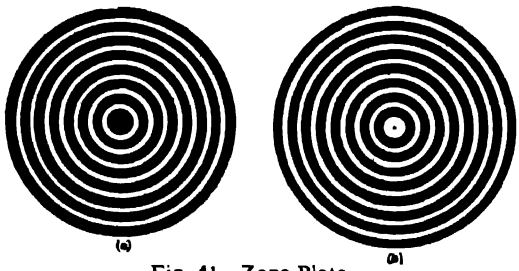


Fig. 41—Zone Plate.

If the second, fourth, sixth etc. zones are intercepted, the resultant amplitude at P will be given by, $D = d_1 + d_3 + d_5 + \cdots$

This is many times greater than $\frac{d_1}{2}$, which is the resultant amplitude due to the wavelets from all the zones when none of them is made opaque. The point P will be, as a result, a point of maximum illumination. The behaviour of the zone plate is thus similar to a convex lens. From eqn. (4), the relation between u and v is given by,

$\frac{1}{v} + \frac{1}{u} = \frac{n\lambda}{r_n^2}$. This result is similar to that of a convex lens, for which $f = \frac{r_n^2}{n\lambda}$. Thus a zone plate acts a convergent lens with *multiple foci* for any particular wavelength of light depending upon the value of n. When $n = 1$, $r_n = r_1 = a_1$ (say), and $f_1 = \frac{a_1^2}{\lambda}$. For a fixed position of the source the number of half period elements contained in an

annulus of the zone plate depends on the position of P from which the effect is observed. If P is brought near the plate AS_1 such that each of the transparent annulus contains three half period zones, then again a maximum illumination is obtained. Here $n=3$ and r_3 of the new half period zone is identical with a_1 above. Hence $f_2=\frac{a_1^2}{3\lambda}=f_1/3$.

Similarly, when the positions of P are such that 5, 7, 9, etc. half period elements are contained in each transparent ring, maximum illumination will also occur. So the positions of the different foci are given by

$$f_1=\frac{a_1^2}{\lambda},\quad f_2=\frac{a_1^2}{3\lambda},\quad f_3=\frac{a_1^2}{5\lambda}\text{ and so on.}$$

The intensity at these secondary foci, however, decreases as their distances from the zone plate decrease.

There is, however, some differences between a zone plate and a converging lens. Light from consecutive zones reaches the point P a complete period later. For a converging lens, if white light is used, violet light comes to the focus nearest to the lens, whereas, for a zone plate red light, having the longest wavelength comes to a focus at the nearest point since f of a zone plate is inversely proportional to the wavelength. It can also be shown that a zone plate may be used as a multiple foci *divergent lens.*

In Fig. 41(*a*) NMO represents a zone plate whose pole is at O and whose opaque zones are represented by the thick lines and the transparent zones by the intervening gaps. If parallel rays are incident on the zone plate, on emergence they will be diffracted in all directions. Some of these diffracted rays converge to the point P on the axis where $OP=f_1=r_1^2/\lambda$. Due to diffraction at these annular apertures some of the rays again proceed in the directions like MR, NS, etc. such that they make equal angles with the direction of the incident ray as do the rays like MP, NP etc. respectively, but on the opposite side of the direction of incidence. So when an eye is placed so as to receive rays like MR and NS, these rays appear to come from a point Q, such that $OP=OQ=f_1$. So the zone plate also behaves as a concave lens and Q is the first focus of this

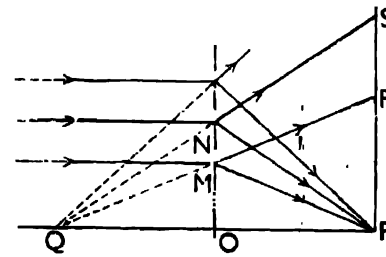


Fig. 41(*a*)

concave lens. Similarly corresponding to other foci of the zone plate used as a convergent lens, there will be multiple foci of the divergent lens too.

48. Cylindrical Wavefront—*Cornu's spiral :—Let us consider a linear source in the form of a long narrow slit illuminated by monochromatic light. Each point of this slit will give rise to a spherical wavefront and the envelope of all these spherical wavefronts originating from the different points of the slit will form a cylindrical surface whose axis is the linear slit. In Fig. 42, $C_1\ C_2\ C_3\ C_4$, represents the cylindrical envelope of the different spherical wavefronts like $C_1\ C_4$ and $C_2\ C_3$ originating from the points S_1 and S_2 of she slit. Here $C_1C_2C_3C_4$ actually forms the new wavefront of the light waves emitted by the linear source. It is obviously a *cylindrical wavefront.*

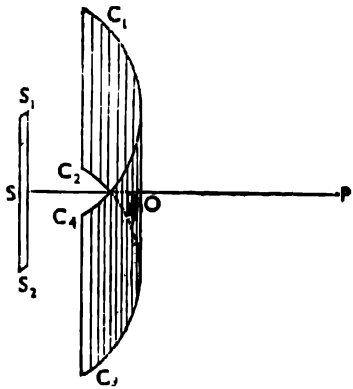


Fig. 42—Cylindrical wavefront

This cylindrical wavefront may be divided into half period strips with respect to the point of observation P lying the plane of the paper. Let O be the point of intersection of the cylindrical

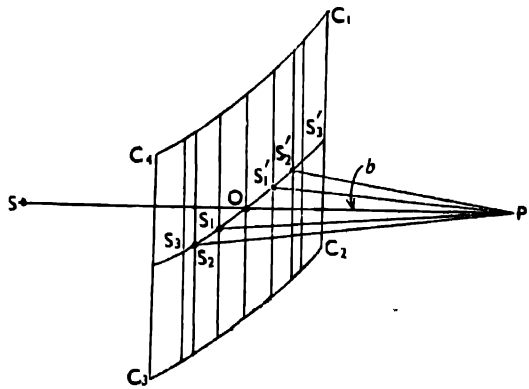


Fig. 43—Half Period strips.

wavefront with the line joining P and its corresponding point S in the slit. If S_1, S_2.....S'_1, and S'_2,.....be the positions

of the boundaries of the half period strips (Fig. 43) in a plane at right angles to the plane of the paper, then,

$$PS_1 = PS'_1 = b + \frac{\lambda}{2} \text{ (where } b = OP),$$

$$PS_2 = PS'_2 = b + \frac{2\lambda}{2} = b + \lambda, \text{ and so on.}$$

The half period strips in a cylindrical wavefront are not equal in area like the half period zones of a plane or a spherical wavefront. Their areas are proportional to their widths which decrease rapidly as the strip recedes from the pole O as is shown in Fig. 43.

The effect of these half period strips of a cylindrical wavefront can be represented by a curve of spiral form as shown in Fig. 44. which was first drawn by Cornu and is known as the Cornu's spiral. In Cornu's spiral the effect of each half period strip is represented by a half turn of the spiral. If the action of the whole wavefront is considered, the spiral will make an infinite number of turns finally subsiding at the centres J and J' of the asymptotic circles. OJ represents the effect (*i.e.* the amplitude contribution) of one half of the wavefront and OJ' that of the other half while JJ' indicates the effect of the complete wavefront. The intensity at P due to strips S_1 and S'_1 on the two sides of the pole O is represented by the square of the line joining S_1 and S'_1. Similarly the effect due to the second, third or fourth strip on one side of the pole will be obtained by joining the ends of the second, third or fourth half turn of the spiral to O. By means of this spiral, it is possible to plot graphically the distribution of intensity in the diffraction patterns formed by cylindrical wavefronts.

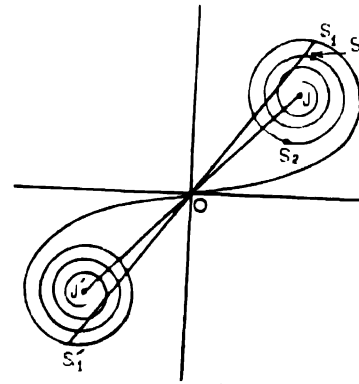


Fig. 44—Cornu's Spiral

49. Classification of Diffraction Phenomena :—Diffraction phenomena are classified in two distinct groups : (A) Fresnel diffraction, and (B) Fraunhofer diffraction. In Fresnel diffraction phenomena, either the source or the point of observation or both should lie at finite distances from the aperture causing diffraction. The phenomena can be very easily observed with naked eye or a magnifier may be used to advantage. The wavefronts producing the diffraction phenomena are divergent in nature and as such

the mathematical treatment involved for explanation is rather complex. It was first investigated by Fresnel.*

In Fraunhofer diffraction phenomena, the source of light and the screen on which the pattern is observed are effectively at infinite distances from the aperture causing the diffraction. It may be observed in practice by rendering light from a source parallel with the help of a lens and focussing it on a screen with another lens placed behind the aperture. This arrangement effectively removes the source and the screen to infinity. This class of diffraction is much simpler to treat mathematically.

A. Fresnel Diffraction

50. Diffraction by a straight edge :—AB is an opaque obstacle in the form of a knife edge or any straight edge in general placed in front of a narrow slit S placed at right angles to the plane of the paper and illuminated by monochromatic light of wavelength λ [Fig. 45]. POQ is the screen placed in front of the obstacle. Due to the line source at S, a cylindrical wavefront CD will be generated. This wavefront will be obstructed by the straight edge AB, and the secondary wavelets coming out of this primary wavefront will give rise to the diffraction effect. On the screen PQ, the following phenomena will be observed.

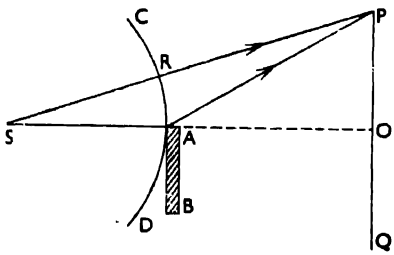


Fig. 45

(i) Above the line AO there will form alternate bands of darkness and brightness which will be unequally spaced. As we proceed more and more away from O, the bands become closer to each other and ultimately they will vanish leading to a general illumination.

(ii) Below the geometrical shadow bounded by line SAO, the illumination will vanish gradually but rapidly.

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* Fresnel's theory though very successful in explaining simple diffraction phenomenon can not explain why the secondary sources do not radiate backwards. It can neither yield the correct phase for the disturbance reaching a particular point. These difficulties have been avoided by Kirchhoff in his analytical formulation of Huygens' principle. As given in Art. 46, eqn. (3), the amplitude contribution at any point due to an arbitrary zone (nth) of a wavefront is $d_n = \text{const.} \times \frac{A_n}{b_n}(1+\cos\theta)$. So in the backward direction θ being 180°, the factor $(1+\cos\theta)$ vanishes. So the intensity contribution in the backward direction due to secondary wavelets would be zero. For Kirchhoff's treatment students may consult an advanced treatise on light.

Let us consider a point P on the screen. Join SP. It meets the cylindrical wavefront CD at R. So the point R is the pole of the wavefront with respect of the point P. Similarly for the point O on the screen, A is the pole of the wavefront. For the point O of the screen, the pole of the wave being at A, one half of the wavefront is exposed. The resultant amplitude at O will, therefore, be half the amplitude due to an unobstructed wavefront, *i.e.* $d_1/2$. As we consider any point below O, the pole of the wavefront corresponding to that point goes below A, and gradually, the first, the first two, the first three, etc. half period elements of the exposed wavefronts are obstructed. As a result, the resultant amplitudes at various points would be $\frac{d_2}{2}$, $\frac{d_3}{2}$, $\frac{d_4}{2}$, etc. As d_1, d_2, d_3...etc. are in descending order of magnitude, the intensities at various places as we go below O rapidly fall. Here d_1, d_2, d_3 etc. are the amplitudes at O due to secondary wavelets reaching O from the 1st, 2nd, 3rd, etc. half period strips on either side of the central line through A in the unobstructed wavefront DC.

As we go above O along the line OP, the pole travels from A towards C. As a result, one after another, the first, the first two, the first three half period elements of the lower half of the wavefront will be exposed. Thus if we consider any point P the illumination at P will be due to the upper half of the wavefront above R, together with half period elements, present within AR. If AR contains an even number of half period elements, their effects will be mutually cancelled. But if it contains an odd number of half period elements, then even after mutual cancellation in pairs, the resultant amplitude and hence the intensity will be greater. Thus above O we get alternate brightness and darkness.

In order to determine the positions of the fringes, let us denote SA by a, AO by b and OP by x. We have then,

$$SP^2 = (a+b)^2 + x^2$$

$$= (a+b)^2\left[1 + \frac{x^2}{(a+b)^2}\right],$$

So, $SP = (a+b)\left[1 + \frac{x^2}{2(a+b)^2}\right] = a + b + \frac{x^2}{2(a+b)}$

But $SP = a + RP$. $\therefore$ $RP = b + \frac{x^2}{2(a+b)}$.

Now, $AP^2 = b^2 + x^2$; or, $AP = b + \frac{x^2}{2b}$.

For maximum brightness at P.

$AP - RP = (2n+1)\lambda/2$;

or, $b + \frac{x^2}{2b} - b - \frac{x^2}{2(a+b)} = (2n+1)\lambda/2$;

or, $x = \sqrt{\frac{b(b+a)}{a}(2n+1)\lambda}$, **for maxima.**

Similarly, $x = \sqrt{\frac{b(b+a)}{a} 2n\lambda}$, for minima.

These equations show that the spacing of the dark and bright bands decreases with the increase of the order number of the bands. Fig. 46 shows the intensity distribution at the boundary of a straight edge against the distance from the edge.

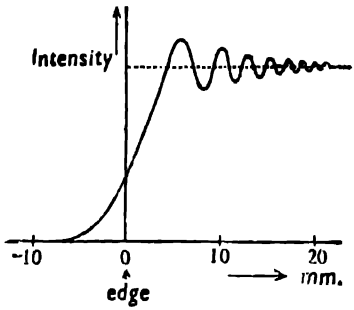


Fig. 46

The figure shows that the intensity falls to zero within a short distance on the negative side of the edge. It sharply rises and becomes maximum within a short distance on the positive side. The intensity curve shows a number of maxima and minima and ultimately becomes parallel to the abscissa. The intensities of these minima are never zero, owing to the general background illumination caused by one half of the unobstructed wavefront.

51. Diffraction by a narrow obstacle :—A narrow obstacle here means a fine wire, a human hair, or any other thing whose length is large compared to its finite but small width. The shaded strip *AB* in Fig. 47 indicates the diameter of the obstacle. Light from a fine slit *S* perpendicular to the plane of the paper is diffracted by this narrow obstacle and falls on the screen, *MON*, on which we want to observe the diffraction pattern. *MN* is the geometrical shadow of the obstacle. Outside the geometrical shadow there are diffraction patterns on each side, which are similar to those formed outside the geometrical shadow of a straight edge. These fringes are called *the exterior fringes.*

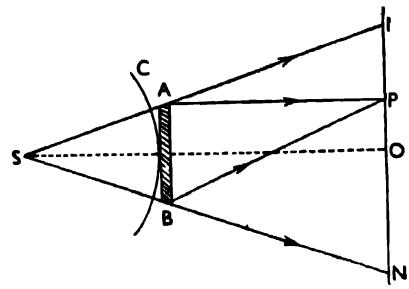


Fig. 47

Within the geometrical shadow *MN*, another system of fringes known as *the interior fringes* are also formed. These are due to interference of the light waves entering into the region of the shadow round the two parallel sides of the obstacle. The two extremities of the diameter, being situated on the same wavefront *CD*, act as two coherent sources. Thus the fringe at any point *P* within

the shadow MN, may be dark or bright according as the path difference of the distances from the two ends of the diameter is an odd or an even multiple of $\lambda/2$. If x be the distance of the point of observation P from the centre O of the geometrical shadow MN and if the diameter of the obstacle be a and the distance of the screen from the obstacle b, then the path difference between BP and AP is given by ax

Hence for a bright fringe $x = \frac{v}{a}n\lambda$ and for a dark fringe, $x = \frac{b}{a}(2n+1)\frac{\lambda}{2}$. The fringe width is evidently equal to $\frac{b\lambda}{a}$. Hence these fringes will be equidistant like the ordinary interference fringes formed by biprism. The central fringe located at O will be obviously bright, though it should be darkest from the geometrical consideration. If the obstacle is of large width, only the exterior fringes are visible. Because light rays from the two extremities of the diameter cannot bend to such an extent as to overlap, the interior interference fringes are not possible here.

It is evident from above that from the observation of the fringe width of the interior pattern, the diameter of a wire or that of a human hair producing the diffraction pattern can be determined.

52. Diffraction by a circular obstacle :—A point source of light S is placed in front of a circular obstacle AB, *i.e.* an opaque circular disc. Light after diffraction by the circular disc is received on a screen MN placed perpendicularly to the axial line. Here Fresnel's method is to be applied to find the diffraction effect. Let the spherical wavefront originating from S be divided into a number of Fresnel zones with respect to the point O. The resultant displacement at O will be half that due to the first of the series of unobstructed zones. Hence there can be no darkness along the central line of the shadow. The result is apparently paradoxical. This effect can be easily understood if we suppose that light bends uniformly round all the sides of the circular obstacle and meets the axial line in the same phase. When the point O is very near to the obstacle, the obliquity factor reduces the illumination to a great extent. If the point of observation is off the axis, the diffracted waves meet the point with a phase difference and as such interference occurs. Hence within the region of geometrical shadow interference rings appear. Outside the region

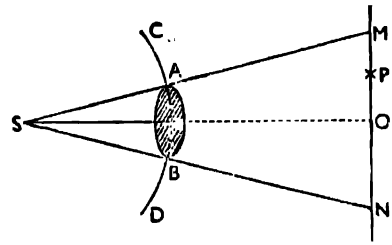


Fig. 48—Circular Obstacle

of the geometrical shadow, circular exterior fringes bordering the outside of the shadow are also found to appear.

53. Diffraction by a circular Aperture :—When there is a circular opening *AB* between a point source of light *S* and a point *O* on the screen *MN* (Fig. 49), then the intensity at *O* will depend upon the size of the opening as well as the distance between the opening and the screen. As usual we shall divide the circular opening into a large number of Fresnel's half period zones and the resultant effect at *O* will be the sum of the contributions of all such zones. If the aperture contains as even number of zones, the intensity at *O* will be minimum, due to mutual cancellation in pairs of the light waves coming from the successive zones. If the aperture contains an odd number of zones, then even after mutual cancellation in pairs, the resultant intensity will be greater. Thus the intensity at *O* will be *maximum or minimum according as the opening includes odd or even number of zones.* If we consider any point off the axis, the diffracted waves produce darkness or brightness depending upon the phase difference. Hence the diffraction pattern caused by a circular aperture consists of a central spot, bright or dark depending upon the size of the aperture and the position of the screen, surrounded by a number of alternate dark and bright rings.

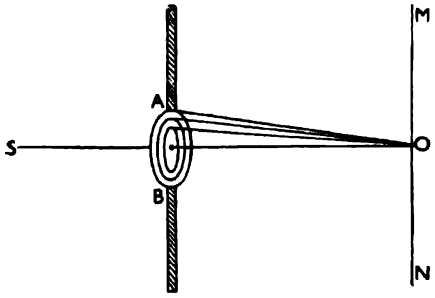


Fig. 49

54. Fresnel Diffraction by a single slit :—Here *AB* represents the width of the diffracting slit in a plane perpendicular to the plane of the paper. It is illuminated by monochromatic light coming through a narrow slit *S* parallel to the diffracting slit. The screen or the plane of observation *MN* is at a finite distance from *S* and as such Fresnel diffraction takes place. In Fig. 50, *CD* represents the cylindrical wavefront originating from *S*. Light diffracted through the slit of width *AB* is received on the screen *MN*.

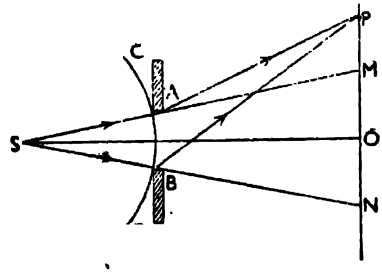


Fig. 50

The cylindrical wavefront at *AB* can be divided into half period elements with respect to the point of observation on the screen. The resultant effect of these half period strips at that

point is then to be calculated. The region MN where light directly reaches without diffraction is almost uniformly illuminated when AB is narrow and no *interior* fringes are observed in this region. But for a point P outside the geometrical boundary one half of the wavefront is cut off and there will be maximum or minimum illumination according as AB contains an odd or even number of half period strips. So beyond the geometrical boundary diffraction fringes of gradually decreasing intensity will be observed. The number of half period elements contained in AB depends on the path difference $\delta = BP - AP$.

$\therefore$ For maximum intensity, $\delta = (2n+1)\frac{\lambda}{2}$,

and for minimum intensity, $\delta = n\lambda$

If the slit width $AB = a$ and the distance of P from the centre O of the screen be $OP = x$, then,

for bright bands, $x = \frac{b}{a}(2n+1)\frac{\lambda}{2}$,

and for dark bands, $x = \frac{b}{a}n\lambda$,

where b is the distance of O from the slit AB and n is any integral number.

The exterior fringes will be equispaced and independent of the position of the source S.

As stated above, when the slit is sufficiently narrow ($a = 0{\cdot}5$ *mm.* nearly), the region MN is uniformly illuminated and no fringe is formed in that region. But when the slit width is increased to about 1·8 *mm.*, over and above the exterior fringes, interior diffraction fringes also appear in the central region of MN. But when the width is still further increased to about 4 *mm.*, the diffraction pattern is the resultant of the patterns due to two straight edges facing each other. Here the fringes are limited in a small region near O and the exterior fringes outside the geometrical boundary disappear.

***54. (a) Babinet's Principle :—**

The principle states that two complementary diffraction screens (i.e., screens in which the transparent portions of one occupy the same positions as the opaque portions of the other) produce the same illumination due to diffraction at a point which would show zero illumination if both the screens were absent, i.e. if no diffracted light were allowed to reach the point. This becomes apparent if we consider the wave disturbances ϕ_1 and ϕ_2 due to two complementary diffraction screens at the same point. If both the screens are simultaneously present, the resultant disturbance of the point will then be $\phi_1 + \phi_2$. But the superposition of two complementary screens

indicates that no diffracted rays would reach the point. This is as good as absence of any diffraction screen there at all.

$$\therefore \quad \phi_1 + \phi_2 = 0$$
$$\text{or,} \quad |\phi_1|^2 = |\phi_2|^2.$$

Hence the intensity will be identical with each of the diffraction screens. Since the simultaneous placing of the two screens or absence of any should produce zero illumination at the point, such a point must always be outside the geometrical projection of the aperture on the screen and sufficiently apart from this projection so that no light can normally reach there directly or by diffraction.

B. Fraunhofer Diffraction

55. Fraunhofer diffraction due to a single slit :—A single slit is set up vertically on a spectrometer table having its systems focussed for parallel rays. The opening of the slit is made exactly parallel to the slit of the collimator. Fig. 51 shows a cross-section of the slit by a plane at right angles to its length. A plane wavefront *i.e.* a beam of parallel rays, after emergence from the collimator is incident normally on the single slit of width a. The rays are then diffracted in all directions and are focussed on the focal plane of the convergent system of lens L. The direct rays are converged to the point O known as the *principal* maximum. All the direct rays start from the slit in the same phase and are therefore reinforced to produce brightness at O.* Let us consider

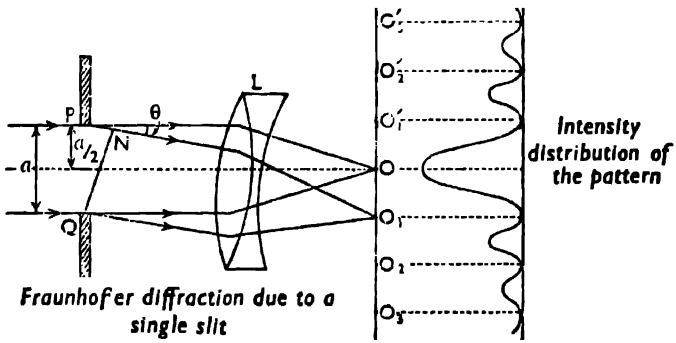


Fig. 51

any other set of diffracted rays, which are diffracted at angle θ and are brought to a focus at O_1, lying in the focal plane of the lens system L. The point O_1 will be a maxima or a minima depending on the phase difference of the extreme rays. Their path difference δ is equal to PN, shown in Fig. 51.

* Note :—In Fraunhofer diffraction the central band is always bright. But in Fresnel diffraction this band may be dark as well depending on the number of half period elements in the slit.

$\therefore \quad \delta = a \sin \theta \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$

Therefore the phase difference,

$$\phi = \frac{2\pi}{\lambda} \times \delta = \frac{2\pi}{\lambda} \cdot a \sin \theta \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

If in Fig. 52, $PN = \delta = \lambda$ (say), then the rays from the corresponding point (A and B) separated by a distance $\frac{a}{2}$ in the two halves of the slit will have a path difference $\lambda/2$ and will produce the first minimum at O_1.

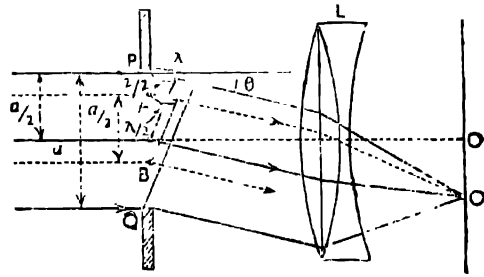


Fig. 52

Hence the first minimum on either side of O will occur in the direction for which the path difference of the extreme rays is $\delta = \lambda$; or, $\lambda = a \sin \theta$; or, $\sin \theta = \frac{\lambda}{a}$. Hence, $a \sin \theta_n = n\lambda$, gives the direction of the nth minimum, where n is an integer. For $n = 1$, 2, 3, etc. we shall get the corresponding directions of 1st, 2nd, 3rd,...minima on either side of the principal maximum. Since a (about 1 mm.) is usually very large compared to $\lambda (\approx 5000 \times 10^{-8}$ cm.), the value of $\sin \theta$ and hence θ is very small. Therefore $\sin \theta$ may be written as equal to θ. The condition for minimum in the direction of θ thus reduces to $a\theta = n\lambda$. Therefore we see that the minimum are equispaced.

Besides the principal maximum at O, there are other maxima known as secondary maxima which are formed between any two minima. The secondary maxima will be at those points for which the phase difference between the light waves coming from the corresponding points separated by a distance $a/2$ in the two halves of the slit, *i.e.* $\frac{\pi a \sin \theta}{\lambda}$ is given by 0, $1{\cdot}43\pi$, $2{\cdot}46\pi$, $3{\cdot}47\pi$,.....etc. The condition for maxima as obtained on rigorous calculation shows that the maxima are not situated in the middle of any two consecutive minima. The intensities of these secondary maxima will be much smaller than that of the primary one formed at the centre, the first secondary maximum having an intensity which is only $\frac{1}{23}$ of the intensity of the principal maximum. The intensity will diminish rapidly with order number and will not be practically perceptible after a few orders on either side of the principal maximum. A photograph of the nature of fringes formed by a single slit is shown in **Plate II (c)**.

***55(a). Intensity distribution in the case of a single slit :—**

To find out an expression for intensity distribution at a point in the diffraction pattern due to single slit of width a, let us consider the slit-width to be divided into a very large number, m, of identical elements each of width $\frac{a}{m}$. If we consider the diffraction effect in a direction making angle θ with the incident rays (supposed to have fallen normally on the slit) then the total path difference between the top-most ray (at P) and the lower-most ray (at Q) in Fig. 51 will be $a \sin\theta$ and the corresponding phase difference is $\frac{2\pi a \sin\theta}{\lambda} = 2\phi$ (say).

Let the wave disturbance reaching the point of observation Q_1 (Fig. 51) from the first elementary strip be represented by $r\cos pt = re^{ipt}$ (real part only) p being the pulsatance of the waves. The subsequent elements would also send identical waves to O_1, which would only differ in phase. This phase difference between any two adjacent slits will be $\delta = 2\phi/m$. Hence considering m elementary strips, the resultant wave disturbance at O_1 is given by,

$$re^{ipt} + re^{i(pt-\delta)} + re^{i(pt-2\delta)} + \cdots\cdots \text{to } m \text{ terms}$$

$$= re^{ipt}\cdot[1 + e^{-i\delta} + e^{-i2\delta} + e^{-i3\delta} + \cdots \text{ to } m \text{ terms}]$$

$$= re^{ipt}\cdot\frac{1-e^{-im\delta}}{1-e^{-i\delta}}, \quad [\because (m-1)\approx m]$$

So the complex amplitude of the disturbance

$$= r\cdot\frac{1-e^{-im\delta}}{1-e^{-i\delta}},$$

and the resultant intensity at the point of observation is,

$$I = r^2\left(\frac{1-e^{-im\delta}}{1-e^{-i\delta}}\right)\left(\frac{1-e^{im\delta}}{1-e^{i\delta}}\right) = r^2\frac{(1-\cos m\delta)}{(1-\cos\delta)}$$

$$= r^2\cdot\frac{\sin^2(m\delta/2)}{\sin^2\delta/2} = m^2r^2\frac{\sin^2(m\delta/2)}{m^2\cdot(\delta/2)^2} \quad (\because \delta/2 \text{ is very small})$$

$$= A_0^2\cdot\frac{\sin^2\phi}{\phi^2} \cdots\cdots (1) \quad [A_0 = \text{ampl. of the wave incident on the slit} = mr \text{ and } m\delta = 2\phi].$$

[**Note** :—Expression (1) can be derived by the simple vector addition method or by integration method also].

Now to analyse the intensity distribution at various points in the focal plane of L (Fig. 51), let us find $dI/d\phi$ and equate that to zero.

Then, $$\frac{dI}{d\phi} = 2A_0^2\frac{\sin\phi}{\phi}\cdot\frac{(\phi\cos\phi - \sin\phi)}{\phi^2} = 0$$

Hence either (*i*) $\sin\phi = 0$, or (*ii*) $\phi\cos\phi = \sin\phi$, or $\phi = \tan\phi$. The first condition yields that $\sin\phi = 0$, when $\phi = n\pi$, where $n = 0, \pm 1, \pm 2, \pm 3$, etc. But when $n = 0$, $\phi = 0$ also.

$\frac{\sin\phi}{\phi}$, in the limit ϕ tending to zero, becomes unity. Hence under this condition, (when $\phi = 0$), we get, $I = A_0^2$, which is evidently a maximum. This maximum for $\phi = 0$, is called the *principal maximum.* For values of ϕ, other than zero, for which $\sin\phi$ is zero, *i.e.* when $n = \pm 1, \pm 2$, etc., or, $\phi = \pm\pi, \pm 2\pi$, etc., I is equal to zero. These are the *position of minima* which are *equispaced* and are shown in the intensity diagram (Fig. 53, upper portion) at $\pm\pi$, $\pm 2\pi$ etc.

The second condition $\phi = \tan\phi$, which is satisfied by the equations,

$$y = \phi \text{ and } y = \tan\phi,$$

can be graphically solved by plotting these two equations on the same graph. The $y = \phi$ represents a straight line inclined at 45° to both the axes and $y = \tan\phi$ represents tangent curves, whose asymptotes are at $\pm\pi/2$, $\pm\frac{3\pi}{2}$, $\pm\frac{5\pi}{2}$ etc. But the points of intersection of the straight line with the tangent curves lie at O, $\pm 1{\cdot}43\pi$, $\pm 2.46\pi$, etc. The zero position is evidently the principal maximum. Since at other positions the intensity $I = A_0^2 \frac{\sin^2\phi}{\tan^2\phi}$ $= A_0^2\cos^2\phi$, this intensity is less than A_0^2 *i.e.* less than the intensity at the principal maximum position. These positions are known as *secondary maxima* where intensity is much less than A_0^2. If we substitute the values of ϕ corresponding to these secondary maxima in relation (1), it is found that intensities of 1*st*, 2*nd*, etc. positions of the secondary maxima fall in the ratio 1/22·2, 1/61·7 etc. with respect to the primary maximum. Hence only a few bands are visible with a single slit, the intensity rapidly diminishing with the increase of order.

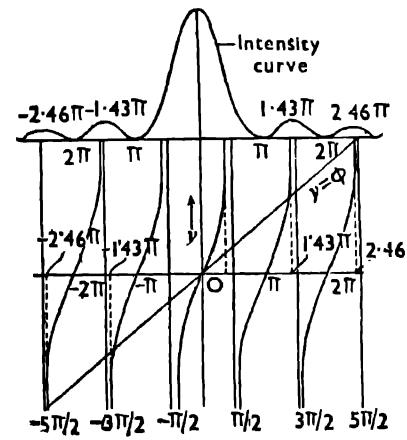


Fig. 53

To express the conditions of the minima and secondary maxima in terms of path difference, we know that for *minima*,

$$\phi = \pm n\pi, \text{ where } \phi = \frac{\pi a \sin\theta}{\lambda}$$

$$\therefore \quad a \sin\theta = n\lambda$$

As θ is small $a\theta = n\lambda$.

For *secondary max.*, $\phi = \pm 1{\cdot}43\pi$, $\pm 2{\cdot}46\pi$, etc.

$$\therefore \quad a \sin\theta = \pm 1{\cdot}43\lambda, \pm 2{\cdot}46\lambda, \text{ etc.}$$

Here also θ being small, $a\theta = \pm 1{\cdot}43\lambda$, $\pm 2{\cdot}46\lambda$, etc.

The *secondary maxima are not* evidently *equispaced.*

56. Fraunhofer Diffraction due to a Circular Aperture :— Airy investigated the Fraunhofer diffraction pattern formed due to a circular aperture. According to Airy's theory, there should be a bright central spot surrounded by a number of concentric rings, alternately dark and bright, when a monochromatic source is used. The intensity of the rings rapidly fades away as they recede from the centre. So the pattern will be limited only to a small region near the centre. The radius of the first dark ring surrounding the central bright spot is here found to be slightly greater than half the width of the central bright fringe obtained with a slit of width equal to the diameter of the circular aperture. According to Airy, the angular radius θ of the first dark ring is given by the relation,

$\sin\theta = \frac{1{\cdot}22\lambda}{a}$, where a is the diameter of the circular aperture.

When θ is small, $\theta = \frac{1{\cdot}22\lambda}{a}$.

If the circular aperture be the objective lens of a telescope and if F be the focal length of the objective which is evidently the distance of the point of image formation (*i e.* screen distance) from the centre of the aperture, then the linear radius of the first dark ring is given by $r = \theta f = \frac{1{\cdot}22\lambda f}{a}$. So if the aperture increases, the diffraction rings gradually become smaller and smaller in size.

Hence if we use a lens to concentrate parallel beam of rays, it acts as a circular aperture, then it is found that a definite ring-shaped diffraction pattern is formed in the focal plane instead of the parallel rays converging to a point focus. This pattern becomes sharper as the rings decrease in radius. As referred to above, this pattern is bright in the centre and alternate dark and bright rings of unequal spacings surround this central bright zone. The intensities of these rings also fall very rapidly as their distances from the centre of the pattern increase.

***57. Fraunhofer diffraction due to double slit :**—A double slit consists of two narrow slits arranged parallel to each other. As in the case of a single slit, the double slit is set up on the table of a spectrometer. A parallel beam of light from the collimator is made normally incident on the double slit (each of width a and separated by an opaque space b). The distance of separation between any two corresponding points of the two slits is $(a+b)$. In the focal plane of the telescope, the pattern shows the diffraction effect of each slit, as described earlier, as well as the interference effect of the two beams diffracted from each of the slits. Let $\psi_1 = A_1 e^{ipt}$ (real part) represent the resultant vibration in the focal plane of L due to the rays coming through the first slit where $A_1 \propto A_0 (\sin \phi/\phi)$. Now the path difference between the two rays coming from two corresponding points A and B of the two slits is evidently given by $(a+b) \sin \theta$. Therefore the vibration from the point A of the first slit differs in phase from that coming from the point B of the second slit by $2\beta = \frac{2\pi}{\lambda}(a+b) \sin \theta$. So the equation of the resultant vibration due to the rays from the second slit is given by $\psi_2 = A_1 e^{i(pt-2\beta)}$ Hence the resultant vibration due to the two slits is given by

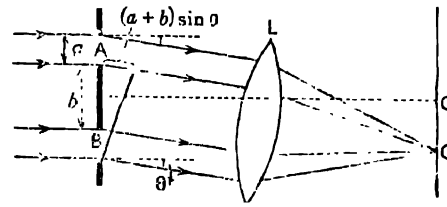


Fig. 54

$$\psi = \psi_1 + \psi_2 = A_1 e^{ipt}[1 + e^{-i2\beta}]$$

The complex amplitude of the resultant vibration is given by $A_1[1 + e^{-i2\beta}]$.

Therefore, the resultant intensity is given by,

$$I = A_1^2[1 + e^{-i2\beta}]\ [1 + e^{i2\beta}]$$
$$= 4A_1^2 \cos^2 \beta \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

Two systems of fringes appear in the field of view, of which one system is due to diffraction in a single slit and is governed by the term $A_1^2 \left(= A_0^2 \frac{\sin^2 \phi}{\phi^2}\right)$. The nature of such fringes has been described in Art. 55. The other system of fringes is due to interference and is governed by the term $\cos^2\beta$. The two systems superpose. The interference minima occur when $\beta = (n + \frac{1}{2})\ \pi$, where n is any integer including zero. For these minima we have,

$$\beta = \frac{\pi}{\lambda}(a+b)\sin\theta = (n+\tfrac{1}{2})\pi\ ;$$

or, $(a+b)\sin\theta = (n+\frac{1}{2})\lambda$.

For the interference maxima, we have, $\beta = n\pi$;

or, $(a+b)\sin\theta = n\lambda$.

As evident from these relations, both interference maxima and minima are equispaced.

Out of all these expected interference maxima positions, some of the bands will be, however, missing due to the fact that at these positions the diffraction minima will occur and as such there will be no diffraction light to produce interference bands there (see Art. 59).

Fig. 55 gives the intensity variation across the pattern for the case $b = 2a$. Eqn. (1) shows that the resultant intensity will be zero, if either factor is zero, *i.e.* at a minimum of either system there is darkness. The first term of the eqn. (1) gives the intensity distribution in the Fraunhofer diffraction pattern for a single slit and in the present case gives the slowly varying envelope within which lie the fringes given by the second term, *i.e.* $\cos^2\beta$ fringes which is characteristic of the interference between two points or line sources $(a+b)$ apart.

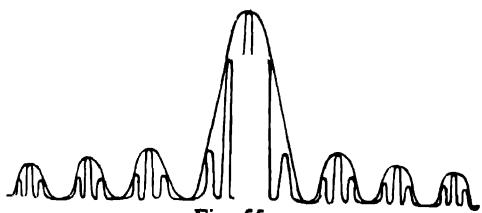


Fig. 55

58. Plane Diffraction Grating (Transmission type):—A plane diffraction grating consists of a large number parallel and equidistant slits. It is thus a structure of alternate transparent and opaque spaces. Present day gratings usually consist of equidistant diamond rulings on optically plane parallel glass plates or mirrors or replica of such ruled gratings. The gratings made on mirrors are known as *reflection gratings* and those on glass as *transmission gratings*. We shall here deal with a transmission grating. Each ruling on a grating behaves as an opaque line while the transparent portion between any two consecutive rulings behaves as a slit. If a be the width of each aperture and b that of each ruling, $(a+b)$ is called the *grating constant or grating element*. Any two points in the consecutive clear spaces whose distance of separation is $(a+b)$ are called the *corresponding points.*

The grating is set up on the table of a spectrometer with its rulings parallel to the slit of the collimator. A parallel beam of light rays is allowed to be incident on the grating, say, *normally*. When the diffracted beams are collected by the telescope we find a

number of intensely bright lines in the field of view. It is to be noted that here the diffraction effect of the slits is unimportant ; only the interference effect of all the diffracted beams coming from the slits are of importance and that is responsible for the observed pattern in the field of view. The path difference between the beams coming from any two corresponding points in the grating and diffracted at an angle θ is given by $(a+b)\sin\theta$. These rays which are diffracted at the angle θ will produce brightness when the path difference is an even multiple of $\lambda/2$. Thus we have, for bright bands, $(a+b)\sin\theta = n\lambda$ ... ... (1)

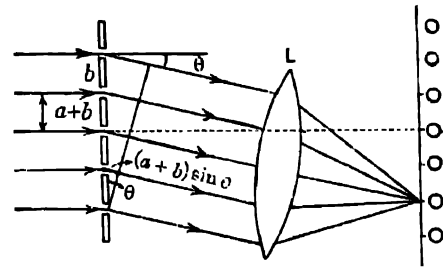


Fig. 56

Similarly, for dark bands $(a+b)\sin\theta = (2n+1)\lambda/2$ ... (2)

Eqn. (1) is called the *grating equation*. For $n=0$, we shall get the *central maximum* at O, which is the brightest of all. For $n=1$, 2, 3 etc. we get maxima of *first order, second order, third order*, etc. symmetrically disposed on either side of central maximum at O. The intensity of different orders, however, diminishes as the order increases. These are called the *principal maxima* of the different orders.

***Intensity Calculation :—**

The intensity of the resultant may be calculated in the following way. The path difference between the light coming from the corresponding points of the grating is $(a+b)\sin\theta$; so the phase difference is $\frac{2\pi}{\lambda}(a+b)\sin\theta = 2\beta$. Let the equation of vibration of the wave due to a single aperture of the grating reaching the focal plane of the telescope be $\psi_1 = Ae^{ipt}$(real part) where $A = A_0\frac{\sin\phi}{\phi}$ (for single slit). Then the equation of vibration due to the next aperture will be $\psi_2 = Ae^{i(pt-2\beta)}$ and so on. Hence the resultant vibration in the focal plane of the telescope is

$$\psi = \psi_1 + \psi_2 + \cdots \text{to } N \text{ terms} = Ae^{ipt}\,[1 + e^{-i2\beta} + e^{-i4\beta} + \cdots \text{to } N \text{ terms}]$$

$$= Ae^{ipt}\frac{(1-e^{-iN2\beta})}{(1-e^{-i2\beta})},$$ where N is the number of slits in the grating and $(N-1)\simeq N$.

Here the complex amplitude is $A\frac{(1-e^{-iN2\beta})}{(1-e^{-i2\beta})}$.

$$\therefore \quad \text{The resultant intensity} = A^2 \frac{(1-e^{-iN2\beta})(1-e^{iN2\beta})}{(1-e^{-i2\beta})(1-e^{i2\beta})}$$

$$= A_o^2 \frac{\sin^2\phi}{\phi^2} \cdot \frac{\sin^2 N\beta}{\sin^2\beta} - (1)$$

The first factor, $A^2 = A_o^2 \frac{\sin^2\phi}{\phi^2}$, is simply due to diffraction at a single slit which we have already discussed. The second factor $\left(\frac{\sin^2 N\beta}{\sin^2\beta}\right)$ essentially accounts for the interference effect. The modification of the diffraction pattern due to the multiplication of the slit number can be discussed by taking into account this interference factor, $J = \sin^2 N\beta/\sin^2\beta$. To study the max. and min. values of J, we differentiate J and equate it to zero.

$$\frac{dJ}{d\beta} = \frac{2 \sin N\beta}{\sin^2\beta} [N \sin\beta \cos N\beta - \cos\beta \sin N\beta) = 0.$$

Hence, either (i) $\sin N\beta = 0$; or, (ii) $N \tan\beta = \tan N\beta$.

Considering the first condition, $\sin N\beta = 0$ when $N\beta = m\pi$, $m = 0$, 1, 2,... etc. Again, $\sin\beta$ has the value zero for $\beta = 0, \pi, 2\pi, 3\pi$....., $m\pi$.

Thus $N\beta = m\pi$ is the *condition of minima* for all the values of m except 0, N, $2N$,.....etc., for which $J = 0/0$ is indeterminate. When the numerator and denominator of J are both zero, the ratio is finite and J can be shown to be equal to N^2. This represents the *principal maxima.*

Thus for principal maxima $N\beta = m\pi$ where m is any multiple of N including zero ; let m be equal to nN.

$$\therefore \quad \beta = n\pi ;$$

i.e. $(a+b) \sin\theta = n\lambda$ is the condition of principal maxima for different orders of the diffraction pattern.

Since the condition for a *minimum* is $\beta = m\pi/N$ (excepting for $m = nN$) and for $m = 0$ and $m = N$ there are principal maxima as we have seen above, we find that in between two principal maxima, there will occur $N-1$ minima or zero intensity points and necessarily $N-2$ *secondary maxima* between any two neighbouring principal maxima. These secondary maxima satisfy the relation $N \tan\beta = \tan N\beta$. So the value of J for secondary maxima will be,

$$J_s = \left(\frac{\sin^2 N\beta}{\sin^2\beta}\right)_s = \frac{N^2}{1 + (N^2-1)\sin^2\beta}$$

(by the substitution of above relation)

$$\therefore \quad \frac{J_s}{J \text{ principal}} = \frac{1}{1 + (N^2-1)\sin^2\beta}.$$

So the intensities of the secondary maxima become too feeble to be visible, as N becomes very large. As N increases the principal maxima becomes sharper and more intense and the secon

maxima become more feeble. Fig. 57 shows the distribution of intensity in a grating spectrum. The secondary maxima are of negligibly small intensity compared to the principal maxima and as

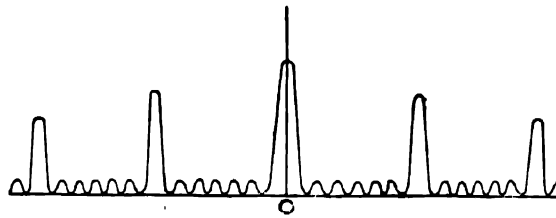


Fig. 57

such in a grating of large number of lines they become practically imperceptible and the spectral lines as obtained by a grating are nothing but the principal maxima.

Formation of Spectrum by a Diffraction Grating.—It follows from the grating equation, $(a+b)\sin\theta = n\lambda$, that for a particular order number n, the angle of diffraction θ will be greater for a light of longer wavelength. As a result if the incident light is not monochromatic in character, we shall get a spectrum in all orders excepting the central one. In the central band, n is zero for all wavelengths, and as a result no spectrum will be formed, and the band will be of the same colour as that of the incident light. As the wavelength of red light is longer than that of violet light, θ will be greater for red than for violet, and as such, red will be in the outermost position while violet will be in the innermost position of all orders other than the central one.

Measurement of Wavelength of Light

For the measurement of wavelength of light and for the study of spectrum, the diffraction grating is the most efficient means from all considerations, as it produces spectra of great dispersion and purity, and also because the quantities involved can be measured with considerable accuracy. For the study of spectrum a spectrometer or a spectroscope can be used. The spectrometer is first levelled and focussed for parallel rays. The grating is set up with its rulings parallel to the slit, which should be sufficiently narrow. The principal maxima appear as sharp lines parallel to the rulings of the grating. To measure the wavelength of light, the angle of diffraction of the spectrum of a particular order is measured on both the sides of the central image, and the mean is taken. The grating constant $(a+b)$ is determined by dividing the total length of the ruled surface by the total number of lines in it. Then from the equation $(a+b)\sin\theta = n\lambda$, the value of λ can be calculated. Thus an absolute measurement of the wavelength can be made. Usually the grating constant is determined from the spectrum of a radiation

of known wavelength, say, sodium light. The wavelength of the radiation being known, the value of $(a+b)$ can be calculated from the grating equation. Next, this value of $(a+b)$ is to be substituted in the grating equation, while measuring the wavelength of an unknown radiation. This is a sort of relative measurement of the wavelength of light.

Maximum number of orders of spectra with a diffraction grating.—Consider a grating having 1000 lines per cm. and $\lambda = 5000$ Å $= 5000 \times 10^{-8}$ cm. So, $a+b = \frac{1}{1000} = 10^{-3}$cm.

The first order spectrum will be formed at an angle θ_1 given by $\sin\theta_1 = \frac{n\lambda}{a+b} = \frac{1 \times 5 \times 10^{-5}}{10^{-3}} = 0{\cdot}05.$

$\therefore \quad \theta_1 = 2°52'.$

The second order spectrum will be formed at an angle θ_2 given by $\sin\theta_2 = \frac{2\times\lambda}{a+b} = \frac{2 \times 5 \times 10^{-5}}{10^{-3}} = 0{\cdot}1$

$\therefore \quad \theta_2 = 5°44'.$

Maximum value of n occurs when $\theta = 90°$

$\therefore$ From grating equation, we have,

$$n = \frac{(a+b)\sin\theta}{\lambda}$$

$$= \frac{10^{-3} \times 1}{5 \times 10^{-5}} = 20.$$

Thus the maximum number of orders that can be observed with such a grating is 20.

If again the number of lines per cm. be increased to 10,000, the maximum number of orders that will be visible with such a grating is evidently 2.

Example

Monochromatic light of wavelength, 5000A is diffracted by a grating of 2500 lines per cm. Is the 16th order possible? If not, what is the highest possible order of the image?

Ans. We know, $n\lambda = (a+b)\sin\theta$; or, $\sin\theta = \frac{n\lambda}{a+b}$

Here $n = 16$, $\lambda = 5000 \times 10^{-8}$ cm. and $(a+b) = \frac{1}{2500}$

$\therefore \quad \sin\theta = 16 \times 5000 \times 10^{-8} \times 2500 = 2$, which is impossible.

Hence the 16th order is not possible.

Highest possible order occur when $\theta = 90°$.

$$\therefore \quad n = \frac{(a+b)\sin\theta}{\lambda} = \frac{\sin 90°}{2500 \times 5000 \times 10^{-8}} = \frac{1}{125 \times 10^{-3}} = \frac{1000}{125} = 8.$$

Hence the maximum order possible is 8.

59. Absent spectra :—It is sometimes observed that certain orders of spectra are absent even if they satisfy the condition of maxima. For example, for a particular grating, the first and second order may be clearly visible, while the third order may not be visible at all and the fourth order may again be visible. We have seen that the intensity of illumination due to a grating in any direction of diffraction is governed by two factors, one due to diffraction by a single slit and the other due to interference. It may so happen that for a particular grating the direction in which the second factor should give us maximum, the first factor may be zero. It means that every slit by itself will produce darkness in that direction and as a result there can be no illumination even if the condition of reinforcement is satisfied by the second factor.

The direction of zero illumination due to a single slit is given by

$$a \sin \theta = m\lambda.$$

Again, the condition for maximum illumination in the same direction due to the grating is given by

$$(a+b) \sin \theta = n\lambda.$$

If these two conditions are simultaneously satisfied, the nth order spectrum will be missing.

$$\therefore \quad \frac{a+b}{a} = \frac{n}{m}.$$

If $a = b$, *i.e.* if the transparent and opaque parts of the grating are equal,

$$\text{we have, } \frac{2}{1} = \frac{n}{m}$$

$$= \tfrac{3}{1}, \tfrac{4}{1}, \tfrac{6}{1}, \tfrac{8}{1} \text{ and so on.}$$

That is, 2nd, 4th, 6th, 8th, etc. and in fact all even order spectra will be absent.

If again $2a = b$, *i.e.* if the transparent part is equal to half the opaque part,

$$\text{we have, } \frac{3}{1} = \frac{n}{m}$$

$$= \tfrac{3}{1}, \tfrac{6}{2}, \tfrac{9}{3} \text{ etc.}$$

That is, 3rd, 6th, 9th, etc. orders will be absent.

These discussions are valid equally in the case of a double slit.

60. Dispersive power of a diffraction grating :—When a composite pencil of light is incident on a grating, the different colours are separated from each other forming the diffraction spectrum. This separation of colours or wavelengths is known as **dispersion.**

If there be rays of wavelengths λ and $\lambda + d\lambda$ in the incident pencil, the corresponding angles of diffraction in the nth order of the

bright bands would be θ and $\theta+d\theta$ respectively. The dispersive power of a grating is defined to be the ratio $\frac{d\theta}{d\lambda}$, *i.e.* the rate of variation of the angle of diffraction with wavelength. The bright band of the nth order for the wavelength λ will be formed in the direction of diffraction θ, given by,

$$(a+b)\sin\theta = n\lambda \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

Differentiating the equation with respect to λ, we get,

$$(a+b)\cos\theta\frac{d\theta}{d\lambda} = n\,;$$

$$\text{or,}\quad \frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta} = \frac{mn}{\cos\theta} \quad \ldots \quad \ldots \quad (2)$$

where m = number of rulings per cm. $= \frac{1}{(a+b)}$.

Eqn. (2) is a measure of the spreading out of the spectrum. From eqn. (1) we have, $\frac{n\lambda}{a+b} = \sin\theta$.

Therefore, eqn. (2) may be written in the form,

$$\frac{d\theta}{d\lambda} = \frac{\sin\theta}{\lambda\cos\theta} \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

The eqn. (3) shows that the dispersion is independent of the grating constant. The large or small values of $(a+b)$ are compensated for by the higher or lower order number. However, for a particular order, the dispersion depends on the grating element, and if $\cos\theta$ can be taken to be constant ($\theta<6°$), $d\theta$ becomes proportional to $d\lambda$. Such a spectrum, for which $\frac{d\theta}{d\lambda}$ is constant, is known as a *rational* or *normal spectrum.*

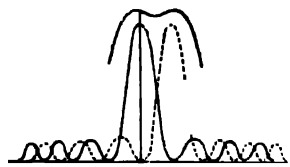
From equation (2) the rate of variation of dispersion is

$$\frac{d}{d\theta}\left(\frac{d\theta}{d\lambda}\right) = \frac{d}{d\theta}\left(\frac{mn}{\cos\theta}\right) = -\frac{mn}{\cos^2\theta}\sin\theta.$$

If $\frac{d\theta}{d\lambda}$ is constant. the above expression should be zero. This is zero when $\theta = 0$, or approximately so ; hence the name *normal spectrum.*

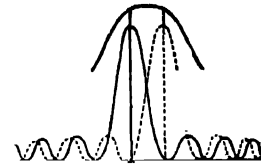
61. Resolving power of a diffraction grating :—The resolving power of a grating may be defined as its power of separating two very closely situated spectral lines. According to Lord Rayleigh two adjacent spectral lines appear to be just separated when the principal maxima of one falls on the first minima of the other, *i. e. when the distance between the centres of two images is half the width of the principal maximum of either.* In that case the two lines

do not completely get separated, but there is only an intensity variation in the middle of an otherwise broad line as shown in Fig. 58. If the lines come closer than the above limiting value, the resultant in-



(a)

Fig. 58



(b)

Fig. 59

tensity curve will have no kink in the middle and the lines cannot be distinguished [Fig. 59].

Let the two spectral lines be of wavelengths λ and $\lambda + d\lambda$. From the grating equation, we have,

$$(a+b) \sin \theta = n\lambda \quad \cdots \quad \cdots \quad (1)$$

where n represents the order of the spectrum.

Owing to dispersion, the angular separation between two successive lines (*i.e.* the two principal maxima of wavelengths λ and $\lambda + d\lambda$) is given by,

$$\Delta\theta = \frac{n}{(a+b)\cos\theta} \cdot \Delta\lambda \quad \cdots \quad \cdots \quad (2)$$

The angle between the principal maximum and the first minimum may be obtained as follows : For a grating of total number of rulings N, the principal maximum of order n is determined by, $\beta = n\pi$, where $\beta = \frac{\pi}{\lambda}(a+b) \sin\theta$, and the condition for minima is $\beta = \frac{m\pi}{N}$, [for $m \neq 0$ and nN] ;

or, $NB = m\pi$.

$\therefore$ $d\beta \frac{\pi}{N} \Delta m$. Hence the first position of zero intensity on either side is given by the above variation of $d\beta$, when $\Delta m = 1$.

$\therefore$ $d\beta = \frac{\pi}{N}$;

or, $a \frac{[\pi(a+b) \sin\theta]}{\lambda} = \frac{\pi}{N}$; or, $\frac{(a+b)\cos\theta . d\theta}{\lambda} = \frac{1}{N}$;

or, $d\theta = \frac{\lambda}{N(a+b)\cos\theta}$ $\quad \cdots \quad \cdots \quad (3)$

According to Rayleigh's criterion, the limit of revolution will be obtained, when the angular separation between the two principal

maxima of wavelengths λ and $\lambda + d\lambda$ is equal to the angular separation of the principal maxima and the first minima of wavelength λ, for a particular order ; or, when $\triangle\theta = d\theta$.

$\therefore$ From eqns. (2) and (3),

$$\frac{n}{(a+b)\cos\theta}\, d\lambda \qquad \frac{\lambda}{N(a+b)\cos\theta} ;$$

$$\text{or,} \quad \frac{\lambda}{d\lambda} = N.n.$$

The quantity $\frac{\lambda}{d\lambda}$ is called the *resolving power*, where $d\lambda$ is the smallest difference of wavelength that can be resolved at the wavelength λ. It is equal to the the product of *total number of rulings N* and the *order number n*.

The wavelengths of sodium D_1 and D_2 lines are 5896 and 5890 A. U. respectively. Therefore $d\lambda = 6$Å, or $\frac{\lambda}{d\lambda}$ is approximately 1000. *D*-lines will be just separated in the second order with a grating of 500 lines and in the first order with a grating of 1000 lines. It should be noted that the actual resolving power is always less than the theoretical resolving power, as the slit having a finite breadth necessarily illuminate only a portion of the grating, thereby reducing the effective number of total lines in the grating.

62. Resolving power of a prism spectrometer :—Let *ABC* be a prism and let a plane wavefront *PB* from a collimator be incident on the face *AB* of the prism making an angle *i*, the prism being adjusted for minimum deviation. After refraction through the prism the rays emerge as *AR* and *CR'*. *CQ* is drawn perpendicular to *AR* from *C*. Evidently *CQ* is the emergent wavefront. The path $PA + AQ$ in air is

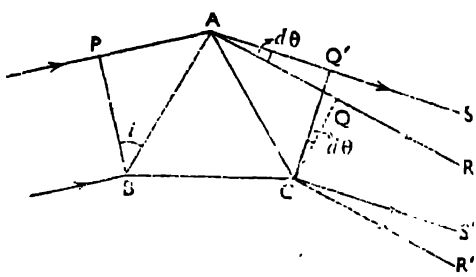


Fig. 60

equal to path *BC* in the prism.

Hence $PA + AQ = \mu . BC$

$$= \mu . t. \text{ (say)} \qquad \ldots \qquad \ldots \qquad (1)$$

where $t = BC$ and μ is the *R. I.* of the prism material.

If the incident light contains another wavelength $\lambda + d\lambda$, the emergent rays corresponding to that will be *AS* and *CS'* and *CQ'*

is the emergent wavefront corresponding to the light of that wavelength. Then, we shall have,

$$PA + AQ' = (\mu - d\mu)\, t, \qquad \ldots \quad (2)$$

where $\mu - d\mu$ is the refractive index corresponding to the wavelength $\lambda + d\lambda$, the negative sign before $d\mu$ indicates that as λ increases, μ diminishes,

$\therefore$ From (1) and (2), we have,

$AQ - AQ' = QQ' = t.\ d\mu$, approximately.

The angle between the rays AR and $AS = \angle QCQ' = \dfrac{QQ'}{QC}$ approx.

$$= \frac{t d\mu}{QC} = d\theta.$$

This is the angle of dispersion between the wavelengths λ and $\lambda + d\lambda$, *i.e.* the angular separation between the two images of the slit for the wavelengths λ and $\lambda + d\lambda$. The prism may be considered as a rectangular slit of width QC, for which the angular half-width $d\theta'$ (*i.e.* the distance between the centre of a maximum and the centre of the adjacent minimum on either side), of the maxima of each of the wavelengths is given by,

$$d\theta' = \frac{\lambda}{QC}. \quad \left[\text{see Art. 55, for 1st minimum } n = 1.\right.$$

$$\left.\therefore \quad \sin\theta = \theta = \frac{\lambda}{a}\right]$$

Now, limiting resolution will be obtained when $d\theta = d\theta'$.

$$\therefore \quad \frac{\lambda}{QC} = t.\ \frac{d\mu}{QC},$$

$$\text{or,} \quad \frac{\lambda}{d\lambda} = t.\ \frac{d\mu}{d\lambda}.$$

Thus the resolving power of a prism depends on the thickness of the base of the prism, if the entire prism is utilised.

If $\dfrac{d\mu}{d\lambda} = 1{,}000$ for very dense flint glass for a wavelength 6×10^{-5} cm., and $t = 1$ cm., for such a prism the resolving power is 1000, and as such it is just sufficient to resolve the D-lines of sodium, as $\dfrac{\lambda}{d\lambda}$ for sodium D-lines is 1000 approximately.

63. Resolving power of a Telescope :—The objective of an astronomical telescope behaves like a circular aperture. The image formed by it is due to the diffraction at the circular aperture of its objective. Thus a distant point source of light, such as a star, will not give a point image at the focal plane of the telescope, but it will produce a circular bright disc surrounded by alternate

dark and bright rings. It was found on rigorous calculation by Airy that the angle which the radius of the first dark ring subtends at the objective is given by $\frac{1{\cdot}22\lambda}{d}$, where λ is the wavelength of the light and d is the diameter of the telescope objective. According to the Rayleigh criterion of resolution the images of two such stars or point objects lying close to each other are said to be resolved if the central maximum of one falls on the first minimum of the other. So when the centre of the first bright circle of one falls on the first dark ring of the other as shown in Fig. 61, the two images are just resolved. Thus it follows that two stars will be just distinguished by a telescope as separate if their angular separation is $d\theta = 1{\cdot}22\,\frac{\lambda}{d}$.

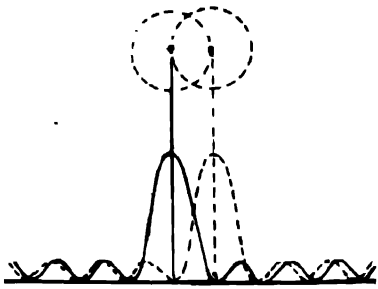


Fig. 61

A telescope having a larger diameter will reveal a greater detail of the object. This power of showing two point sources distinctly is known as *the resolving power* and is measured by the reciprocal of the smallest angular separation between the points which can be distinguished. So the resolving power of the telescope is given by $\frac{1}{d\theta} = \frac{d}{1{\cdot}22\lambda}$. A telescope of 1″ diameter will just resolve two stars at a distance of 5 seconds, whereas a telescope 200″ diameter will be able to resolve two stars at a distance of 0·025 seconds.

64. Resolving power of a microscope :—The rays from a point O in front of a microscope objective produces an image at I. Obviously, the rays which starting from O meet at I are in the same phase. If we have another point object O' very near to O, then the rays $O'A$ and $O'B$ are diffracted by the aperture of the lens and will meet at I'. The path difference between these two extreme rays $O'B$ and $O'A$ is $2 \times OO' \sin\theta$, where θ is the semi-angle subtended by the objective at the point object. We wish to know the smallest distance between two object points O and O' which will produce images I and I' that are just resolved. Each image consists of a disc and

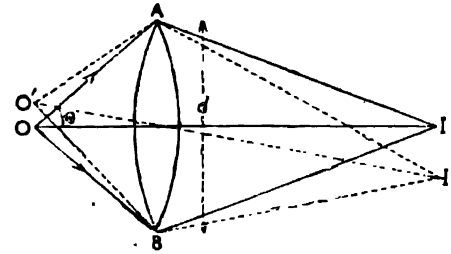


Fig. 62

a system of rings and the angular separation of the two discs, when they are on the limit of resolution, is $\theta = \frac{1{\cdot}22\lambda}{d}$. When this condition holds, the two extreme rays $O'A$ and $O'B$ differ in path by $1{\cdot}22\lambda$. We, therefore, have,

$$2 \times OO' \sin\theta = 1{\cdot}22\lambda ;$$

$$\text{or,} \quad OO' = \frac{1{\cdot}22\lambda}{2 \sin\theta}.$$

Abbe investigated the problem in detail and showed that a good working formula for the limit of resolution is obtained by omitting the term 1·22 in the above expression because in the case of a microscope the point objects O, O' are not self-luminous as was supposed in Airy's analysis. If μ be the refractive index in the object space, then $\mu = \frac{\lambda_0}{\lambda}$, where λ_0 is the wavelength of light in vacuum and λ that in the object space. Therefore,

$$OO' = \frac{\lambda_0}{2\mu \sin\theta}$$

Thus the smallest distance resolvable by a microscope can be diminished by using a light of smaller wavelength and increasing μ, which is done in oil immersion objectives. This also increases the brightness of the field of view as the amount of light lost by reflection in the first lens surface is diminished. The factor $\mu \sin\theta$ was called by Abbe the *numerical aperture* (*N. A.*) which is characteristic of a particular objective.

65. Comparative study of the prism spectra and the grating spectra :—

(1) Ordinary prism spectra are impure, though with proper adjustments and arrangements they can be made pure. Each of the component colours in a diffraction spectrum is pure, *i e.* unmixed with any other colour.

(2) In a prism only one spectral sequence is obtained whereas in a grating quite a number of spectral orders are observed on either side of the central band.

(3) The spectra due to different prisms of identical dimensions but of different materials will be dissimilar, the deviation depending upon the angle of the prism and the refractive index of the material. Here the deviation is more for the violet and less for the red. But in a grating spectrum deviation is directly proportional to the wavelength and inversely proportional to the grating element. So the spectrum is independent of the nature of material

of the grating. The spectra of two grating of identical number of lines per cm. will be identical. Moreover, here the red component suffers greater deviation than the violet one.

(4) The dispersive powers of a prism is greater for shorter wavelengths than for longer wavelengths. So the violet portion of the spectrum being more drawn out becomes less intense than the red portion. Moreover, the relative dispersion of any two colours to that of the extreme colours are found to be different for prisms of different materials. This is known as the *irrationality of the prism spectrum.* The dispersive power of a grating spectrum is given by

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}.$$

If θ does not exceed 6°, $\cos\theta$ is fairly constant. So for a particular order of the spectrum, the angular separation $d\theta$ between two lines differing in wavelengths by $d\lambda$ remains constant and is doubled as the order number is doubled. Moreover, though the dispersion increases or decreases as the grating element decreases or increases, the relative dispersion here remains constant. Hence the spectrum formed by any grating is exactly similar to that formed by any other, one being an exact copy of the other on a larger or smaller scale. For this reason the grating spectrum is called a *normal or rational spectrum.*

(5) The resolving power of a prism is small and is given by $t\frac{d\mu}{d\lambda}$. So it depends on the *effective length* of the base and $\frac{d\mu}{d\lambda}$. In a grating the resolving power is given by Nn, where n represents the order and N the total number of lines in the grating. So it does not depend on μ or λ. Moreover, due to its high resolving power the doublets and triplets are also resolved and as such the spectrum becomes almost a pure one.

(6) But so far as the intensity of the spectra is concerned the prism spectrum is far superior to the grating spectrum.

66. Interference and Diffraction spectra compared :—

(1) We know that interference is the result of superposition of light coming from two different wavefronts originating from the same source or from different parts of the same wavefront. But the diffraction fringes are formed due to the interaction of light coming from the same wavefront only.

(2) In interference fringes the minimum intensity positions are found to be completely dark whereas in diffraction the minima positions are not perfectly dark.

(3) The interference fringes may or may not be equispaced. In a diffraction pattern equispaced fringes are never formed.

(4) In an interference pattern all the bright bands are of uniform intensity whereas in a diffraction pattern the intensity of the bands are not uniform.

67. Plane Reflection Grating :—A plane reflection grating is made exactly in the same way as the plane transmission grating excepting in the fact that here the rulings are performed on a polished metal (usually speculum metal) surface. The observations are made with reflected light. The rays coming from various points of each grating element here have complicated phase-relationships depending on the form of the grooves. In a reflection grating the spectra are differently spaced and the central maximum occurs in the direction of regular reflection, on both sides of which the higher order spectra are distributed. The problem is essentially similar to that of a transmission grating when used with oblique incidence.

Plane reflection gratings are used in the study of ultra-violet rays as well as in the absolute determination of the wavelength of *X*-rays. The angle of incidence is, however, to be changed according to the range of wavelength in which investigations are to be made. In the case of *X*-rays almost a grazing incidence is desirable.

***68. Concave grating :—**

In performing experiments with plane gratings a serious difficulty is encountered ; the lens of the observing telescope cannot have the same focal length for all wavelengths, so that the lines to be examined cannot be brought exactly to a focus simultaneously. To avoid this difficulty Rowland constructed (1882) a concave grating of the reflection type in which focussing is done by a concave mirror, upon which the grating itself is ruled. Concave gratings are made by ruling lines on concave spherical surfaces of speculum metal. This metal reflects light of all wavelengths and as such the grating can be used for the infra-red and ultra-violet regions also. The rulings are made in such a way that the distance between the rulings measured along a chord is constant. It can be shown that if R is the radius of curvature of the spherical surface of the grating, a circle with radius $r\left(=\frac{R}{2}\right)$ may be drawn tangent to the midpoint of the grating, and this circle defines the locus of all points where the spectrum will be focussed, provided the source slit lies on that circle. This circle is known as the *Rowland circle*. The concave grating thus combines the image-forming power of a mirror with the spectrum-producing power of a plane grating. It does away with the necessity of using a focussing device, such as a lens, thus

eliminating the chromatic aberrations altogether. Rowland's discovery thus marked a conspicuous advance in the domain of spectroscopy.

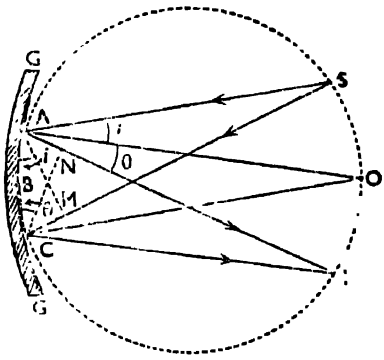


Fig. 63

Let GG (Fig. 63) represent a concave grating. Suppose that the width of each reflecting space $AB = a$, and that of each ruling $BC = b$. A and C are the corresponding points at a distance $a + b$ apart. Let O be the centre of curvature of the grating surface GG and S a monochromatic source of light (say, an illuminated slit) perpendicular to the plane of the paper and parallel to the rulings of the grating. Light from this monochromatic source S incident on the grating element AC will produce the nth order spectrum at I if the path difference

$$(SC + CI) - (SA + AI) = n\lambda \quad \ldots \quad (1)$$

Draw CN and AM perpendiculars on AI and CS respectively.

Since, $CI = NI$ and $SA = SM$ (approximately, the grating element being very small), we have the path difference as,

$$MC - AN = n\lambda \quad (2)$$

But from figure, $MC = (a + b) \sin i$,
and $AN = (a + b) \sin \theta$.

Therefore, from eqn. (2),

$$(a + b)(\sin i - \sin \theta) = n\lambda \quad (3)$$

If S and I lies on the same side of O, the above equations take the form

$$(a + b)(\sin i + \sin \theta) = n\lambda \quad (4)$$

Equations (3) and (4) are the grating equations. To get the relative positions of the spectral lines with respect to the source let us differentiate eqn. (3) or (4) and get,

$$\cos i \, di \pm \cos \theta \, d\theta = 0 \quad \ldots \quad (5)$$

($\because$ $n\lambda$ = constant for a particular order.)

If angles at S, O and I be respectively α, β and γ,

$\angle SCO = i'$ and $\angle ICO = \theta'$, then,

$di = i' - i$ and $d\theta = \theta' - \theta$.

From Fig. 63, $\alpha + i = \beta + i'$; or, $di = i' - i = (\alpha - \beta)$ ... (6)

Again $\beta + \theta = \gamma + \theta'$; or, $d\theta = \theta' - \theta = (\beta - \gamma)$ ... ... (7)

Substituting in (5), $(\alpha - \beta) \cos i \pm (\beta - \gamma) \cos \theta = 0$ ... (8)

If the source be on a circle of diameter R, the centre of the grating is tangent to this circle and as $OA = OC = R$, we have $\beta = (a+b)/R$. Since S and O are on the same circumference, the angles α and β, subtended by the same chord AC must be equal. $\therefore \quad \alpha = (a+b)/R$.

Substituting in (8), $\left(\frac{a+b}{R} - \frac{a+b}{R}\right) \cos i \pm \left(\frac{a+b}{R} - \gamma\right) \cos \theta = 0$

$\therefore \quad \gamma = (a+b)/R = \alpha = \beta.$

Since γ is also subtended by the chord AC at I, I must also lie on the same circumference as S and O. Hence image is also formed on the Rowland's circle. Since I shifts its position for different values of m and λ, we get a series of spectra on either sides of O, all distributed along the circumference of Rowland's circle.

***69. Mountings of concave gratings :—**

(a) Rowland Mounting.—In Rowland mounting (shown in Fig. 64), the grating G and the plate holder P are fixed at the opposite ends of a rigid beam of length R equal to the diameter of the Rowland circle. This beam is supported on swivel trucks which can slide along the arms SX and SY at right angles to each other. The position of the slit S remains stationary. Since the angle $\angle XSY$ is always 90°, G, P and S always lie on the Rowland circle as the beam is slided one way or other. It is obvious that as the beam slides, the angle of incidence $\angle SGP$ varies while the angle of diffraction remains always close to zero. Thus, the nth order line of wavelength λ will be focussed on the plate P when,

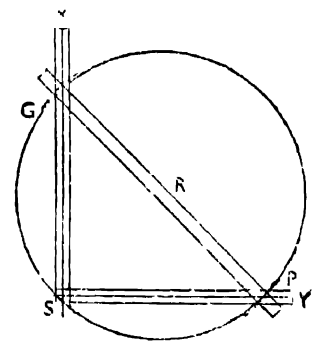


Fig 64—Rowland mounting.

$$(a+b)(\sin i + \sin 0) = n\lambda$$

or, $\quad (a+b) \sin i = n\lambda$;

or, $\quad (a+b) \frac{PS}{R} = n\lambda.$

Thus $PS \propto \lambda$. Since $R > PS$, $\frac{n\lambda}{a+b}$ cannot exceed unity and wavelengths greater than $\frac{a+b}{n}$ cannot be examined in the nth order. The spectrum obtained here is normal and the wavelengths may be permanently marked on the rail PS because of the simple linear

proportionality. This method has the serious drawback of showing astigmatism in higher orders.

(b) Paschen Mounting :—This method also utilises the properties of a Rowland circle. The slit is set upon the Rowland circle and the light that strikes the grating surface is diffracted in different directions and forms the spectra of various orders. Here the grating and the slit mounted on separate piers are fixed and the plate is curved to fit any convenient portion of the Rowland circle. Several orders of spectra can be photographed simultaneously which is not possible in any other mounting. The spectrum obtained with it is not normal and as a result the dispersion on the plate is not linear. Dispersion is, however, practically constant for a considerable region near the normal.

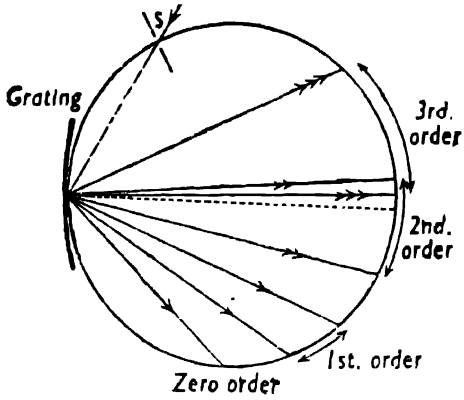


Fig. 65—Paschen mounting.

Other types of mountings (Eagle mounting, Wadsworth mounting etc.) are also adapted in a concave grating for special types of spectroscopic work. But the above two mountings are rather more common.

***70. Echelon Grating** :—It is a very useful form of grating first devised by Michelson, which enables us to observe the spectra of very high orders. The resolving power of ordinary plane or concave gratings are not very high and the spectra of orders higher than the 5th or 6th cannot be observed with them without overlapping. In Michelson's echelon gratings spectra of the 10,000th order may also be observed. He suggested that a grating of this type could

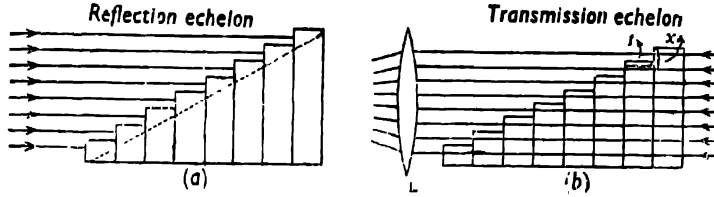


Fig. 66

be produced by arranging a number of glass plates in step formation, as shown in Fig. 66(*a*), the steps being silvered. The glass plates

are of exactly equal thickness and the width of the steps are also exactly equal. Another type of echelon grating was constructed, known as a transmission echelon [Fig. 66(*b*)]. A parallel beam of light is incident as shown in the figure and passes normally through the echelon and each step acts as an aperture of width x, which produces its own diffraction pattern. Since the light passing through different apertures traverses different paths in glass, a constant phase difference is introduced between them on transmission. The path difference between the rays of light passing through adjacent steps is $t\ (\mu_{glass} - \mu_{air})$, whereas for reflection echelon it would be $2\mu_{air}t$, *i.e.* 3 to 4 times greater. In the focal plane of the lens L we shall get a single-slit-aperture pattern which provides the envelope within which the pattern resulting from the interference between the disturbances passing through the different steps is formed.

Let us now calculate the path difference between the corresponding rays of a transmission echelon (Fig. 67), consisting of a pile of plate parallel plates of constant thickness t with each plane projecting a distance x beyond the one just below it. The path difference between the rays AX and BY, diffracted at an angle θ is evidently,

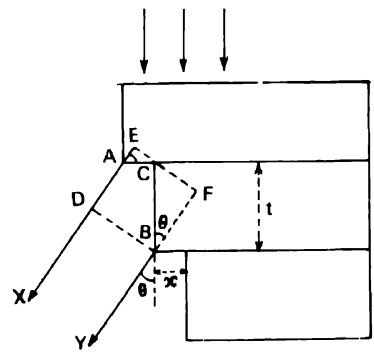


Fig. 67

BC (in glass) $- AD$ (in air)
$= BC$ (in glass) $- (DE - AE)$ (in air)
$= BC$ (in glass) $- (BF - AE)$ (in air)
$= \mu.t - (t \cos\theta - x \sin\theta)$
$= (\mu - \cos\theta)t + x \sin\theta.$

If θ is taken to be very small, $\sin\theta = \theta$, and $\cos\theta = 1$, and hence the path difference $= (\mu - 1)t + x\theta$.

If the light diffracted in that direction gives a maxima of the nth order, we have,

$$(\mu - 1)t + x\theta = n\lambda.$$

This is the working equation for an echelon grating. The plates have ordinarily a thickness of about 1 or 2 cm. and are put together such that $x = 1$ or 2 mm. Taking $\theta = 0$, $\mu = 1{\cdot}5$ and $t = 1$ cm. and $\lambda = 5000$ Å.U., we have,

$$n = 10{,}000.$$

The resolving power is given approximately by the same expression as for a ruled grating, *i.e.* $\frac{\lambda}{d\lambda} = nN$, N being the number of plates.

If there are 50 plates, the resolving power becomes,

$$\frac{\lambda}{d\lambda} = 10{,}000 \times 50 = 500{,}000.$$

Since the separation between the consecutive orders of spectra is very small, the effect of overlapping is enormous, and the grating is unsuitable for examining a great length of spectrum. Even a sodium light will be too complex for it. It is only suitable for the study of *fine or hyperfine structure* of spectral lines. With an echelon grating made of quartz, lines in the ultra-violet region can also be studied.

Examples

1. Monochromatic light of wavelength 6560 Å. U. falls normally on a plane transmission grating 2 cm. wide. The second order spectrum is observed at an angle of 38° 14′ from the normal. What is the total number of lines on the grating ?

Ans. If $(a+b)$ be the grating element,

$(a+b) \sin\theta = n\lambda$.

Here, $\theta = 38°14'$, $n=2$, $\lambda = 6560 \times 10^{-8}$ cm.

$$\therefore \quad (a+b) = \frac{n\lambda}{\sin\theta} = \frac{2 \times 6560 \times 10^{-8}}{\sin 38°14'} = 2{\cdot}119 \times 10^{-4}$$

$\therefore$ The number of lines per cm. is $\frac{1}{a+b} = \frac{1}{2{\cdot}119 \times 10^{-4}}$.

Hence the total number of lines in the grating $= \frac{2}{2{\cdot}119 \times 10^{-4}} = 9434$.

2. Calculate the wavelength of light which gives a deflection of 9°32′ for the third order spectral lines with a grating of 1012 lines per cm.

Ans. Here number of lines per cm. $= \frac{1}{a+b} = 1012$;

$$\text{or,} \quad (a+b) = \frac{1}{1012}.$$

From $(a+b) \sin\theta = n\lambda$, we have,

$$\lambda = \frac{(a+b)\sin\theta}{n}.$$

But $n=3$, $\theta = 9°32'$ and $(a+b) = \frac{1}{1012}$

$$\therefore \quad \lambda = \frac{\sin 9°32'}{1012 \times 3} = 5456 \times 10^{-8} \text{ cm.} = 5456 \text{ Å. U.}$$

3. A parallel beam of sodium light is allowed to be incident normally on a plane grating having 4250 lines per centimetre and a second order spectral line is observed to be deviated through 30°. Calculate the wavelength of the spectral line.
(Bihar U. 1954)

Ans. Here $(a+b) = \frac{1}{\text{no. of lines per cm.}} = \frac{1}{4250}$; $\theta = 30°$, $n=2$.

$\therefore$ From the relation, $(a+b) \sin\theta = n\lambda$, we have,

$$\lambda = \frac{(a+b)\sin\theta}{n} = \frac{1 \times \frac{1}{2}}{4250 \times 2} = \frac{1}{4 \times 4250} = 5883 \text{ Å. U.}$$

4. A parallel beam of light consisting of two wavelengths 4000 Å and 7500 Å falls normally on a grating for which the distance between grating elements is 0·001 cm. Calculate the difference in the angular deviations of these two spectral lines in the second order. *(C. U. 1964)*

Ans. Since $(a+b)\sin\theta = n\lambda$,

differentiating $(a+b)\cos\theta\, d\theta = n\, d\lambda$

As $(a+b)$ is comparatively large, θ will be very small and $\cos\theta = 1$.

$$\therefore \quad d\theta = \frac{n.d\lambda}{(a+b)} = \frac{2\times(7500-4000)}{0{\cdot}001}$$

$$= \frac{2\times 3500\times 10^{-8}}{10^{-3}} = {\cdot}07 \text{ radians}$$

$$= \frac{{\cdot}07\times 180^{0}}{3{\cdot}14} = 4^{\circ}(\text{appr.})$$

5. The Mount Wilson Observatory telescope has an objective of diameter 100 inches. Assuming the mean wavelength of light to be $5{\cdot}5\times10^{-5}$ cm., estimate the smallest angular separation of two stars which can be resolved by it.

(Agra U. 1949)

Ans. The diameter of the objective aperture $= d = 100$ inches
$= 2{\cdot}54\times 100$ cm. $= 254$ cm. ; $\lambda = 5{\cdot}5\times10^{-5}$ cm.

$\therefore$ The smallest resolvable angle or the angular separation between the stars is $\theta = \dfrac{1{\cdot}22\lambda}{d} = \dfrac{1{\cdot}22\times5{\cdot}5\times10^{-5}}{2{\cdot}54} = 2{\cdot}642\times10^{-7}$ radians $= 0{\cdot}0544$ second.

6. A Rowland grating (concave) has 15000 lines to the inch. Calculate the angular separation of the two mercury lines 5770A. and 5461A. in the spectrum of first order. *(All. Sup 1957)*

Ans. Eqn. of a concave grating is $(a+b)(\sin i \pm \sin\theta) = n\lambda$.

For the 1st. order $n = 1$. Hence differentiating this eqn. and putting $n = 1$.

we have, $d\theta = d\lambda/(a+b)\cos\theta$.

For Rowland's grating $\theta = 0$, $\cos\theta = 1$. $d\lambda$ (from data) $= (5770 - 5464)A$

$= 309\times10^{-8}$ cm., $(a+b) = 2{\cdot}54/15000$.

Hence. $d\theta = 309\times10^{-8}\times15000/2{\cdot}54 = 1{\cdot}83\times10^{-3}$ radians.

Questions (Chapter III)

1. Explain the rectilinear propagation of light on the basis of the wave theory. (Agra U. 1943 ; B.U. 1944, '47 ; P.U. 1955, '51, '50, '49 ; C.U. 1955, '65)

2. (*a*) What is a zone plate ? Compare its function with that of a lens. (Luck. U. 1954 ; C.U. 1953 ; Ag. U. 1946)

(*b*) Explain the phenomenon of diffraction of light. (Del. U. 1967)

3. Explain clearly the difference between interference and diffraction. (C. U. 1943, '45, '52, '53, '63 ; Gau. 1949 ; P. U. 1953, '55)

4. (*a*) What is the difference between Fresnel and Fraunhofer classes of diffraction ? (Del. U. 1968 ; C. U. H. 1949)

(*b*) Explain the essential difference between a diffraction grating and a prism spectrograph used for the study of line spectra. (C. U. 1957)

5. Discuss the diffraction pattern produced by a knife edge when monochromatic light is used. (*cf.* Del. U. 1967 ; P. U. 1952, '53, '55)

6. Give an elementary treatment of diffraction of monochromatic light by a thin wire. Explain the changes produced in the diffraction pattern as the diameter of the wire is increased. (P. U. 1949)

7. Explain the formation of diffraction pattern when a monochromatic beam of light passes through a narrow rectangular aperture. (P. U. 1943)

8. How from the observations on the diffraction pattern formed by a straight wire, the diameter of the wire can be measured ?

9. Explain the action of a diffraction grating and describe how to use it to measure the wavelength of light.
(C. U. 1943, '45, '48, '50, '56, '58, '64, '67 ; Del. U. '69 ; Gau. U. 1949)

10. (*a*) How many lines per cm. are there in a grating which gives an angle of diffraction equal to 30° in the first order for light of wavelength 6×10^{-5} cm ?
(C. U. 1948)
[*Ans.* 8333·3]
(*b*) What would be the effects if the distances between the rulings in a grating were (*a*) very large, (*b*) very small compared with the wavelength ?
(C. U. 1950)

11. Develope the theory of a diffraction grating and derive the conditions for absent spectra. What is the condition which must be satisfied if the second order spectra is to be suppressed ?
[P. U. 1951 (*S*) ; Ban. U. 1953]

12. Give the theory of a plane transmission grating and describe how it is used to determine the wavelength of any source of light.
(*cf.* Utk. U. '70*S* ; Bihar U. 1954 ; C. U. '68)
What do you mean by the resolving power of a grating ?
(C. U. 1968 ; Utk. U. '66)

13. Explain with theory the formation of spectrum by a grating. How does a grating spectrum differ from a prism spectrum ? When would you prefer the use of the former ? (Agra U. 1950 ; P. U. 1941, '49)

14. A plane transmission grating having 3000 lines per cm. gives an angle of diffraction of a line by 30° in the third order. Find the wavelength of the given line. (C. U. 1956)
[*Ans.* 5556×10^{-8} cm.]

15. Deduce an expression for the resolving power of a spectroscope.
(C. U. 1952)

16. Write notes on diffraction gratings and their uses. (C. U. 1947, '51, '54)

17. Deduce an expression for the resolving power of a grating spectroscope. On what does this resolving power depend ? (*cf.* Agra U. 1953 ; C. U. 1659)

18. What is a normal spectrum ? (P. U. 1956)

19. Describe Rowland's concave grating and a method of mounting it for measuring the wavelength of light. Deduce the formula used. (Agra U. 1954)

20. Show that the resolving power of a telescope objective depends upon its diameter and wavelength of light. (P. U. 1954)

21. Distinguish between the dispersive and resolving power of a grating.
Deduce the expression for the resolving power of a plane transmission grating.
(*cf.* Del. U. '69 ; P. U. Sup. 1955)

22. A parallel beam of white light strikes at right angles to an opaque plate in which a small circular hole is bored and falls on a screen after passing through the hole. Describe the appearance on the screen. How does it vary as the distance between the plate and the screen is altered and why ?
If the plate with the hole in it were replaced by a small circular disc at right angles to the incident light, what would be the appearance on the screen ? Give reasons of your statement. (Gau. U. 1957)

23. Find an expression for the angular dispersion of a plane diffraction grating. (C. U. 1958)

24. What changes will you observe in the grating spectra as you pass from one order to another, if the radiation is not strictly monochromatic ?

How many orders will be visible, if the wavelength of the incident radiation is 5000Å and the number of lines on the grating be 7620 to an inch ? (Agra U. 1950)

[*Ans.* upto six orders]

25. A vertical slit, illuminated by sodium light, is placed in front of a vertical screen and in between them is placed (*a*) a narrow vertical aperture (*b*) a narrow vertical obstacle. Describe fully and explain carefully the effects observed on the screen in each case making suitable comparisons.

What will happen if the sodium light is replaced by arc light ? (Gau. U. 1951)

26. What are half period zones ? Explain how the rectilinear propagation of light has been accounted for by the wave theory.
(Raj. U. 1954 ; *cf.* Utk. U. 1949, '70S)

27. Explain what is meant by the resolving power of a diffraction grating. Show that the resolving power is given by the product of the total number of rulings and the order number of the spectrum.

A grating has 1000 lines ruled on it. In the region of wavelength $\lambda = 6000 \times 10^{-8}$ cm., what will be the difference between two wavelengths which would just appear to be separated in the first order spectrum. (C. U. 1966)
[*Ans.* 6 A.]

CHAPTER IV || POLARISATION OF LIGHT POLARIMETERS

71. Introduction :—Huygens in 1690 discovered the phenomenon of polarisation while observing the behaviour of a pencil of natural light transmitted through a crystal of Iceland spar. When natural light passes through such a crystal it shows a kind of asymmetry in optical properties with respect to its direction of propagation. This lack of symmetry is due to the acquirement of *two-sidedness* or *polarity* by the pencil of the rays. This phenomenon wherein a pencil of natural light acquires a definite two-sidedness as a result of transmission through a transparent crystal like Iceland spar or tourmaline is known as **plane polarisation.**

So far as the phenomena of interference and diffraction are concerned, they can be completely interpreted from the standpoint of the wave theory of light without any assumption as to the nature of light waves. We know that the waves in general may be either longitudinal or transverse. Both these types of waves actually exhibit the phenomena of interference and diffraction. But the phenomenon of polarisation cannot be properly interpreted unless a definite character is assigned to the light waves. We know that the characteristic property of a longitudinal wave is that each particle of the medium of propagation executes simple harmonic vibrations about some mean position of rest in the direction of propagation of the waves. Sound is propagated by longitudinal wave motion. In a transverse wave the particles of the medium of propagation execute simple harmonic vibrations about their positions of rest in a direction at right angles to the direction of propagation of the wave. Waves formed on the surface of water are transverse in nature. We shall have to judge from the nature of the phenomenon of polarisation whether light waves belong to the first (longitudinal) type or to the second (transverse) type of waves.

72. Transverse nature of light waves :—When a pencil of ordinary light is allowed to fall on a plate of tourmaline* crystal cut

*Tourmaline is a double refracting crystal which cleaves in the form shown in Fig. 68(*a*). The crystal has a lightly bluish tinge. A pencil of natural light incident on a tourmaline crystal is split up into two components. But a moderate thickness of the crystal will selectively absorb the light polarised in a plane containing the optic axis [*o.a.* in Fig. 68(*a*)] *i.e.*, the ordinary ray. The other component *i.e.* the extra-ordinary ray gets through the crystal. The *dichroic* property renders the crystal to be used as an analyser or as a polariser. The transmitted polarised light is green in colour.

parallel to its optic axis (*o.a.*) and the transmitted light is observed by a naked eye, the pencil of light will appear to have slightly changed in colour owing to selective absorption within the crystal. The

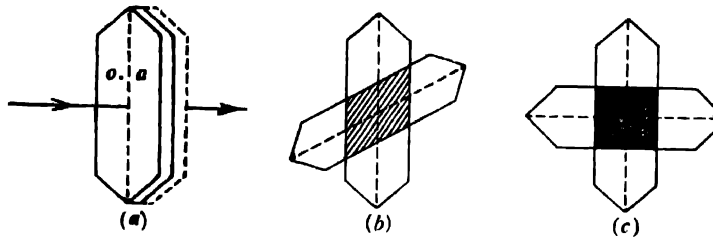


Fig. 68

character and intensity of the light remains the same even if the plate is rotated. If now the light transmitted through this plate is allowed to pass through a second such plate of tourmaline which is also cut parallel to the axis of the crystal, the light is observed to pass freely through the second plate without diminution in intensity provided the axis of the second plate is parallel to that of the first [Fig. 68 (*a*)—the second crystal shown by dotted lines]. If, however, the second plate is gradually rotated with its plane always perpendicular to the pencil of rays so that the axes of the two crystals are now inclined at a finite angle [Fig. 68(*b*)], the intensity of the light emerging from the second plate will gradually diminish. When the axes are at right angles to one another (*i.e.* crossed) [Fig. 68(*c*)] the light from the first plate is entirely refused transmission through the second plate and is completely extinguished. If the rotation is continued past 90°, the light reappears and continuously increases in intensity with the progress of rotation, till another 90° is completed when the light appears as bright as at the start. The same phenomena will be observed if the second plate is kept fixed and the first one is rotated as above. It is immaterial which plate is turned ; but turning both of them in the same direction and to the same extent will produce no effect.

The explanation of the above phenomenon points to the actual mechanism of the propagation of light. That light is a kind of wave motion set up in the hypothetical medium, ether, establishes itself from the observations on interference and diffraction of light. But no conclusion whatsoever can be arrived at regarding the nature of vibrations in ether from the study of these phenomena. Huygens and Young thought that the light waves are longitudinal in nature as is the case with sound waves. But from the fact that there is one position of the second plate which entirely cuts off the light transmitted through the first (in above experiment), it has been con-

cluded that light waves cannot be longitudinal, for if they were so, the rotation of the second plate with respect to the first would have produced no effect. A simple experiment will illustrate the meaning implied in the last sentence. Let a stretched rubber cord *CD* be threaded through a narrow slit *S* in a cardboard *AB* (Fig. 69). Now if a longitudinal wave motion is set up in *CD* by moving the end *C* or *D* to-and-fro along the length of the cord, it is evident the longitudinal wave along *CD* passing through the slit cannot be disturbed by turning the slit round the string. But if *C* is moved up and down in a vertical plane so that transverse waves pass along the cord *CD*, each point of the cord moves up and down as the wave passes. In this case it is evident that if the slit is vertical as *SV* (Fig. 70) the waves will easily get through this slit. If however, the slit is made horizontal as *SH* by rotating the cardboard (Fig. 71), the waves will be stopped by the slit. Thus we see the fact that all the particles that vibrate in a vertical plane automatically single out this particular plane as one with respect to which the waves have distinct properties as compared with other planes. From the above simple experiment it is also evident that a transverse wave whose vibrations are confined to a single plane can under suitable conditions be stopped by a slit.

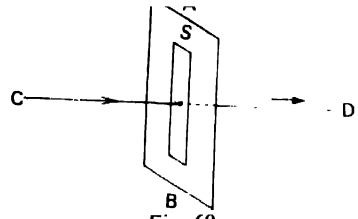


Fig. 69

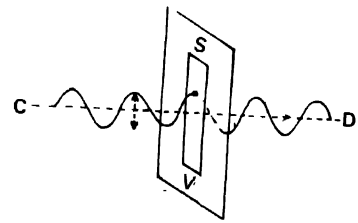


Fig. 70

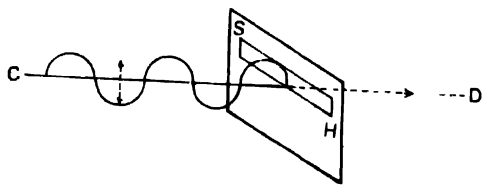


Fig. 71

The complete extinction of light by a properly oriented second tourmaline plate automatically implies that here an optical mechanism similar to the above simple mechanical arrangement may be involved.

It was Fresnel who for the first time thought that the light waves are transverse in character and on the supposition of this transverse nature of the light waves he succeeded in explaining the

variation of intensity in the experiment with tourmaline crystals. According to Fresnel, (*a*) ordinary light consists of the waves set up in hypothetical ether medium wherein the ether particles vibrate in all directions normal to the direction of propagation of the waves, (*b*) tourmaline crystals possess the property of allowing the vibrations parallel to a particular direction (*i.e. optic axis*) only to pass through them. *So during the passage through such crystals, the vibrations of ether particles become restricted to a particular plane only or, in other words, they become plane polarised.*

As in a pencil of ordinary light the vibrations take place equally in all directions at right angles to the direction of the ray, the intensity of the light transmitted through a single tourmaline plate remains constant as the plate is rotated about an axis normal to the plane of the plate. The privileged direction of transmission of the

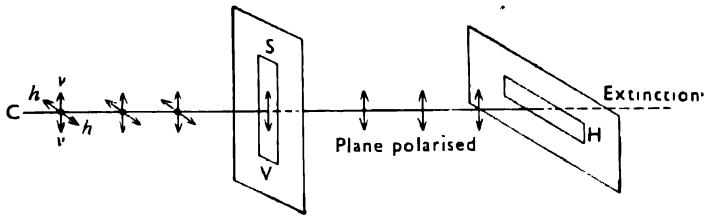


Fig. 72

vibrations is the optic axis of the tourmaline plate. In all possible orientations, this optic axis being always similarly placed with respect to an equal number of rays, the transmitted intensity remains unaltered during the rotation of the plate. In the case of two crystals, the vibrations occur in one plane only after passing through the first plate and when the second plate is placed normally to the path of light and rotated gradually with respect to the first, this plane polarised light is also gradually stopped and is finally extinguished when the axes of the crystals are crossed just as the transverse vibrations of a rubber cord freely taking place along a long slit will gradually be damped and ultimately altogether stopped by a second such slit when the later is gradually turned and finally set at right angles to the first (Fig. 72). This is evident from the following simple considerations. Let *PQ* (Fig. 73) represent the direction of propagation of the light and *AB*, the optic axis of the first tourmaline plate. The vibration of the light waves transmitted through the first plate will evidently be restricted only in the direction

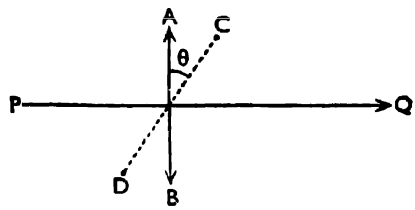


Fig. 73

AB. When this transmitted light falls on the second plate with its optic axis along *CD* inclined at an angle θ with *AB*—the planes containing *AB* and *CD* being evidently parallel—the second plate does not allow the vibrations along *AB* to pass through it completely. Only a component of *AB* along *CD*, *i.e.* *AB* cos θ, is allowed to get through it. So the intensity of the finally transmitted light varies with cos θ. When *CD* is crossed with respect to *AB* (*i.e.* *AB* and *CD* are at right angles), cos θ becomes zero and as such no vibration is transmitted through the second plate. So a complete extinction is secured by a pair of crossed tourmaline plates.

(*ii*) If a plane polarised light falls on a reflector and the latter is turned round the beam as the axis, the angle of incidence remaining the same (see Biot's polariscope), the intensity of the beam reflected from the reflector passes through two maxima and two minima as the reflector (and hence the plane of incidence) is rotated through 360°. These observations can neither be explained on the assumption of the longitudinal nature of the light waves. But by assuming the light waves to be transverse in character, these observations can be readily explained.

(*iii*) The transverse nature of light waves is also corroborated by the observations on *the interference of polarised light*. A beam of monochromatic light is first passed through a tourmaline crystal *P* and the transmitted plane polarised beam is then allowed to fall on a fine slit *S*. The light from *S* is then focussed by means of two half-lenses L_1 and L_2 on two points T_1 and T_2 (Fig. 74). These two images T_1 and T_2 of *S* are two coherent sources and would under ordinary circumstances produce interference fringes on a screen placed in the proper overlapping region. Now if light from the images T_1 and T_2 be allowed to pass through two tourmaline plates as shown in Fig. 74, with their optic axes in the plane of the plate and parallel to each other, the interference fringes are cast on the screen as usual. These interference fringes exhibit colour effects similar to the colours of thin films in ordinary light. But if one of the plates, T_1 or T_2 be turned through 90°, the fringes vanish altogether. This is due to the fact that in the later case, the vibrations in the interfering waves, are not coincident, but at right angles to each other. These two rectangular vibrations combine in the overlapping region as usual and do not produce interference fringes but give general illumination on the screen

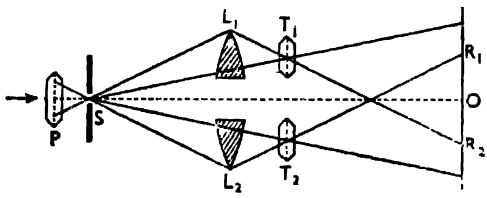


Fig. 74

whose intensity is equal to the sum of the intensities of the component vibrations. The light in the overlapping region is found to be elliptically polarised. As a result of the systematic investigations of Fresnel and Arago on the interference of plane polarised light, it was found that two plane polarised rays interfere and produce fringes as ordinary light if they are polarised in the same plane and originally belonged to the same polarised pencil. This observation confirms the theory that the vibrations of the ether particles in a light wave are transverse to the direction of propagation of the light. In the case of a longitudinal wave the nature of vibration of the interfering components would not have been affected by the rotation of any of the tourmaline plates and for any relative orientations of the plates interference fringes would have been produced. This is not in accordance with observed facts and as such the light waves cannot be longitudinal in nature.

(*iv*) Moreover, the facts that two light rays polarised in different planes combine in general to yield elliptically polarised and in particular cases circularly polarised light, also indicate that light waves are transverse in nature.

(*v*) Zeeman effect, wherein the spectral lines emitted by atoms placed in a strong magnetic field are split up and polarised, suggests that the light waves are transverse in nature.

Though in the above discussions it has been indicated that light waves are transverse in nature, the above observations do not necessarily rule out the possibility of component longitudinal vibration in the light waves. Even if this longitudinal component be there, it does not in any way help us in explaining the observed facts that have been discussed above.

73. Polarisation : Polarised Light : Plane of Polarisation :— Ordinary light consists of transverse waves set up in the ether medium such that the vibrations of the ether particles take place in all arbitrary directions transverse to the direction of propagation of light. If by some act these vibrations can be confined to a particular plane, then the light is said to be *polarised.* Thus *polarisation* (linear) means the act of making the ether particles vibrate in a particular plane only (instead of all arbitrary planes), the vibrations being restricted in a direction transverse with respect to the ray. As has been

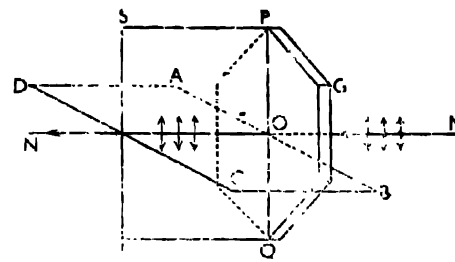


Fig. 75

mentioned in the introduction, polarisation arises due to a kind of asymmetry introduced in natural light by some artificial means. This asymmetry is due to the fact that the vibrations of the ether particles are so restricted that each of these particles in the wavefront continues to describe identical paths. If this path be a straight line, the light is then said to be *plane polarised or linearly polarised*; if the path is a circle, light becomes *circularly polarised*; and if the path described is an ellipse, the light is then said to *elliptically polarised.* Circular polarisation is produced when two plane polarised beams, vibrating a right angles to each other, equal in amplitude and frequency but differing in phase by 90°, are combined together. But if two plane polarised beams, polarised at right angles to each other and differing both in phase and amplitude are combined together elliptically polarised light will be produced.

Plane polarised light can be produced either by reflection or by refraction through a crystal. As has been shown earlier, a simple method of producing plane polarised light consists in allowing a pencil of natural light *MON* to pass normally through a tourmaline plate *PGQ* (Fig. 75) which is cut so that the refracting faces of the crystal are parallel to the axis *POQ*. Before passing through the plate the ordinary light waves may be supposed to be comprised of transverse vibrations in all possible directions at right angles to the direction of propagation. For convenience, these vibrations may be resolved into two mutually perpendicular vibrations, one in the plane of paper (as indicated by the double-headed arrows) and the other normal to this plane (as indicated by the black dots midway between the arrows) (Fig. 75). The optic axis *POQ* of the tourmaline plate also lies in the plane of the paper. So on transmission through the plate the vibrations in the plane of the paper (*i.e.* parallel to the optic axis) will be only allowed to get through whereas the vibrations perpendicular to the plane of the paper will be cut off. Thus the transmitted light will be plane polarised wherein vibrations occur only in the plane of the paper, *i.e.* parallel to optic axis of the crystal. The *plane PQRS containing the ray and parallel to the optic axis of the crystal is called the* **plane of vibrations** *and the plane ABCD drawn through the direction of the ray and at right angles to the plane of vibration is called the* **plane of polarisation** *of the transmitted light.*

Note :—It must be remembered that if the thickness of the tourmaline plate be very small, actually two pencils of rays will come out of the crystal due to the phenomenon of double refraction and that these two pencils are plane polarised at right angles to each other. One of them is the ordinary component and the other the extraordinary component. But as most of the tourmaline crystals used in the laboratory are of considerable thickness, the

ordinary component is selectively absorbed and we get only one type (extraordinary) of transmitted plane polarised light.

74. **Polarisation by Reflection : Polarising Angle :**—Ordinary light may be polarised by reflection. Malus observed for the first time that the light reflected from the surface of a glass plate gets polarised It has been subsequently found that polarisation occurs by reflection from any polished surface—opaque or transparent. But the proportion of the polarised light in the reflected beam has been found to depend on the angle of incidence. Experimentally it has been observed that as the angle of incidence is changed from 0° to 90°, the quantity of polarised light in the reflected beam increases progressively at first with the angle of incidence, reaches *a maximum value at some particular angle of incidence depending on the nature of the reflecting surface*, and finally decreases again.

For each reflecting substance there is a particular angle of incidence for which maximum polarisation takes place in the reflected light. This angle is called the *angle of polarisation* or *polarising angle* of that reflecting substances. Results of M. Jamin's investigations show that only a few substances having refractive index of about 1·46, can polarise light completely by reflection. For them, evidently the angle of polarisation is the angle of incidence producing complete polarisation. For all other substances, the angle of polarisation means the angle of incidence corresponding to maximum polarisation.

Polarisation by reflection can be demonstrated by a simple experiment. Let RR_1 (Fig. 76) be a plane reflector with its reflecting surface normal to the plane of the paper. PO is an ordinary incident ray in which the transverse vibrations of the ether particles may be supposed to be resolved into two mutually perpendicular components. One of these vibrations occur in the plane of the paper (shown by double-headed arrows) and the other in a plane parallel to the plane of the reflector (shown by black dots). Since ON is the normal at the plane of incidence, the plane of the paper is evidently the plane of incidence. Now if the reflected ray OE be allowed to pass through a tourmaline plate T cut parallel to its optic axis, then it will be found that when the optic axis of T is in the plane of incidence the transmitted beam has the minimum intensity. For any other direction the intensity is greater than this minimum value. So it is obvious that in the reflected beam there is a portion which

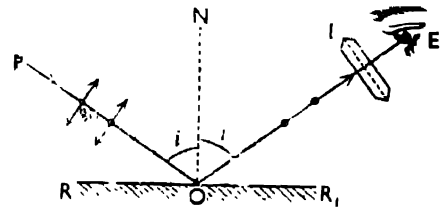


Fig. 76

is plane polarised and whose plane of vibration is normal to the plane of incidence. Again if the tourmaline plate is placed with its optic axis parallel to plane of incidence and the angle of incidence is gradually varied, it will be found that for a particular value of the angle of incidence, the intensity of the light received by the eye E is minimum. Hence the angle corresponds to the maximum polarisation (in a plane parallel to the optic axis), and is known as the polarising angle.

75. Plane of polarisation : Plane of vibration (in reflection) :— We know that the *plane of incidence* is the plane passing through the incident ray and the normal at the point of incidence and that the *plane of reflection* is the plane passing through the reflected ray and the normal drawn on the reflecting surface at the point of reflection. Both these planes of incidence and reflection are obviously identical.

When plane polarised light is obtained by reflection, the *plane of polarisation* of the reflected ray is the plane of reflection *i.e.* the plane of incidence also. According to Fresnel the vibrations of polarised light take place in a plane perpendicular to the plane of polarisation. Hence the *plane of vibration* of light polarised by reflection is perpendicular to the plane of incidence, *i.e.* the direction of vibration of the ether particles is parallel to the plane of the reflecting surface. In Fig. 77, AB is the trace of a reflecting surface whose plane is perpendicular to the plane of the paper. A ray PO in the plane of the paper is incident at O on AB and ON is the normal at the point of incidence. Then

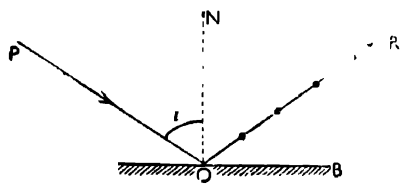


Fig. 77

the plane of incidence is the plane containing PQ and ON, *i.e.* the plane of the paper. Since the reflected ray OR also lies in the same plane, the plane of paper is also the plane of reflection. The reflected ray OR is polarised in the plane of reflection and hence the plane of polarisation of OR is also the plane of the paper. The plane of vibration of the ether particles in the reflected ray is normal to the plane of paper and is consequently parallel to the plane of the reflecting surface AB. In the figure the black dots indicate that the vibration takes place up and down in straight lines at right angles to the plane of the paper through the position of each of these dots.

76. Production and detection of plane polarised light by reflection : Biot's Polariscope :—The production and detection of plane polarised light by reflection are most conveniently done in the laboratory with the help of an apparatus known as Biot's polariscope.

Biot's polariscope consists of two mirrors A and B of polished black glass fitted at two ends of a vertical frame work as shown in Fig. 78. The mirror A can be rotated about a horizontal axis and the angle of rotation can be conveniently noted from a circular scale attached to the vertical stand. The mirror B can also be rotated about a horizontal axis and the supports carrying B are fixed to a metal ring which is capable of rotation about a vertical axis over a scale graduated in degrees. To understand the working principle of the apparatus let us suppose that the mirrors A and B are initially set parallel to each other. Let A be kept fixed in position and a pencil of ordinary light MO be incident on it. If the mirror B be now rotated round the vertical axis of the apparatus keeping the inclination of B with respect the axis of rotation constant, it will be found that the amount of light reflected from B will diminish and become a minimum when the mirror is rotated through 90° so that A and B are crossed. It will be observed that the minimum amount of light which is reflected when the mirrors are crossed depends on the angle of incidence of the light and for one particular value of this angle (for glass) the light reflected from B almost vanishes. This angle is the polarising angle for glass which is found to be nearly 57°.

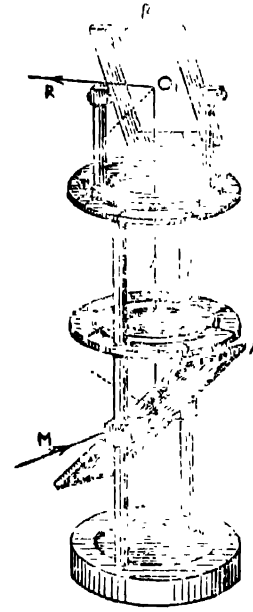


Fig. 78

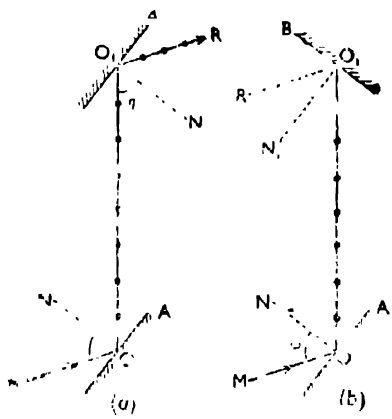


Fig. 79

Suppose a ray of light MO is incident on the mirror A at an angle ϕ with the normal [Fig. 79(a)]. The reflected ray OO_1 will be polarised in the plane of the paper which is the plane of incidence. If the reflected ray is received on the second mirror B also at an angle ϕ and if the plane of incidence of OO_1 on B be the same as the plane of incidence on A the polarised light will be reflected along O_1R.

Now rotating B round the vertical axis of the apparatus, the angle of incidence of OO_1 on B remains the same (*i.e.* ϕ) but the plane of incidence of OO_1 on B changes and since OO_1 is polarised in the plane of the paper, the intensity of the reflected ray will diminish at B is gradually rotated round OO_1 (*i.e.* the axis of the apparatus). When the plane of incidence of OO_1 on B is perpendicular to the plane of the paper (Fig. 79(b)). *i.e.* when the plane of the mirror B is at right angles to that of A, the intensity of the reflected ray O_1R will be reduced to a minimum. If by trial ϕ is adjusted for the polarising angle, this intensity practically reduces to zero, which indicates that, at the polarising angle, the reflected ray OO_1 has been almost completely polarised in the plane of incidence of MO. Hence at the crossed position of B, the light is practically extinguished. A and B in this experiment serve respectively as a polariser and analyser.

17. Brewster's Law :—Sir David Brewster, from a series of experiments made with different transparent reflectors, discovered that complete extinction might be expected when the refracted ray is at right angles to the reflected ray. The particular angle of incidence at which the reflected ray and refracted ray are mutually perpendicular (*i.e.* when extinction is possible) is the polarising angle for the reflector used.

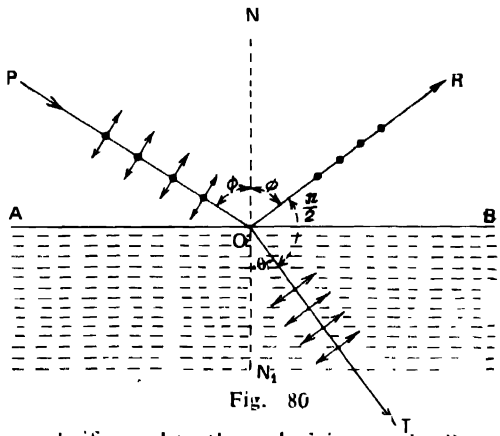


Fig. 80

If PO, OR and OT (Fig. 80) be the incident, reflected and refracted rays, when the angle of incidence, ϕ, if equal to the polarising angle, Brewster's law states that

$$\angle ROT = \frac{\pi}{2}.$$

If θ be the angle of refraction, then $\phi + \theta = \frac{\pi}{2}$ also.

But $\mu = \frac{\sin \phi}{\sin \theta}$, where μ is the refractive index of the material of the reflector.

As $\sin \theta = \sin\left(\frac{\pi}{2} - \phi\right) = \cos \phi$,

$$\mu = \frac{\sin \phi}{\sin \theta} = \frac{\sin \phi}{\cos \phi} = \tan \phi \qquad (1)$$

This relation (1) which states that the refractive index of the medium of the reflector is equal to the tangent of the polarising angle is another form of enunciating Brewster's law. The refractive index depends of the colour of the incident light but for ordinary glass or water the dispersion is so small that the variation in the angle of polarisation over the whole range of visible spectrum is almost negligible.

For ordinary glass $\mu = 1{\cdot}5$, hence from Brewster's law, the polarising angle for glass will be ϕ where $\tan \phi = 1{\cdot}5$,

$$\text{or,} \quad \phi = \tan^{-1} 1{\cdot}5 = 57° \text{ nearly.}$$

78. Pile of Plates :—When light falls on a polished surface of glass only a small fraction of the light is reflected, the major part being refracted and ultimately transmitted through the transparent glass medium. Even when the light is incident at the polarising angle the reflected light is only partly plane polarised. The transmitted light is also far from being completely polarised. As shown in Fig. 80, the incident unpolarised light PQ has its vibrations both in the plane of the reflector (*i.e.* normal to the plane of the paper) as also in the plane of incidence (*i.e.* in the plane of the paper). On reflection from the surface AB only the vibrations parallel to the plane of the reflector (represented by the black dots) will be present in the reflected beam and the remaining component, *i.e.* the vibration in the plane of incidence (represented

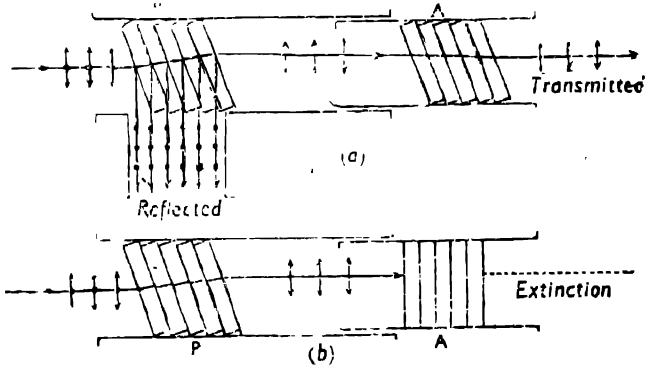


Fig. 81

by double-headed arrows) will be refracted and ultimately transmitted. But even when the light is incident at the polarising angle, this division is not perfect. Neither the reflected nor the transmitted component will be perfectly plane polarised. So to obtain perfectly plane polarised light, it is usual to make use of a *pile of plates*, which consists of a number of identical parallel faced glass plates placed together as a pile or bunch. At each succeeding reflection the light becomes richer

in waves vibrating at right angles to the plane of incidence. With a pile of more than ten plates, practically the whole of the waves vibrating in a plane at right angles to the plane of incidence will get reflected and the transmitted component will be highly plane polarised in a plane normal to the plane of incidence. This transmitted polarised light is generally studied and used for practical purposes. The reflected plane polarised light may also be utilised by arranging a side-tube at the position of the polarising pile as shown in Fig. 81 (*a*). With twenty plates in the pile, the transmitted light becomes plane polarised to the extent of 80% nearly. Since such a pile transmits only, an almost completely plane polarised beam, it can be used as a *polariser*. If two such piles are mounted on two coaxially fitted tubes as shown in Fig. 81 (*a*) & (*b*), the first pile acts as the *polariser* and the second as the *analyser*. If the surfaces of the piles are parallel as in Fig. 81 (*a*) the plane polarised light transmitted through the polarising pile will be completely transmitted through the second pile also. But if the second pile be rotated about the beam of light as the axis, the intensity of the transmitted light gradually falls till at an angle of rotation of 90°, the beam is almost completely extinguished by the analyser [Fig. 81 (*b*)].

79. Malus' cosine-squared Law :—Let OA (Fig. 82) be the direction of vibrations which are reflected by a reflecting analyser or transmitted by an analysing pile of plates. Let OC be the plane of vibration of the plane polarised light falling on the analyser where $\angle AOC = \theta$. If a be the amplitude of the incident light, the cosine component (OA) of a will only get through the analyser and the sine component (OB) will be debarred from reflection or transmission as the case may be. The amplitude of the light coming out of the analyser is evidently $a \cos\theta$ and hence the intensity of that light as given by $I = a^2 \cos^2\theta = I_0 \cos^2\theta$, where I_0 is the intensity of the incident light. This result was empirically arrived at by Malus in 1810 and is known as Malus' cosine-squared law. Malus' law states that *the intensity of the light emerging from an analyser is proportional to the square of the cosine of the angle between the polariser and the analyser*.

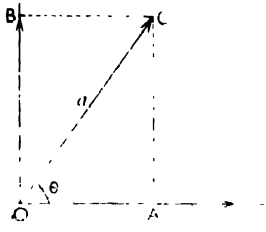


Fig. 82

80. Calcite crystal : its Optical Properties :—Calcite is a transparent crystalline form of Calcium Carbonate. It is also known as calspar or Iceland spar. Though it crystallises in various forms, it can always be reduced by cleavage to a

fundamental rhombohedral form (Fig. 83). The rhombohedron is bounded by six parallelograms each of which has two acute angles of 78°5′ and two obtuse angles of 101°55′. The dihedral angles between the faces are 105°5′ and 74°55′. Out of the eight corners, solid angles at six corners are formed by one obtuse and two acute angles. But the solid angles at the other two opposite corners *A* and *B* are formed by three obtuse angles. The direction which makes equal angles with the three faces either at *A* or at *B* is known as the **optic axis** of the crystal. It must be realised that the optic axis of a crystal is not a particular line in the crystal. It actually indicates a definite direction. In case the edges of the rhombohedron are equal, the optic axis will be parallel to the line *AB* as shown in Fig 83. Crystals like calcite, quartz, tourmaline etc. which possess only one optic axis are known as *uniaxial crystals*. Crystals like aragonite, mica, borax, etc. have two optic axes and are therefore called *biaxial crystals*.

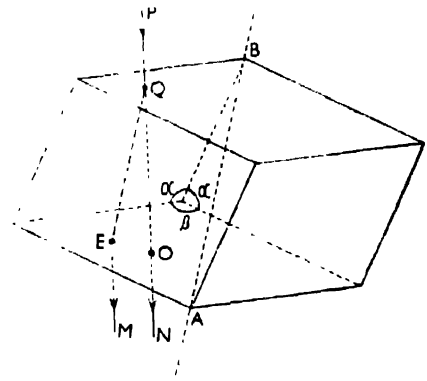


Fig. 83
(Calcite Crystal : $\alpha = 78°5'$, $\beta = 101°55'$, *AB*, the optic axis)

It must be noted that the optic axis of a crystal has distinct and special optical properties which remain unaltered even if the length, breadth or thickness of the crystal be altered. So this axis may be looked upon as an optically favoured direction in the crystal.

81. Principal section :—The plane which passes through the optic axis and is normal to a face of a calcite rhomb is called the **principal section** of the calcite crystal. For every point in the crystal three principal sections can always be obtained. In a rhombohedron of calcite, a principal section always intersects the surfaces in a parallelogram of approximate angles. 109° and 71° as shown in Fig. 84(*a*). An end-on view of a calcite rhomb is shown in

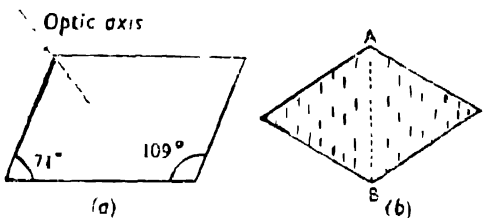


Fig. 84

Fig. 84(*b*) in which *AB* represents the principal section. All planes through the rhomb parallel to *AB* are also principal sections as indicated by the dotted lines parallel to *AB*.

82. Double Refraction : Polarisation by Double Refraction :— Erasmus Bartholinus in 1669 discovered that a pencil of natural light falling normally on the surface of a calcite crystal is divided into two distinct pencils within the crystal due to refraction. This phenomenon is known as **double refraction** or *birefringence*. To study the effect of double refraction a rhomb of calcite (Iceland spar) is taken and a pencil of natural light is made normally incident on the surface of the rhomb. It is found to split up into two pencils of equal intensity inside the rhomb and to emerge out in parallel directions. In Fig. 85(*a*), *PQ* is the incident pencil and *QO* and *QE* are the two refracted pencils in the rhomb which ultimately emerge out as *ON* and *EM*. To understand the phenomenon thoroughly, let us consider a principal section of the rhomb passing through the point of incidence *Q*. Let *ABCD* be this section [Fig. 85(*a*)] and the incident pencil *PQ* fall normally at *Q* on the face *AB*. Due to refraction in the rhomb the pencil breaks up into two components of which one (*QO*) passes without deviation through the rhomb and emerges out as *ON*. The second component *QE* is deviated within the rhomb and thereby suffering a lateral displacement ultimately emerges out of the crystal in the direction *EM* parallel to *QON*. The pencil *QON* which emerges undeviated (as the incidence is normal) obeys the

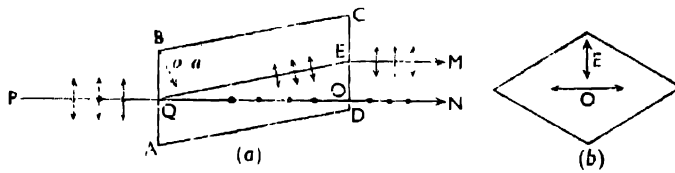


Fig. 85

ordinary laws of refraction and is known as the **ordinary ray**. The refractive index of the crystal for this ordinary ray has always a fixed value irrespective of the manner in which the crystal is cut or of the way in which the ray is incident on it. But in the case of the pencil *QE*, Snell's law is not obeyed. Here the angle of refraction is not generally in the same plane as the angle of incidence. The refractive index of the crystal for this ray is neither a constant quantity. So this ray *QE* is called the **extraordinary ray**.

The **principal plane** of a refracted ray (ordinary or extraordinary) in the plane containing the ray and the optic axis The principal plane and principal section are, however, identical for normal incidence.

When the emergent rays are examined by a suitable analyser of any type it is found that the *ordinary ray is polarised in the*

principal plane with its vibration occurring perpendicularly to that plane and that the extraordinary ray is polarised perpendicularly to the corresponding principal plane. Hence the *ordinary ray vibrates perpendicularly to the optic axis* and the *extraordinary ray vibrates parallel* to it [Fig. 85 (*a*)]—the plane containing the two rays being parallel to the direction of the optic axis. Fig. 85 (*b*) shows an end on view of the section of the calcite rhomb wherein the vibrations of the ordinary and extraordinary rays are indicated respectively by the double-headed arrows *O* and *E*.

As a result of double refraction an object looked at through a double-refracting crystal will produce two images—one ordinary and the other extraordinary. If a black dot is marked on a piece of white paper and a calcite rhomb be placed on it, two dots will be seen through it. On rotating the rhomb one of these dots remains fixed while the other rotates round the stationary image. The fixed image is due to the ordinary rays and the rotating one due to the extraordinary ray. When viewed through the rhomb the ordinary image seems to be nearer the eye than the extraordinary image. This is due to the fact that the refractive index of calcite for the ordinary ray is greater than that for the extraordinary ray ($\because \mu = \frac{\text{real thickness of slab}}{\text{apparent distance of the spot}}$, the apparent distance being smaller, μ will be greater). Again as the refractive index is defined by the relation $\mu = C_v/C_m$, where C_v = velocity of light in vacuum and C_m is that in a material medium, we have the *ordinary refractive index* $\mu_o = C_v/C_o$ and the *extraordinary refractive index*, $\mu_e = C_v/C_e$, C_o and O_e representing the velocities of the ordinary and the extraordinary components of light in the medium. Now as in calcite, $\mu_o > \mu_e$, the velocity of the extraordinary ray C_e will be greater than the velocity of the ordinary ray C_o. The crystals wherein the velocity of the ordinary ray is less than the velocity of the extraordinary ray are known as *negative crystals*. Calcite, tourmaline, ruby, etc. belong to this variety. The crystals in which the velocity of the ordinary ray is greater than that of the extraordinary ray are known as *positive crystals*. Quartz, ice, etc. belong to this variety. For Iceland spar, $\mu_e = 1{\cdot}49$ and $\mu_o = 1{\cdot}66$ and for quartz, $\mu_e = 1{\cdot}55$ and $\mu_o = 1{\cdot}54$.

Dichroism :—Dichroism is the property possessed by some crystals of selectively absorbing light vibrations in one plane, whilst allowing the vibrations at right angles to pass through. So the transmitted light being plane polarised the dichroic crystals act both as polarisers and analysers. Tourmaline and herapathite are dichroic in nature

83. Huygens' observations on Double Refraction :—Huygens found that when a pencil of natural light passes through a calcite rhomb *in any direction other than its optic axis* it always splits up into two component pencils, one ordinary and the other extraordinary, which are of identical intensity but are polarised in

mutually perpendicular planes. Each of these two rays can again be split up into two components of unequal intensity by passing them through a second calcite rhomb except when the principal sections of the two rhombs are either parallel or mutually perpendicular. In these two cases each of the above rays will be transmitted through the second rhomb without further bifurcation. If the angle between the principal sections of the rhombs be 45°, the two rays after splitting will be of equal intensity. When the principal sections are parallel, the ordinary ray from the first rhomb passes through the second as ordinary ray and so does the extraordinary ray as an extraordinary ray. When the two principal sections are mutually perpendicular, the ordinary ray from the first rhomb passes through the second as the extraordinary ray and the extraordinary ray emerges out as the ordinary ray. The actual state of affairs is represented in Fig. 86 (*a*), (*b*) & (*c*). As one of

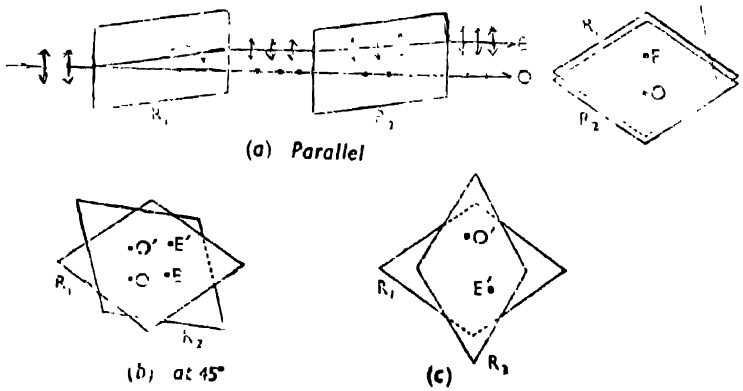


Fig. 86

the rhombs is rotated keeping the other fixed, the appearance of the images changes showing the asymmetry of the rays about the direction of propagation.

84. Huygens' Theory of Double Refraction :— In order to explain the phenomenon of double refraction Huygens extended his theory of formation of secondary wavelets. Since the wave-surface in substances like water or glass is spherical and as one of the rays in Iceland spar obeys the ordinary laws of refraction, Huygens assumed that the corresponding wave surface should also be spherical. From a detailed study of the laws that govern the other ray, he supposed that the extraordinary wavefront should, in the simplest case, be an ellipsoid of revolution, *i.e.* a spheroid. The optic axis being an axis of symmetry, the axis of revolution of this spheroid was supposed to coincide with the optic axis. If a pencil of light falls on the face of a calcite rhomb and the angle is gradually varied, it is found that the ordinary and extraordinary rays subtend smaller

and smaller angles with one another as they become more and more parallel to the optic axis. Along the optic axis these two rays become coincident. So the ordinary and the extraordinary velocities are equal in the direction of the optic axis. To explain this Huygens assumed that the spheroidal and spherical wave surfaces touch each other in the direction of the axis of revolution of the generating ellipse. In the case of *calcite the generating ellipse rotates about its minor axis* and the spheroid is *oblate* in nature. So the spherical wave-surface lies within the spheroidal one. This type of crystals wherein the spherical wave surface lies within the spheroidal one is known as a *negative* crystal. For substances like ice, quartz, etc. the spheroidal surface lies within the spherical one and the spheroid is generated by rotating the ellipse about its major axis. Here the spheroid is *prolate* and the crystal is *positive* in nature. Fig. 87(*a*) and (*b*) respectively show the sections of wave-surfaces in negative and positive crystals along with their corresponding optic axes. To understand how Huygens used his new ideas regarding the ordinary and extraordinary wavelets to find the direction of the refracted rays let us consider a few simple cases.

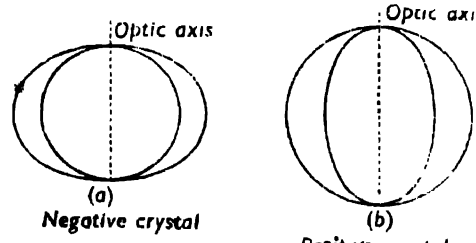


Fig. 87

(*i*) *When the optic axis lies in the plane of incidence but is inclined to the refracting face of the calcite crystal.*

Let SS be the refracting face of a uniaxial negative crystal (*e.g.* calcite) and PA and QBC be two rays obliquely incident on SS, AB being the corresponding plane wavefront. The optic axis (o, a_1) of the crystal is in the plane of incidence but is inclined to SS as shown in Fig. 88(*a*).

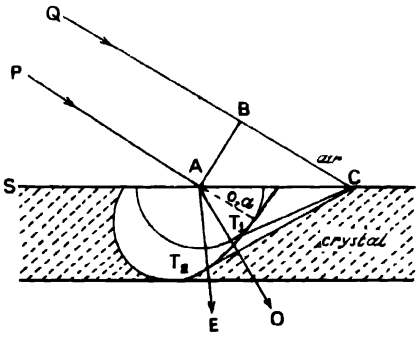


Fig. 88 (*a*)

To find the direction of the refracted ordinary ray, a circle is to be drawn with A as centre and BC/μ_o as radius. From C, a tangent CT_1 is drawn to this circle, CT_1 obviously represents the ordinary refracted wavefront and the direction, AT_1O represents the direction of the refracted ordinary ray.

In the case of the extraordinary ray, the maximum refractive index of the crystal for the extraordinary ray is equal to μ_e and the

principal refractive index of the *E*-ray is the minimum value of the refractive index which is definitely less than μ_o in a negative crystal. So the extraordinary wavefront in the sectional diagram will be an ellipse whose minor axis coincides with the optic axis and whose major axis is given by BC/μ_e where μ_e is the principal refractive index for the *E*-ray. The major axis and the minor axis being known and the position of the latter being along the optic axis (*o.a.*), the ellipse can be drawn. The tangent CT_2 on this ellipse gives the extraordinary refracted wavefront and AT_2E represents the refracted extraordinary ray. Thus the *E*-ray and the *O*-ray come out from the crystal in different directions.

(*ii*) *When the optic axis lies in the plane of incidence and is parallel to the refracting face of the calcite crystal.*

In this case since the optic axis is parallel to the face of the crystal and also lies in the plane of incidence it must be along *SS*. By an exactly similar method as in case (i), the positions of the extraordinary wavefront *CE* and the ordinary wavefront *CO* can be drawn, supposing the rays are obliquely incident on *SS*. Here *AO* and *AE* respectively represent the refracted *O*-ray and *E*-ray as shown in Fig. 88(*b*). These rays travel with different velocities in different directions. Here the sphere and the spheroid (or in the sectional diagram the circle and the ellipse) touch along *SS* (*i.e.* along the optic axis). Since the point *C* is on the minor axis of the ellipse, the line joining *O* and *E*, *i.e.* *EO* will intersect the line *AC* at right angles (See any text book on Co-ordinate Geometry).

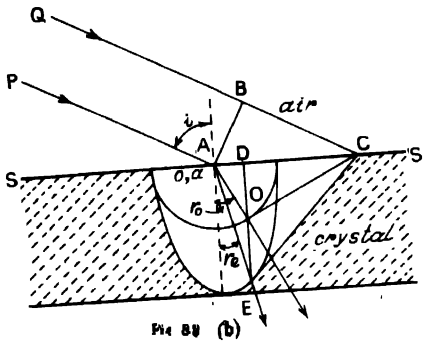


Fig. 88 (b)

Let r_o and r_e be the angles of refraction of the ordinary and extraordinary rays and μ_o and μ_e be the ordinary and extraordinary refractive indices of the crystal.

Then, $\tan r_o = \dfrac{AD}{DO}$, and $\tan r_e = \dfrac{AD}{DE}$.

$$\therefore \quad \frac{\tan r_o}{\tan r_e} = \frac{DE}{DO}.$$

But $\mu_o \propto \dfrac{1}{DO}$, and $\mu_e \propto \dfrac{1}{DE}$.

$$\therefore \quad \frac{\mu_o}{\mu_e} = \frac{DE}{DO} = \frac{\tan r_o}{\tan r_e}.$$

This relation was experimentally established by Malus, thereby proving the validity of Huygens' theory of double refraction.

If the rays PA and QB be normally incident on SS, after refraction the ordinary components AO and BO' will reach the surface of the sphere simultaneously and will form the ordinary wavefront OO' During the same interval the extraordinary components AE and BE' will form the extraordinary wavefront EE' which is parallel to OO' [Fig. 88(*c*)]. So the O-ray and the E-ray travel in the same direction with different velocities and a phase difference is created between them. This phase difference is of particular importance in the construction of half-wave and quarter-wave plates.

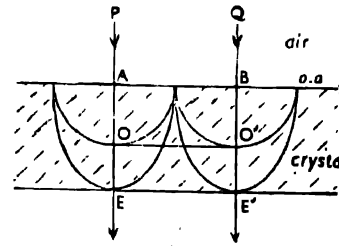


Fig. 88 (c)

(*iii*) *When the optic axis is perpendicular to the plane of incidence and lies in the refracting face of the calcite crystal.*—

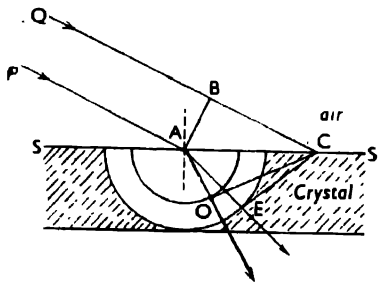


Fig. 88(*d*)

Here since the optic axis is perpendicular to the plane of incidence, the sections of the ordinary and extraordinary wavefronts in the plane of incidence will evidently be circles as shown in Fig. 88(*d*). In the case of oblique incidence the O-ray and E-ray emerge out in the directions AO and AE respectively and the corresponding O-wavefront and E-wavefront are represented in the figure by CO and CE. AO and AE travel in different directions with different velocities in the case. But if the incidence be normal the two rays will travel in the same direction but with different velocities. Here the extraordinary ray has the same velocity in all directions in the plane of incidence and hence obeys Snell's law. This property of the E-ray has been utilised in the determination of refractive index for the extraordinary ray.

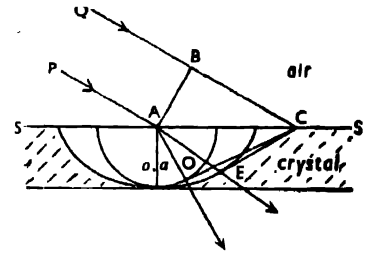


Fig. 88 (e)

(*iv*) *When the refracting face of the calcite crystal is perpendicular to the optic axis.*— In Fig. 88(*e*) the ordinary and the extraordinary wave surfaces are drawn in the same way as

in case (*i*). *o.a.* represents the optic axis and *CO* and *CE* the ordinary and the extraordinary wavefronts for oblique incidence. The corresponding *O*-ray and *E*-ray are represented by *AO* and *AE* respectively. These rays travel with different velocities in different directions.

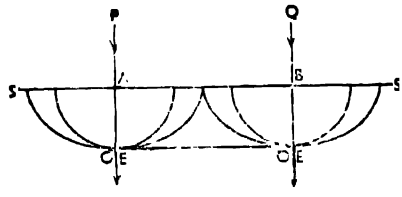


Fig. 88 (*f*)

In the case of normal incidence [Fig. 88(*f*)], both the extraordinary and ordinary wavefronts are coincident and are represented by the line *OE*, *OE*. So the *O*-ray and *E*-ray travel with the same velocity in the same direction.

(*v*) *Wavefronts in a positive crystal when the optic axis is in the plane of incidence but inclined to the refracting face.*

Here the ellipse will lie inside the circle in the sectional diagram and they will touch along the major axis of the ellipse which is the optic axis of the crystal. The circle and ellipse can be drawn exactly in the same manner as mentioned in case (*i*). Here *AO* and *AE* are the *O*-ray and the *E*-ray respectively. They travel with different velocities in different directions. The *O*-wavefront and *E*-wavefront are respectively given by *CO* and *CE* in Fig. 88(*g*).

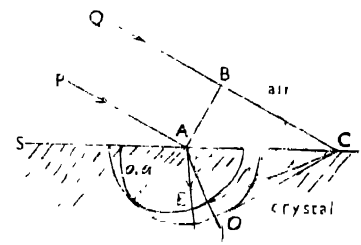


Fig. 88(*g*)

***Note :**—Huygen's construction is in accordance with the electromagnetic theory of light, wherein the equation of the wave-surface in an anisotropic biaxial crystalline medium is given by,

$$\frac{v_x^2 x^2}{r^2 - v_x^2} + \frac{v_y^2 y^2}{r^2 - v_y^2} + \frac{v_z^2 z^2}{r^2 - v_z^2} = 0$$

where v_x, y_y, v_z represent the velocities in the three co-ordinate directions inside the crystal ; x, y, z are the co-ordinates of a point in the direction of the ray velocity at a distance r from the origin. This equation yield the equation of the wave-surface of a uniaxial crystal exactly of the same nature as given in Huygen's theory. Let z-axis be the direction of the optic axis. Then $v_x = v_y = v_0$ and $v_z = v_e$. Putting these values in the above equation, it is reduced to the form.

$$(x^2 + y^2 + z^2 - v_0^2)[v_0^2(x^2 + y^2) + v_e^2 z^2 - v_0^2 v_e^2] = 0.$$

This gives, $x^2+y^2+z^2=v_0^2$ ... (1)

and $\frac{x^2}{v_e^2}+\frac{y^2}{v_e^2}+\frac{z^2}{v_0^2}=1$ ... (2)

Eqn. (1) represents a *sphere* of radius v_o and eqn. (2), an *ellipsoid of revolution* about z-axis touching each other at points where the optic axis intersects them. The ellipsoid of revolution may be thought of as generated by rotating an ellipse $\frac{x^2}{v_e^2}+\frac{z^2}{v_0^2}=1$ about the z-axis. If $v_e>v_o$, then the spheroid is oblate in form and if $v_0>v_e$, it is prolate in form.

Thus the general aspects of Huygens' theory are exactly in accordance with those of the electromagnetic theory.

85. Determination of Refractive Indices for the ordinary and extraordinary rays :—

(*a*) *Ordinary Refractive Index.*—Identical prismatic slabs are cut off from a calcite crystal such that the orientations of these slabs are arbitrary with respect to the optic axis. The slabs are arranged one above the other to form a composite prism. This prism is then mounted on the prism table of a spectrometer and the usual experiment is performed to determine the refractive index. Each of the slabs gives two refracted rays (except when the light in any one travels along the optic axis). Since the incident rays are parallel, it is found that one of the rays in each slab comes out in the same direction and does not depend on the direction of the optic axis. For this ray, $\sin i/\sin r$ is a constant for all the slabs. This ray is the ordinary ray which obeys Snell's law. The refractive index for this ray is then measured by the usual procedure from the relation, $\mu_o=\sin\frac{(A+D_m)}{2}\Big/\sin\frac{A}{2}$.

(*b*) *Extraordinary Refractive Index.*—The extraordinary refractive index of a double-refracting crystal depends on the direction of the ray with respect to the optic axis. In the case of calcite, which is a negative crystal, the extraordinary refractive index is maximum when the *E*-ray travels along the optic axis. This maximum value of μ_e is equal to the ordinary refractive index μ_o of the crystal. Again the extraordinary refractive index is minimum when the ray travels at right

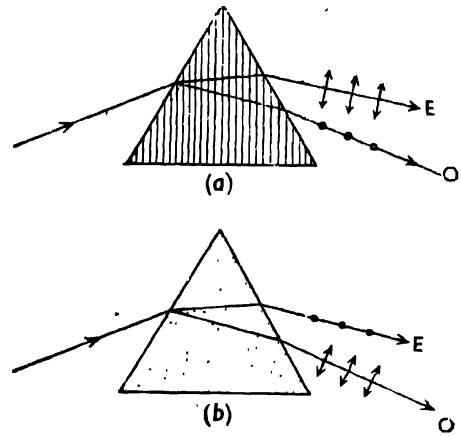


Fig. 89

angles to the optic axis. This minimum refractive index of a negative crystal is known as its *principal refractive* index and is defined by,

$$\mu_e = \frac{\text{velocity in vacuum}}{\text{maximum vel. of the } E\text{-ray}}.$$

[In case of a positive crystal, the principal refractive index for the extraordinary ray is the maximum value of μ_e which is defined as, $\mu_e = \frac{\text{vel. in vacuum}}{\text{min. vel. of the } E\text{-ray}}$]. So in the case of a negative crystal (calcite), if the principal refractive index for the extraordinary ray is to be found out, the refracted *E*-rays must travel perpendicularly with respect to the optic axis. A calcite prism is so cut that its optic axis is either perpendicular to the edge of the prism and lying in the principal section of it [Fig. 89(*a*)] or is parallel to the edge of the prism and normal to the principal section [Fig. 89(*b*)]. If a ray of monochromatic light falls on the refracting face of any one of these prisms, it is split up into *E*-ray and *O*-ray. In both the types of prisms described above the *E*-ray will be perpendicular to the optic axis. In a negative crystal $\mu_e < \mu_0$; so the *E*-ray will be deviated to a lesser extent. With the help of the spectrometer the angle of minimum deviation D_m for the *E*-ray and the angle of the prism *A* are determined. Knowing *A* and D_m the principal refractive index is found from the relation,

$$\mu_e = \sin\left(\frac{A+D_m}{2}\right) / \sin\frac{A}{2}.$$

86. Devices for Production and Detection of Plane Polarised Light :—The method of producing plane polarised light by reflection and of detecting the same by another reflector is not a very convenient method, because by reflection completely polarised light cannot be obtained. By using a pile of plates the beam can no doubt be made more completely polarised but complete polarisation, in the true sense of the term, can hardly be obtained by this method.

By a double refracting crystal two beams of polarised light may be obtained. But as these beams are not sufficiently separated they cannot be used for practical purposes with convenience. Tourmaline crystal of a finite thickness can, however, be used because it absorbs the *O*-ray and transmits only the polarised *E*-ray. But due to the colour of the image formed by tourmaline and partly due to its low intensity, it is neither used in any work requiring measuremental precision. Some of the devices commonly used for the production and detection of plane polarised light are described below.

[A] **Nicol Prism.**—A Nicol prism is a specially devised form of calcite rhomb which is extensively used in many optical instruments for the production and analysis of plane polarised light.

Since due to double refraction both the *O*-ray and the *E*-ray are plane polarised on emergence, it is necessary that one of these rays

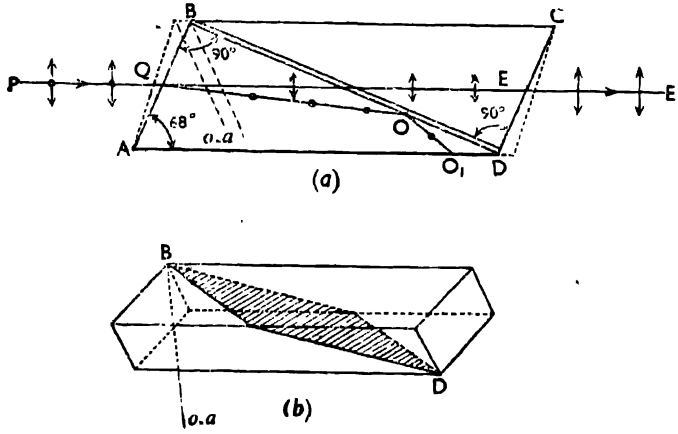


Fig. 90

should be removed so that only a single beam of plane polarised light is obtained on emergence. This was secured by William Nicol in 1828 by a special device which is known as a *Nicol's prism* or simply a *Nicol*. The ordinary component of the refracted rays is removed here by total reflection. A flawless lozenge-shaped rhomb of calcite is chosen, the length of which is about three times its width. Let *ABCD* be the principal section of the rhomb. The two cut-faces *AB* and *CD* are prepared by grinding and polishing so as to make the angles at *A* and *C* 68° each instead of 71°. This rhombic prism is then cut along the diagonal plane *BD* so that the plane is normal to *AB* and *CD* [Fig. 90(*a*)]. After duly polishing the cut-faces, they are cemented together in the original form by means of Canada balsam. The position of the plane *BD* in the original rhomb is shown in Fig. 90(*b*). Canada balsam is a transparent cement of refractive index 1·55 for sodium light. For the same light the ordinary and the extraordinary refractive indices of calcite are given by $\mu_o = 1{\cdot}658$ and $\mu_e = 1{\cdot}486$ respectively.

If ray *PQ* of natural light falls on the face *AB* in a direction nearly parallel to the length of the rhomb, the extraordinary ray QEE_1 will be transmitted through the nicol because $\mu_e < 1{\cdot}55$ and there will be no total reflection in this case. But as $\mu_o > 1{\cdot}55$, the *O*-ray *QO* will suffer total internal reflection if its angle of incidence on the balsam layer exceeds the angle $\theta_c = \sin^{-1}\left(\frac{1{\cdot}55}{1{\cdot}658}\right) = 69°$ approximately. The sides *AD* and *BC* of the nicol are blackened to absorb this totally reflected *O*-rays. If the incident

ray PQ makes an angle greater than PQA [as shown in Fig. 90 (a)], the ordinary ray will as usual be totally reflected and the E-ray will be more and more parallel to the optic axis and its refractive index approaches that of the ordinary ray. So when the angle of incidence is sufficiently greater than PQA, the E-ray will also be totally reflected and there will be no light emergent from the nicol. Again if the angle of incidence be much smaller than PQA, the O-ray strikes the balsam layer at an angle less than the critical angle and hence it will get through the nicol. So the nicol's prism cannot be used for highly divergent or convergent pencils of rays. The angular range of this convergence or divergence is about 28°. Nicol's prisms are widely used as analysers and polarisers in different types of optical instruments. In the article on 'Polariscopes' (Art. 87), the polarising and analysing functions of a nicol will be described in detail.

(B) Rochon and Wollaston Prism :—A Nicol prism allows only the polarised extraordinary ray to pass through it. So only a single polarised beam is obtained in this. Moreover, the Canada balsam used in a nicol absorbs ultra-violet radiations and as such it is unsuitable for use in the ultra-violet range. These difficulties were obviated in Rochon and Wollaston prisms which supply both the polarised rays with a considerable separation between them and can thus be suitably used also in the ultra-violet region of the spectrum.

(i) Rochon Prism :—It is a *double image prism* consisting of two right-angled prisms of quartz. The right-angled prism ABC has its optic axis parallel to the base but perpendicular to the refracting edge [Fig. 91(a)]. The optic axis of other prism ACD is also parallel to the base and is at the same time parallel to its refracting edge (*i.e.*, perp. to the plane of the paper). The light is incident on the face AB normally so that the optic axis and the incident light

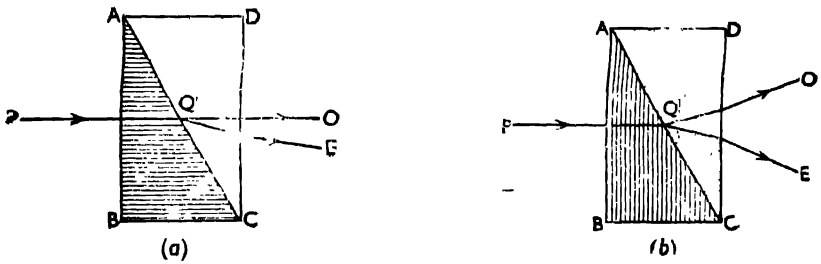


Fig. 91

are parallel. So in the prism ABC the incident light is not split up into two components. But as the light enters the prism

ACD at the point *Q* [Fig. 91 (*a*)], the beam of light is doubly refracted. The *O*-rays passes out of this prism undeviated but the *E*-ray is deviated towards the apex *C* of *ACD*. So the two plane polarised rays emerge out of the Rochon prism with a considerable angular separation between them. The two rays are polarised in planes mutually perpendicular to each other.

(ii) **Wollaston Prism** :—It is also a *double image* prism consisting of two rectangular prisms (*ABC* and *ACD*) of calcite. The optic axis of *ABC* is parallel to the refracting facing *AB* as shown in Fig. 91(*b*). In the prism *ACD*, the optic axis is perpendicular to the plane of the paper, *i.e.* at right angles to the optic axis of *ABC*. When a beam of natural light is incident normally on the face *AB* of this composite block, it passes through *ABC* without splitting because light here travels in a direction perpendicular to the optic axis. On reaching the point *Q*, the beam is split up into the ordinary and extraordinary components. The ordinary ray is deflected towards the base *AD* of the prism *ACD* and the extraordinary ray towards its apex *C*. So on emergence these two rays have a sufficient angular separation.

87. Polariscopes :—A polariscope is an instrument in which there are arrangements for polarising the light and also for analysing the polarised light. So there should be a polariser and an analyser in the instrument. The polariser and the analyser need not be of the same type. A simple reflector, a tourmaline crystal, a pile of plates or a nicol may be conveniently used either as an analyser or as a polariser. A few common types of polariscopes are described below.

(*a*) *Tourmaline Tongs* (Fig. 92) : The tourmaline tongs may be considered as one of the simplest of polariscopes. It consists of two circular tourmaline plates cut parallel to the optic axis mounted in two rings attached to the ends of a wire bent in the form of a pair of tongs. The plates are capable of rotation in the rings. When they are crossed the field of view will be completely dark. When a body is to be examined in polarised light, it is to be held in between the two plates.

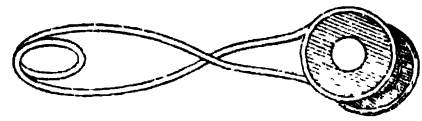


Fig. 92

(*b*) *Reflecting Polariscopes* :—Here two reflecting mirrors of polished black glass are mounted at a distance. The mirrors can

be inclined at any desired angle in any direction. Biot's polariscope is an example of reflecting polariscopes.

(*c*) *Nicol Polariscope* :—It consists of two nicols, the first of which is used as the *polariser* on which the incident natural light falls and the second acts as the *analyser* which analyses the nature of the polarised beam. When the principal sections of the two nicols P and A are parallel as in Fig. 93 (*a*), the extraordinary ray transmitted through P will emerge out of A also as the extraordinary

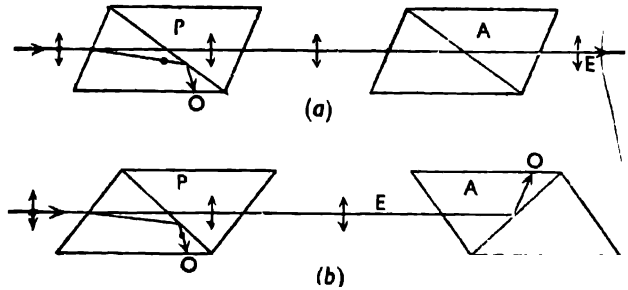


Fig. 93

ray. But if one of the nicols be rotated, the transmitted light gradually falls in intensity. When in course of rotation the principal section of one is perpendicular to that of the other [Fig. 93 (*b*)], the E-ray from P enters A as the O-ray and is totally reflected at the Canada balsam layer of A. So no light is transmitted through the analyser and the nicols are said to be crossed.

88. Theory of the Production of Circularly and Elliptically Polarised Light :—When natural light falls normally on a thin plate of a uniaxial crystal, whose face is cut parallel to the optic axis both the ordinary and the extraordinary components travel along the same path but each with a different velocity. Even when a plane polarised light falls on such a crystal plate normally, it is split up into the ordinary and extraordinary components travelling in the same direction but with different velocities. If t be the thickness of the crystal plate and if μ_e and μ_o be the extraordinary and ordinary refractive indices of the crystal, then the two rays on emergence will have a path difference of $t(\mu_e \sim \mu_0)$. In the case of calcite or tourmaline $\mu_0 > \mu_e$, while for quartz $\mu_e > \mu_o$. Due to the path difference the phase difference between the two emergent rays will be $\frac{2\pi t}{\lambda}(\mu_e \sim \mu_o) = \phi$ (say), where λ is the wavelength of the incident ray.

Let PA be the direction of vibration of the plane polarised wave incident at the point P of the crystal whose optic axis lies along XX_1 (Fig. 94). Let this incident vibration be $A \sin \omega t$ which now splits up into two components inside the crystal. If we consider the crystal to be calcite, the ordinary vibration along PY is given by $A \sin \theta \sin \omega t$ and the faster moving extraordinary vibration along PX is given by $A \cos \theta \sin \omega t$ where θ is the angle between PA and PX. On emergence a phase difference ϕ is introduced between the two components. So, on emergence, we get,

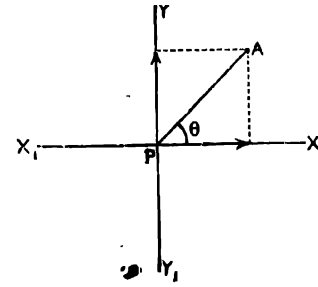


Fig. 94

$$x = A \cos \theta \sin \omega t = a \sin \omega t \qquad (1)$$

and $$y = A \sin \theta \sin (\omega t + \phi) = b \sin (\omega t + \phi) \qquad (2)$$

where $$\phi = \frac{2\pi t}{\lambda}(\mu_o - \mu_e).$$

These two vibrations which are mutually at right angle to each other combine to give a resultant vibration whose equation can be derived in the following way.

Eqn. (2), $y = b \sin (\omega t + \phi)$ can be written as,

or, $$\frac{y}{b} = \sin \omega t \cos \phi + \cos \omega t \sin \phi.$$

But from eqn. (1), $\sin \omega t = \frac{x}{a}$ and $\cos \omega t = \sqrt{1 - \frac{x^2}{a^2}}$

$$\therefore \quad \frac{y}{b} = \frac{x}{a} \cos \phi + \sqrt{\frac{a^2 - x^2}{a^2}} \sin \phi.$$

Squaring and rearranging, we get,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy \cos \phi}{ab} = \sin^2 \phi \qquad \ldots \qquad \ldots \qquad (3)$$

For a given value of ϕ, eqn. (3) represents an ellipse whose axes do not coincide with the co-ordinate axes.

Case I. If $\phi = \frac{\pi}{2}$, eqn. (3) reduces to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, which represents an ellipse whose axes coincide with the co-ordinate axes. So the emergent beam is elliptically polarised.

Case II. If $\phi = n\pi$ where $n = 0, 1, 2 \ldots$etc. eqn. (3) reduces to

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0;$$

or, $$\left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0.$$

This represents two coincident plane polarised vibration passing through the origin P and inclined to the XX_1 axis (*i.e.* the *o.a.* of the crystal) at an angle $\theta = \tan^{-1}\frac{b}{a}$.

Case III. When $\phi = (2n+1)\pi$ where $n = 0,\ 1,\ 2,\ 3\cdots$etc., $\cos\phi = -1$.

$\therefore$ Eqn. (3) reduces to $\frac{x^2}{a^2} + \frac{x^2}{b^2} + \frac{2xy}{ab} = 0$;

or, $\left(\frac{x}{a} + \frac{y}{b}\right)^2 = 0$.

This also represents two coincident plane polarised vibrations passing through the origin and having a slope $\tan^{-1}\left(-\frac{b}{a}\right)$. So the plane vibration under Case II and that in the present case are inclined to each other at an angle 2θ, where $\theta = \tan^{-1}\frac{b}{a}$.

Case IV. If the thickness of the crystal plate be such that $\phi = \frac{\pi}{2}$ and if $a = b$, eqn. (3) reduces to $x^2 + y^2 = a^2$, which represents a circle. So the emergent light is circularly polarised.

If the crystal plate satisfies the above condition, *i.e.* if the thickness be such that $\phi = \frac{\pi}{2}$, or, $\frac{2\pi t}{\lambda}(\mu_o - \mu_e) = \frac{\pi}{2}$ or, $t(\mu_o - \mu_e) = \frac{\lambda}{4}$, the plate is known as a **quarter-wave plate.** Similarly a crystal plate which can introduce a path difference of $\frac{\lambda}{2}$ between the ordinary and the extraordinary rays is called a **half-wave plate.**

89. Devices for Producing Circularly and Elliptically Polarised light :—

(A) Quarter-wave Plate.—A quarter-wave plate is a thin plate generally made of quartz with its optic axis parallel to the refracting face of the crystal. The thickness of the plate should be such that the path difference introduced between the ordinary and the extraordinary rays on emergence through the plate will be $\frac{\lambda}{4}$ or the corresponding phase difference $\frac{\pi}{2}$. If t be the thickness of the plate, then, since $\mu_e > \mu_o$ in quartz, the phase difference introduced is $\frac{2\pi t}{\lambda}(\mu_e - \mu_o)$.

Hence in a quarter-wave plate, $\frac{2\pi t}{\lambda}(\mu_e - \mu_o) = \frac{\pi}{2}$; or, $t = \frac{\lambda}{4(\mu_e - \mu_o)}$.

The quarter-wave plates available in the market are mounted on circular rims and the direction of the optic axis is generally etched on it. If a plane polarised ray falls on it in such a direction that the plane of vibration of the ray makes an arbitrary angle with this optic axis (but not 45°), then on emergence the ray will be elliptically polarised. If the plane of vibration makes an angle of 45° with the optic axis, then the ray becomes circularly polarised on emergence. According to the theory of production of circularly polarised light, the conditions of circular polarisation demand that $\phi = \frac{\pi}{2}$ and $a = b$ (See last Article). First condition is automatically satisfied in a quarter-wave plate. Now if the angle between the optic axis and the direction of vibration of the incident ray be $\theta = 45°$, then $a = A \cos \theta = A \cos 45°$ and $b = A \sin \theta = A \cos 45°$, *i.e.* $a = b$. Thus for this particular direction of incidence, the production of circularly polarised light is corroborated by the theory.

The main defect of the quarter-wave plate is that a particular plate can only be used for a particular λ. Hence other general types of devices are to be found out.

(B) Fresnel's Rhomb :—From theoretical considerations on the nature of polarisation Fresnel concluded that for plane polarised light totally reflected from a glass-air surface (of $\mu = 1{\cdot}51$) at an angle of nearly 55° (actually 54°37′), the plane of vibration being at 45° to the plane of incidence, the reflected light may be supposed to be made up of two rectangular vibrations differing in phase by $\frac{\pi}{4}$. On this principle Fresnel constructed a rhomb of glass known as Fresnel's rhomb which is a good substitute for a $\frac{\lambda}{4}$ plate and has an additional advantage that it can be used for all wavelengths. The rhomb has two opposite angles equal to 55°. When a plane polarised light PQ is incident normally on the face AB, it gets internally reflected at Q, the angle of incidence there being 55° (Fig. 95). If PQ be previously so adjusted that its plane of vibration is at 45° to the plane of incidence (*i.e.* the plane of the paper), then on reflection a phase difference of $\frac{\pi}{4}$ would be introduced between its two supposed

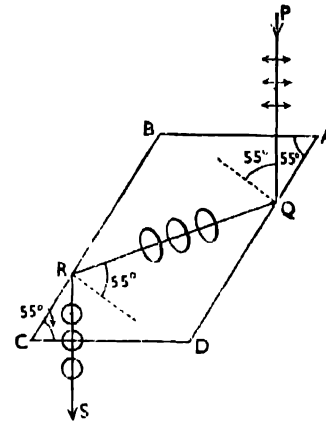


Fig. 95

components. So the reflected light QR will be elliptically polarised. When QR suffers a second reflection at R again at an angle of 55°, the final emergent ray RS will have its components differing in phase by $\frac{\pi}{2}$. So the emergent ray will be circularly polarised. The function of the rhomb is exactly similar to that of a $\frac{\lambda}{4}$-plate.

(C) **Babinet's Compensator** :—Babinet's compensator essentially consists of two thin quartz wedges in the form of right-angled prisms, the hypotenuse of one being placed exactly over that of the other as shown in Fig. 96. The optic axes of the two wedges are mutually at right angles but they are parallel to the refracting faces in both the wedges.

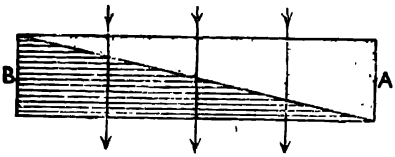


Fig. 96

If plane polarised light falls on one of the faces of the compensator normally, it will split up into ordinary and extraordinary components in the first wedge A (say). When these rays pass from the first to the second wedge B, the ordinary ray becomes extraordinary and the extraordinary ray becomes ordinary. As the refractive indices for the two rays are different, on emergence, the phase difference between them will be given by,

$\phi = \frac{2\pi}{\lambda}(t_1 - t_2)(\mu_e - \mu_0)$, where t_1 and t_2 are the thickness of A and B at the place of traverse. At the centre of the plate $(t_1 - t_2) = 0$. So the ray emerging out at that point remains plane polarised with its vibration plane parallel to that of the incident light. This is also the case at all points along the plate where

$$\phi = 2n\pi \text{ ; or, } (t_1 - t_2)(\mu_e - \mu_0) = n\lambda, \ (n = 0, 1, 2, 3, \text{etc.}).$$

So if a plane polarised monochromatic beam is incident on the compensator and if the emergent light be viewed through an analysing nicol with its principal section perpendicular to the plane of vibration of the incident light, then a series of dark bands will be observed corresponding to the points at which the relation, $(t_1 - t_2)(\mu_e - \mu_0) = n\lambda$ holds. In between two dark bands, ϕ may assume any value and as such the light emerging out from those regions may be elliptically or circularly polarised depending on the value of ϕ. [as shown in Fig. 96(a)] Fig. 96(a) shows the nature of polarisation of the emergent rays on either side of the central $(t_1 = t_2)$ line, when the inclination between the direction of vibration of the incident beam and the optic axis is taken to be 45° (*i.e.* $\theta = 45°$). This is in accordance with the mathematical analysis given in Art. 88. When the analyser is turned through a certain angle, generally 90°,

the above dark fringes will vanish and will be replaced by another set of such fringes half way between the previous fringes where

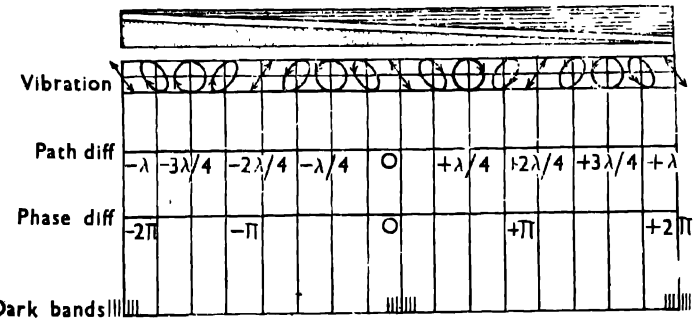


Fig. 96(*a*)

$\phi = (2n+1)\pi$. Here the vibration is also plane polarised but is inclined to the former plane vibration at that particular angle. So by rotating the analyser through that particular angle complete extinction has been secured.

For the purpose of measurements Jamin modified Babinet's compensator such that one of the wedges can be moved by a micrometer screw parallel to itself. The amount of this displacement of the wedge can be adjusted by a graduated screw-head which moves this wedge. So the Jamin-Babinet type is commonly used in all practical measurements with compensators.

Calibration :—Before using the Babinet's compensator, it is to be properly calibrated. To do this a beam of white light is made plane polarised by means of a polarising nicol. This plane polarised light is then allowed to fall normally on the compensator plate. The emergent beam is then seen through an analysing nicol crossed with respect to the polariser by means of an eye-piece. A central dark band is observed at position where $t_1 = t_2$. On both sides of this there will be bright coloured fringes of varying widths. The cross-wire of the eye-piece is fixed on this dark band and the eye-piece is kept fixed at that position. Now monochromatic light is used in place of white light. In the crossed setting of the nicols a number of dark bands are observed in the field of view of the eye-piece—the central dark band being still on the cross-wire. By the movement of the graduated screw-head the next dark band is brought on the cross-wire and the screw-head rotation in linear scale $2b$ is noted. This $2b$ now corresponds to a phase change of 2π or a path difference λ. Once this calibration is made, any phase difference between the rays can be introduced by the movement of the screw-head of the compensator. If the displacement of the screw-head be x then the corresponding phase difference will be $\phi = \frac{2\pi}{2b} \times x = \pi x/b$.

Analysis of Elliptically polarised light :—

(i) Position and ratio of the axes :—Initially the compensator is to be so adjusted as to act as a quarter-wave plate. To do this the compensator is placed between two crossed nicols their principal sections being set at 45° with the optic axis of the quartz wedges. By adjusting the micrometer screw-head the cross-wire is brought exactly to the position of the central dark band formed by the monochromatic light which can be preferably verified by using a white light for illumination. At this position of the dark band, the phase difference between the *O*-ray and *E*-ray is zero. The micrometer screw is then displaced through a linear distance $b/2$ (where $2b$ corresponds to the phase difference 2π). Hence a phase difference of $\pi/2$ or path difference of $\lambda/4$ is introduced between the two emergent rays and the compensator behaves as a $\lambda/4$-plate.

To find the position of the axes of the elliptic vibration of a beam of light, the polarising nicol is removed and the elliptically polarised light is allowed to fall on the compensator. The analyser and the compensator are then slowly rotated about the direction of incident beam independently of one another till the central dark fringe once again comes on the cross-wire of eye-piece. Under this condition, the *axes of the elliptic vibration coincide with the privileged directions of the compensator i.e.* they are parallel to the edges of the quartz crystal in the compensator.

In the above adjustment for setting the central dark band on the cross-wire, the phase difference introduced between two component vibrations of the ellipse is $\pm\pi/2$. This phase difference added to that between the two emergent vibrations through the compensator under the cross-wire makes the total phase difference 0 or π and emergent vibration linear. So by the analyser it could be extinguished. In this position, the tangent of the angle which the principal section of the analyser makes with one of the privileged directions of the compensator gives the ratio of the axes of the elliptic vibration. ($\because \tan\theta = b/a$).

(ii) Phase difference between the Components :—In order to determine the phase difference between the two components of the elliptic vibration, the compensator is placed between two crossed nicols and the cross-hair set on the central dark band. Now removing the polarising nicol, the elliptically polarised light is allowed to fall on the compensator and the micrometer screw is adjusted (*i.e.* wedge position changed) till the central dark band again coincides with the cross-hair. Then evidently the phase difference introduced by the compensator just compensates the phase difference between the components. If x be the displacement of screw-head, then the phase difference introduced is $\frac{\pi x}{b}$ (where displacement $2b$ corres-

ponds to phase change 2π). Hence the phase difference between the component *S.H.* vibrations constituting the elliptic vibration is given by $\pi x/b$.

90. Production and Analysis of different types of Polarised Light :—

(A) Plane Polarised Light.—Plane polarised light may be produced by *reflection* of a beam of natural light from a mirror at the polarising angle. Biot's polariscope, a pile of plates etc. are the common devices meant for this purpose.

By *refraction* also plane polarised beams can be produced as has been shown in a pile of plates. Commonly, however, plane polarised light is produced by *double refraction* through a crystal, the most effective device for this purpose being a nicol prism.

The plane of polarisation of a beam of plane polarised light can be found out with a suitable analyser like a mirror, a tourmaline plate or a nicol. When the beam is completely quenched simply by an analyser it indicates the plane polarised nature of the beam—the plane of vibration of the beam being perpendicular to the principal plane of the analyser. Natural, circularly polarised and elliptically polarised light cannot be quenched by a single analysing nicol whereas plane polarised light can be completely extinguished. This feature distinguishes the plane polarised light from other types of polarised light.

(B) Circularly Polarised light.—Circularly polarised light can be produced by sending a plane polarised beam of light through a quarter wave plate, whose optic axis is set at an angle of 45° to the plane of polarisation of the incident light. The theory of production of the circularly polarised light by a $\frac{\lambda}{4}$-plate has been dealt with in the last article under "Quarter-wave plate." It must be realised that a $\frac{\lambda}{4}$-plate can be used only for a particular wavelength and not for light of all colours. For light of all colours, however, a Fresnel's rhomb can be effectively used for the production of circularly polarised light.

As the circular vibration is symmetrical all around, no change in intensity is observed in any direction when a beam of circularly polarised light is examined by an analysing nicol. In the case of natural light also the same thing happens. Linearly and elliptically polarised lights differ from the circularly polarised light in this respect. In order to analyse the circularly polarised light a quarter-wave plate or a Fresnel's rhomb is used such that the polarised beam before entering the analyser passes through this plate or rhomb. They are to be so oriented that the phase differences between the two components on emergence reduces

to 0 or π. So a plane polarised beam will emerge out of this plate or rhomb which can be quenched by the analysing nicol for a particular direction. In the case of the natural light no such quenching is possible because after emergence through a $\frac{\lambda}{4}$-plate the emergent beam will be elliptically polarised.

(C) Elliptically Polarised Light.—Elliptically polarised light is produced by the double refraction of a plane polarised beam of light provided the extraordinary and ordinary components in the doubly refracting crystal move in the same direction without separation but with different velocities. When plane polarised light is normally incident on a quarter-wave plate such that the plane of vibration of the former makes any angle with the optic axis of the $\frac{\lambda}{4}$-plate other than 45°, 0° or 90°, the emergent light will be elliptically polarised. When the inclination is 45°, the emergent vibration is circular and when it is 0° or 90°, the ellipse degenerates into straight lines. So elliptic vibration is the general case, the other two are merely particular cases of the elliptic vibration. Elliptically polarised light is also obtained in metallic reflection as also in the case of reflection from a surface of high refractive index.

Elliptically polarised light cannot be completely quenched by a nicol. When the vibration direction in the nicol is parallel to the major axis of the ellipse, the intensity in nicol is maximum and when vibration direction in the nicol is parallel to the minor axis, the intensity falls to a minimum. In between these two positions the intensity gradually decreases from the maximum to the minimum position. In this respect elliptically polarised light is similar to a mixture of ordinary light and plane polarised light (*i.e.* partially polarised light). This partially polarised light may, however, be distinguished from the elliptically polarised light with the help of a $\frac{\lambda}{4}$-plate, which at a particular orientation converts the elliptic vibration into linear vibration and the latter can be extinguished by an analysing nicol. The partially polarised light cannot be quenched in a similar way. For the identification of elliptically polarised light, a quarter-wave plate is rotated in front of the analysing nicol in such a way that ultimately its principal axis is parallel to one of the axes of the ellipse. Then the path difference between the *E*-ray and *O*-ray on emergence reduces to 0 or π and the emergent light becomes plane polarised. This can be quenched by the analysing nicol. So rotating the $\frac{\lambda}{4}$-plate about its normal and the analyser also about its axis, when complete extinction is secured in the analyser, the positions of the axes of the ellipse is known from the knowledge of

the optic axis of the $\frac{\lambda}{4}$-plate. The ratio of the axes is obtained from the angle between the principal planes of the analyser and the plate.

The above analysis of the elliptically polarised light can also be done with a Babinet's compensator more conveniently. Here the position of the compensator against the cross-wire is to be so adjusted that the thickness corresponds to that of a quarter-wave plate. Then rotating the compensator about its normal as also the analysing nicol, the positions of the axes and their ratio can be determined exactly in the same way as in a $\frac{\lambda}{4}$-plate. The phase difference between the *E*-ray and *O*-ray that combine to form elliptic vibration can also be determined with a compensator accurately (See Art. 89).

***91. Colours with Crystalline Plates in Polarised Light :—** When a beam of plane polarised light falls on a plate of uniaxial crystal cut parallel to the optic axis, the extraordinary and the ordinary light will, on emergence, have a definite phase difference between them. As these two rays are polarised in different planes, they cannot exhibit interference effect. If, however, two rays are polarised in the same plane or if two rays are polarised at right angles (originating from the *same beam of plane polarised* light) are subsequently brought to the same plane of polarisation, they may give rise to interference. The interference of polarised light have been thoroughly studied both theoretically and experimentally by various scientists. We shall here, first try to give a simple physical picture of the phenomenon of interference of polarised light in the subsequent paragraphs. Let *OP* represent a plane-polarised monochromatic beam polarised in a plane passing through *OP* perpendicular to the plane of the paper. Let it be incident on an uniaxial crystal at the point *O* such that *OP* is inclined at an angle 45° to the direction of optic axis *OR* of the crystal. Inside the crystal this incident beam breaks up into an ordinary ray polarised in the plane containing *OR* and an extraordinary ray polarised at right angle to this plane *i.e.* in the plane containing *OQ*. Let the thickness of the plate be such that a path difference of λ is introduced between the *O*-ray and

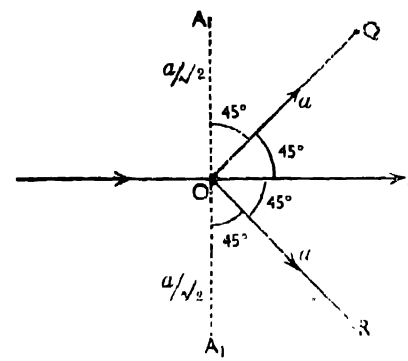


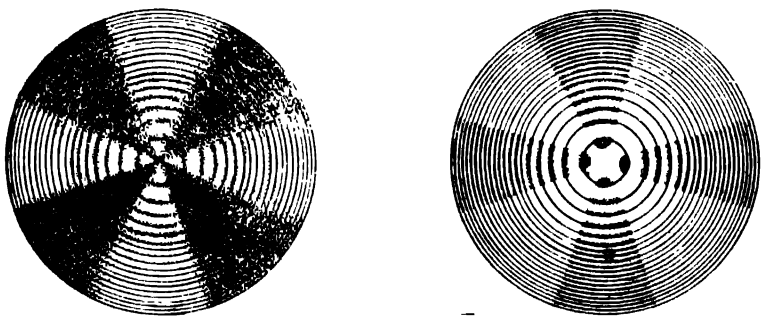
Fig 97(*a*)

E-ray due to passage through it where λ refers to the wavelength of the incident light. Suppose now a nicol is placed such that it allows only the light polarised at right angles to original plane of polarisation to pass through it. So light polarised in the plane containing AA_1 is only transmitted through this analysing nicol.

Both *O*-ray and *E*-ray are polarised at 45° to this plane AA_1. As the amplitudes of these two rays are equal, supposing each to be equal to *a*, their resolved parts along AA_1 are equal to $\frac{a}{\sqrt{2}}$ each[Fig. 97(*a*)]. So the analysing nicol transmits two rays of the same amplitude, polarised in the same plane and also coming from the same beam of light. So they are now in a position to interfere. As is obvious from Fig. 97 (*a*), the phase difference between them will be now 180° and so they will produce interference and no light will be seen through the nicol. Complete extinction, however, occurs only for the wavelength λ. So when white light is used the plate seen through the analysing nicol presents a uniform tinge of colour which is not a pure colour of the spectrum.

When the analysing nicol is rotated, the transmitted beam of wavelength λ increases in intensity till at an angle or 90° the beam of this λ is fully transmitted and its intensity is maximum. At this position the tinge observed through the analyser changes to a colour complementary to the former colour.

If the crystal plate be cut perpendicular to the optic axis, the interference, occurring with white plane polarised light of convergent or divergent character, shows beautiful coloured concentric rings representing the curves of equal inclination. Fig. 97 shows the



(*b*) (*c*)

Nicols crossed Fig. 97 Nicols parallel

appearance of such a calcite crystal when viewed in divergent light. Fig. 97 (*b*) represents the nature of the rings when the analyser and the polariser are crossed and Fig. 97(*c*) represents the case of parallel

setting of the two nicols. The dark and bright crosses appearing along with the rings have also been successfully explained in advanced text books on Light. A simple mathematical analysis of the colour of crystalline plates and rings and brushes together with brief discussions is given below.

(a) **Colours of Crystalline plates** :— Let a beam of plane polarised monochromatic light with direction of vibration parallel to OP [Fig. 98(a)] pass through a double refracting crystalline plate in a direction not coinciding with the optic axis of the plate. Inside the plate the vibration will be split along two perpendicular directions OX and OY in and perpendicular to the principal plane, corresponding to the extraordinary and ordinary rays respectively. If α be the angle which the direction of vibration of the incident beam makes with the principal plane and if $a \sin \omega t$ represents the incident beam the amplitudes of the ordinary and extraordinary vibrations will be [$a \sin \alpha$] and [$a \cos \alpha$] respectively. On emergence the two components acquire a phase difference δ, which depends on the thickness (e) traversed, the inclination of the rays and the velocities of the rays inside the medium. This phase difference is given by the relation $\delta = 2\pi e\,[\mu_e \cos r_e \sim \mu_0 \cos r_0]/\lambda$ where μ_0, μ_e refer to ordinary and extraordinary refractive indices and r_0 and r_e are the angles of refraction of the O-ray and E-ray inside the crystal. Which of the components gains, depends on the nature ($+ve$ or $-ve$) of the crystal. Therefore, supposing the ordinary component to be lagging in phase, the emergent vibrations can be written as,

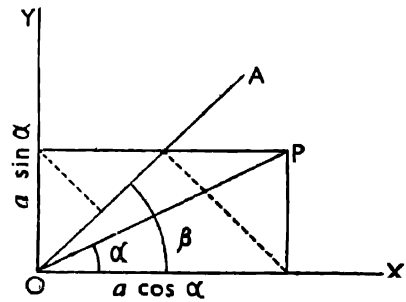


Fig. 98(a)

$x = a \cos \alpha \sin \omega t$ (E-ray) and $y = a \sin \alpha \sin (\omega t - \delta)$ (O-ray). Let these vibrations pass through a nicol whose principal section makes an angle β with the principal plane of the crystal. The amplitudes of the above component vibrations which are transmitted through this nicol (analyser) are,

$R_1 = a \cos \alpha \cos \beta$ and $R_2 = a \sin \alpha \sin \beta$, differing in phase by δ.

The resultant amplitude R is obtained by summing up these two amplitudes R_1 and R_2 vectorially. Hence,

$$R^2 = R_1^2 + R_2^2 + 2R_1R_2 \cos \delta = R_1^2 + R_2^2 + 2R_1R_2(1 - 2 \sin^2 \delta/2)$$
$$= (R_1 + R_2)^2 - 4R_1R_2 \sin^2 \delta/2$$
$$= a^2(\cos \alpha \cos \beta + \sin \alpha \sin \beta)^2 - 4a^2 \cos \alpha \cos \beta \sin \alpha \sin \beta \sin^2 \delta/2$$
$$= a^2 \cos^2 (\alpha - \beta) - a^2 \sin 2\alpha \sin 2\beta \sin^2 \delta/2. \quad \ldots \quad \ldots \quad (1)$$

The intensity of the light transmitted through the analyser is thus proportional to the above expression (1). If instead of monochromatic light, *white light* is used, the values of a and δ for different colours will be different and we shall have to modify expression (1) as,

$$R^2 = \cos^2(\alpha - \beta)\Sigma a^2 - \sin 2\alpha . \sin 2\beta . \Sigma a^2 \sin^2 \delta/2 \quad \ldots (2)$$

where in the second term depends on λ ($\because$ δ depends on μ) and may be called the *colour term* and the first term the *dilution term*. The transmitted light may thus be considered as white minus the colours which have been completely or partially removed from it. So the net effect of transmission is a coloured light.

(*i*) When the polariser and analyser are crossed, $(\alpha - \beta) = 90°$ and the dilution term is zero. So the colour term is only present and the *colour effect is most pronounced*, whose intensity is $I_1 = \sin^2 2\alpha \Sigma a^2 \sin^2 \delta/2$ ($\because \alpha = \beta \pm \pi/2$).

(*ii*) When the nicols are parallel, $\alpha - \beta = 0$, or, $\alpha = \beta$, the *colour term is least pronounced* and the intensity of the transmitted light is,

$$I_0 = \Sigma a^2 - \sin^2 2\alpha \Sigma a^2 \sin^2 \delta/2.$$

Hence $I_1 + I_0 = \Sigma a^2 =$ intensity of the incident light and colour effects in the cases (*i*) and (*ii*) are complementary.

(*iii*) If 2α or 2β are either zero or 180°, the colour term vanishes. The light, transmitted in that case, is entirely white. Here α or $\beta = 0$ or 90°, *i.e.* the principal section of the polariser or analyser is parallel or perpendicular to the principal plane of the crystal. In this case the light is transmitted through the crystal without resolution into ordinary and extraordinary components. Here again if $\alpha - \beta = 0$ (*i.e.* $\alpha = \beta = 90°$), the whole light is transmitted and if $\alpha - \beta = 90°$ ($\alpha = 90°$, $\beta = 0$), no light is transmitted at all.

(b) Rings and Brushes (Colours of Crystals with highly convergent or divergent plane polarised light) :—In order to observe the rings and brushes, a doubly refracting uniaxial crystal cut perpendicular to the optic axis is placed at C in a strongly convergent or divergent beam of rays. In Fig. 98(*b*), S is a strong source of light ; the lens L_1 renders the beam parallel and send it through the polarising nicol N_1 ; L_2 converges the beam on C ; L_3 receives the beam and renders it parallel once again and sends it through the analyser N_2 ; finally L_4 casts the image of the pattern on the screen MN, where the rings and crosses are seen.

Let O be a point where the axis of the convergent beam meets the crystal [Fig. 98(*c*)]. With O as centre describe a circle [Fig. 98(*d*)] and take a point X on it. The incident vibration at X (along XP_1) will be resolved into two components one vibrating

in the principal plane ($\parallel$ to OX) and the other perpendicular to it. The components acquire a phase difference (δ) depending on the thickness of the plate, the inclination of the rays to the axis and the

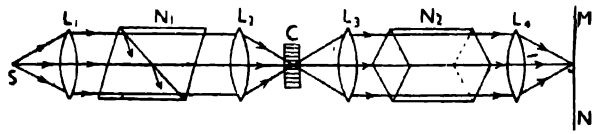


Fig. 98 (b)

wavelength of the incident ray. This δ is, however, constant for a given λ all over the circle. If OP and OA are respectively parallel to the principal planes of the polariser and analyser, then calling $\angle XOP = \angle TXP_1 = \alpha$ and $\angle XOA = \angle TXA_1 = \beta$, we have the intensity of the emergent light.

$I = \cos^2(\alpha - \beta)\Sigma a^2 - \sin 2\alpha \sin 2\beta \Sigma a^2 \sin^2 \delta/2$, as calculated above.

The colour all over the circle (or of its image on the screen) depends only on δ. For another circle the inclination of the rays is different and so also the colour. Hence the general appearance on the screen is a sequence of coloured rings.

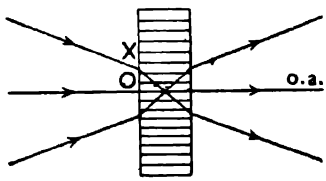


Fig. 98(c)

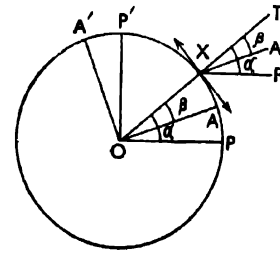


Fig. 98(d)

These rings are not equally bright all over. When X is at P or at P' (OP' is $\perp$ to OP), $\alpha = 0$ or $90°$ and the colour term vanishes and the light is white. This is also the case when $\beta = 0$ or $90°$, *i.e.* when X is at A or at A' [Fig. 98(d)]. Thus coloured rings are crossed by two white brushes parallel and perpendicular to principal planes of the polariser and analyser. If any of them rotates the corresponding cross also rotates. When $\alpha - \beta = 0$ (with either α or $\beta = 0$ or $90°$), the crosses coincide and the light is white. But if $\alpha - \beta = 90°$, the crosses are dark.

If $\sin 2\alpha \sin 2\beta$ is $+ve$, the points of maximum intensity along a given radius between brushes are determined by $\sin^2 \delta/2 = 0$, or, $\delta = 2\pi n$, and the points of minima by $\sin^2 \delta/2 = 1$, or,

$\delta = (2n+1)\pi$. But if sin 2α sin 2β be $-ve$, the relationship is reversed.

The rings and brushes provide a handy method for *testing the sign* of a crystal. If the given crystal, cut perpendicularly to the optic axis, be placed on a standard crystal of known sign and for which rings and brushes have been produced, the rings would contract if the two crystals are of the same sign. If they are of opposite signs, the ring system would dilate.

Examples

1. A ray of light is incident on the surface of a glass plate of refractive index 1·55 at the polarising angle. Calculate the angle of refraction. (Utk. U. 1954)

Ans. From Brewster's law we know, $\mu = \tan\phi$, where ϕ is the polarising angle.

$\therefore$ From the given data, $\tan\phi = 1{\cdot}55$;

or, $\phi = 57°11'$.

Again if θ be the angle of refraction, from the same law we know, $\phi + \theta = 90°$.

$\therefore$ $\theta = 90° - 57°11' = 32°49'$.

2. Calculate the thickness of the quartz half-wave plate for sodium light of wavelength 5893 Å.U. given that the index of refraction of quartz for the ordinary and the extraordinary rays are 1·5442 and 1·5533 respectively. (Agra U. 1949)

Ans. For a *half wave plate* the phase difference between the two emergent rays from the quartz crystal is π.

$$\therefore \quad \frac{2\pi t}{\lambda}(\mu_e - \mu_o) = \pi \; ; \quad \text{or,} \quad t = \frac{\lambda/2}{(\mu_e - \mu_o)}.$$

$$\therefore \text{ The thickness, } t = \frac{5893 \times 10^{-8}}{2(1{\cdot}5533 - 1{\cdot}5442)} = \frac{5893 \times 10^{-8}}{2 \times 0{\cdot}0091} = 3{\cdot}238 \times 10^{-5} \text{ cm.}$$

3. For quartz for sodium light of $\lambda = 5{\cdot}9 \times 10^{-5}$ cm., $\mu_e = 1{\cdot}553$ and $\mu_o = 1{\cdot}544$. Calculate the thickness of the quartz quarter wave plate for sodium light.

Ans. From the given data,

$$t = \frac{\lambda}{4(\mu_e - \mu_o)} = \frac{5{\cdot}9 \times 10^{-5}}{4(1{\cdot}553 - 1{\cdot}544)} = 0{\cdot}0016 \text{ cm.}$$

92. Rotatory Polarisation :—Arago in 1811 discovered that a beam of plane polarised light passing in the direction of the optic axis through a crystal of quartz undergoes a rotation. Here the plane of polarisation of the emergent beam makes a definite angle with the plane of polarisation of the incident beam. This phenomenon is generally known as **optical activity** or **natural optical gyration**. When a quartz plate cut perpendicular to the optic axis is placed between two crossed nicols, due to rotation of the plane polarised light emergent from the quartz plate, the field of view is found to be bright. If the analysing nicol be rotated through an angle, the field of view once again becomes dark. This angle of rotation of the analyser measures the amount of optical rotation.

Some substances like calcite, glass, etc., do not exhibit optical rotation. There is another class of substances that exhibit this pheno-

menon and are known as *optically active* substances. Optically active substances, may be classified into two varieties depending on the direction of rotation of a plane polarised beam through them. One variety known as *dextro-rotatory* or *dextro-gyrate*, rotates the plane of polarisation in the clockwise or right-handed sense as seen by an observer looking in a direction opposite to that propagation of light. In the other variety the rotation is left-handed and as such this variety is known as *laevo-rotatory* or *laevo-gyrate*. In quartz both varieties of crystals are available.

93. Rotatory Power :—Systematic studies of rotatory polarisation by Biot, Arago and others revealed that the optical rotation depends on (1) the thickness of the substance traversed by the polarised light. (2) the density of the substance or the concentration of the solution when a solution is optically active, (3) the wavelength of the light used and (4) the temperature at which the substance is maintained. Moreover, it was found that the combined rotation produced by two different substances of different thickness is the algebraic sum of the rotations suffered by each of them.

Hence it is evident that if the observations are taken with light of a fixed wavelength and if the temperature is kept constant during the observation, the optical rotation, $\theta \propto l.\ \rho$, in the case of a substance in general ; or in the case of a solution, $\theta \propto l.\ c$, where l is the effective thickness of the substance or solution, and ρ and c refer to the density and the concentration as the case may be.

$\therefore$ In the case of a substance, in general, $\theta = \sigma.\ l\ \rho$, where σ is a constant for the substance. This constant when refers to a thickness of one decimeter of the substance is known as the *specific rotatory power* or simply **specific rotation.**

So the specific rotatory power of a substance at a particular temperature and for a given wavelength is defined as the amount of rotation produced by a thickness of one decimetre of the substance of unit density. It is generally designated by S. In the case of a solution, S is defined by the rotation per decimetre length of the solution per unit concentration expressed in grammes per c.c. of the solution. This is generally termed as *the specific rotatory power of a solution.*

Hence $S_{t,\lambda} = \frac{\theta}{l\rho}$, or, $\frac{\theta}{lc}$, (for a solution), where l is the length in decimetre.

Molecular rotation or, **the molecular rotatory power** is defined as the product of specific rotation and the molecular weight.

Instruments devised for the measurement of the optical rotation are known as *polarimeters* or *saccharimeters*, which may be of different types as described in the following articles.

94. Laurent's Half-shade Polarimeter :—The arrangements in a Laurent's polarimeter are shown in Fig. 99. Light from a monochromatic source falls on the polarising nicol P, but, instead of passing immediately afterwards through the tube T containing

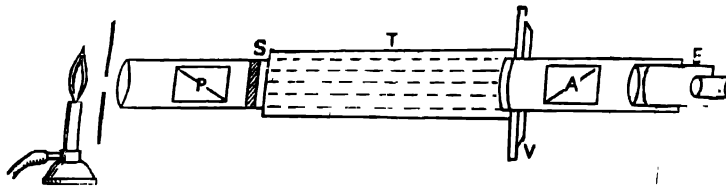


Fig. 99

the solution under test, it is made to pass through a half-wave plate S. On emergence from the solution in T, the light is examined by a telescope E through the analysing nicol A. This plate S consists, of two semicircular plates, one of glass and the other of quartz, mounted in a circular rim as shown in Fig. 100(a). The quartz plate is cut parallel to its optic axis and is of such a thickness that it produces a path difference of $\frac{\lambda}{2}$ in sodium light. So the O-ray gains half a wavelength over the E-ray in passing through this plate, *i.e.* the phase difference between the two becomes π. The glass plate is made of such a thickness that it transmits the same amount of light as the quartz plate. Let OY be the direction of the optic axis of the quartz plate and let its inclination with the principal plane of the polarising nicol be θ (Fig. 100). OR is the principal plane of the polariser. So the vibrations of the light incident normally on the plate will be parallel to OR. The plate splits up this vibrations into two mutually perpendicular vibrations which emerge out of the plate with a phase difference π. So if ON and OX represents the components of OR in the crystal plate, the emergent vibration will be the resultant of ON and OX_1 because OX-component has suffered a phase change of π. Hence OR_1 is the direction of the emergent vibration. Thus as a result of the introduction of the quartz plate the plane of vibration has been rotated through an angle $ROR_1 = 2\theta$.

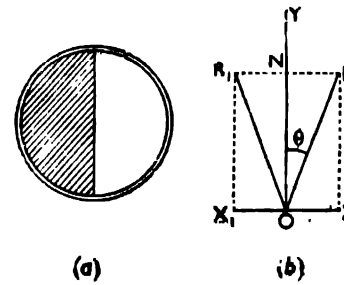


Fig. 100

So the light passing out of the polarising nicol P and falling on the half-shade plate S, will emerge out in such a way that one half of

the light passing through the glass plate comes out without any rotation whereas the other half passing through the quartz plate suffers a rotation of its plane of polarisation by 2θ. When both these halves are viewed through the analyser, the intensity will depend on the inclination of the plane of polarisation of the light and the principal plane of the analyser. The two halves generally appear to be of different brightness. When the principal plane of the analyser is equally inclined to the polarising planes of both the halves, the two halves appear equally bright. Even a slight rotation of the analyser from this positive causes a considerable variation in the brightness of the two halves. So the arrangement is highly sensitive.

In finding out the amount of optical rotation of a solution the position of the analyser when there is equal brightness of the field of view of the telescope E is read off from a circular scale V (Fig. 99) when the tube T contains the pure solvent. Now the solvent is replaced by a solution of known concentration. The matched condition of the field of view will be disturbed. Exact matching is again attained by rotating the analysing nicol. The amount of this rotation is noted from the circular scale V. Once this amount of rotation is obtained, the specific rotatory power of the solution can easily be calculated.

95. Bi-Quartz Polarimeter :—In the construction of the bi-quartz polarimeter advantage is taken of the fact that quartz crystals of different types produce rotations of both clockwise or anti-clockwise nature. Two plates of quartz each of thickness 3·75 mm. and of opposite rotatory powers are made semicircular and cemented together side by side and this circular biplate is mounted on a rim. It is then placed in the position of the half-shade plate in Laurent's polarimeter. When the plane polarised light passes through each half, it is rotated through the same amount by each of them but in opposite directions. In sodium light this rotation is about 90°. For a particular position of the analyser nicol, the two halves will appear exactly similar.

If white light is used and the analyser is set for extinction of the mean yellow rays, a greyish violet tint called the *sensitive tint* is seen in the field of view. The slightest rotation in one way changes this tint to either blue or red. When we are to measure the specific rotation of a solution or a substance, the analyser is set so that the sensitive tint is observed in each of the halves both before and after the insertion of the solution or the substance in T. From the knowledge of angle through which the analyser has been rotated due to insertion of the substance in T, the specific rotation can be calculated.

96. Soleil's Polarimeter :—Soleil modified the above method and used a fixed analyser and two adjustable quartz wedges, which were used to produce an equal and opposite rotation to that of the

substance placed in T. Fig. 101 shows the arrangements in a Soleil's polarimeter. Light passing through the polariser P traverses

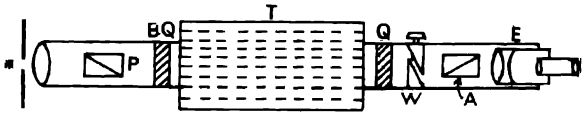


Fig. 101

the bi-quartz plate BQ from which it enters the tube T containing the substance under test. From it light passes through a right-handed quartz plate Q and then through the double-wedge W of left-handed quartz, which is of variable thickness. Light is then received by the fixed analyser A and finally observed through a telescope at E. The analyser A is set so as to produce the sensitive tint when T is empty and the thickness of the double-wedge is equal to that Q. Now putting the substance (or solution) in the tube, one wedge is screwed in or out so as to alter the effective thickness and thereby cancel the rotation produced by the substance If the graduations attached to the screw-head be previously calibrated, the rotation corresponding to the movement of the screw can be calculated and hence the specific rotation of the substance under test can be computed.

***97. Fresnel's Theory of Optical rotation** :—The substances possessing optical rotatory power are found to show a screw-character which may be either crystalline or molecular in origin. As a result, the two types of circularly polarised light have different velocities in an optically active medium. Fresnel put forward his theory of optical rotation on the basis of the above fact that the two types of circular vibrations have different velocities in the medium. According to Fresnel, a plane polarised wave resolves itself into two circularly polarised waves in opposite senses as the former passes through an optically active medium. Due to the different velocities of these components they will acquire a definite phase difference on emergence so that when they again combine together to give a linearly polarised wave, its plane of polarisation will undergo a rotation with respect to the plane of polarisation of the incident wave. The rotation of the plane of polarisation is thus interpreted.

If the incident plane polarised vibration be along the direction of y-axis, it can be represented by the equation, $y = 2a \cos \omega t$. The two component circular vibrations can be put as,

$$y_1 = a \cos \omega t,\ x_1 = a \sin \omega t \text{ (counter-clockwise)}$$

and $y_2 = a \cos \omega t,\ x_2 = -a \sin \omega t$ (clockwise).

Considering the crystal to be left-handed where the anti-clockwise component moves with a greater speed, a phase difference ϕ

will be introduced between the components on emergence. So the equations of the two circular vibrations can be written as,

$y_1 = a \cos(\omega t + \phi)$, $x_1 = a \sin(\omega t + \phi)$,

and $y_2 = a \cos \omega t$, $x_2 = a \sin \omega t$.

On emergence the two rectangular vibrations which arise due to superposition can be represented as,

$$Y = y_1 + y_2 = a \cos(\omega t + \phi) + a \cos \omega t$$

$$= 2a \cos \frac{\phi}{2} \cos\left(\omega t + \frac{\phi}{2}\right),$$

and $$X = x_1 + x_2 = a \sin(\omega t + \phi) - a \sin \omega t$$

$$= 2a \sin \frac{\phi}{2} \cos\left(\omega t + \frac{\phi}{2}\right).$$

The resultant of these two rectangular vibrations is also a plane polarised vibration which is inclined to the Y-axis at an angle θ where $\tan \theta = \frac{X}{Y} = \tan \frac{\phi}{2}$; or, $\theta = \frac{\phi}{2}$.

Hence θ indicates the rotation of the plane of polarisation. Now if t be the thickness of the active medium and if due to the different velocities in the medium, the refractive indices of the left-handed and right-handed circularly polarised light be respectively μ_L and μ_R then the path difference introduced is $t(\mu_L - \mu_R)$

Hence the phase difference, $\phi = \frac{2\pi t}{\lambda_0}(\mu_L - \mu_R)$, λ_0 being the wavelength in vacuum.

$\therefore$ The rotation suffered in the medium is

$$\theta = \frac{\phi}{2} = \frac{\pi t}{\lambda_0}(\mu_L - \mu_R).$$

In media like calcite where the two velocities are identical $\mu_L = \mu_R$. Hence θ vanishes in those media.

Fresnel verified his assumption of two velocities by the following simple arrangements.

A multiple prism in the form of a rectangular parallelopiped was constructed by cementing together several quartz prisms as shown in Fig. 102. The prisms R_1, R_2 were right-handed and L_1, L_2 left-handed. The optic axis of each prism was perpendicular to the end faces of the composite prism. If a plane polarised ray is incident on the left-hand face, it decomposes into two circularly polarised components. The right-handed component moves faster in R_1 and becomes slower in L_1 and is deviated towards the base.

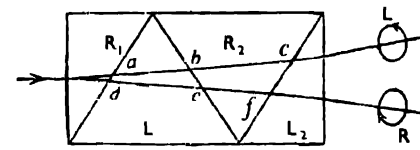


Fig. 102

In R_2 it is again faster and hence deviates from the base and finally in L_2 it deviates towards the base due to its slower velocity. So this component follows the path *defR*. Similarly the left-handed component comes out in the direction *abcL*. So on emergence these two components become completely separated. On analysis with a quarter-wave plate it is found that the two components are circularly polarised in opposite sense and the sense of rotation is quite in accordance with the theory.

98. Applications of Polarimeters :—Polarimeters are extensively used in organic chemistry to ascertain the percentage of optically active substance in a solution. They are also used for quantitative estimations of alkaloids, essential oils etc. They are applied in bio-chemistry for the study of fermentation and other problems. In the sugar industry it is widely used for estimating the percentage of sugar present in a solution. In medical science also it is used for estimating the sugar contents.

Examples

1. A 200-millimetre long tube containing sugar solution when placed in a saccharimeter gives an optical rotation of 11°. If the specific rotation of sugar under the given experimental conditions be 66°, find the strength of the solution.
(Agra U. 1953)

Ans. Since $S_{t,\lambda}=\frac{\theta}{l.c.}$, and here $S_{t,\lambda}=66°$, $\theta=11°$ and $l=200$ mm.$=2$ decimeters, we have,

$$66=\frac{11}{2\times C}\text{;}\quad \text{or,}\quad C=\frac{11}{2\times 66}=0{\cdot}0833 \text{ gm./c.c.}$$

2. For a given wavelength one millimetre of quartz cut perpendicular to the optic axis rotates the plane of polarisation by 20°. Find for what thickness will no light of this wavelength be transmitted when the quartz piece is interposed between a pair of parallel nicols. *(Raj. U. 1950)*

Ans. Here, thickness of the plate is 1 mm.$=0{\cdot}1$ cm. and the angle of rotation is 20°.

We know, other factors remaining constant, the angle of rotation is proportional to the thickness. Now in the arrangement suggested, the two nicols are set parallel. Hence in order to have no transmitted light, the rotation given to the incident vibration must be 90°.

∴ The required thickness of the plate;

$$=\frac{1}{20°}\times 90°=4{\cdot}5 \text{ mm.}=0{\cdot}45 \text{ cm.}$$

3. A tube 20 cm. long, filled with a solution of 15 gms. of cane sugar in 100 c.c. of water is placed in the path of polarised light. Find the angle of rotation of the plane of polarisation if the specific rotation of cane sugar is 66 degrees. (C.U. 1965)

Ans. Since, $\theta=s\,l\,c$ and $S=66°$, $l=20$ cm.$=2$ decimetres and $c=15/100=·15$ gm./c.c., we have,

$$\theta=66\times 2\times ·15=66\times ·3=19°\ 8.$$

Questions (Chapter IV)

1. Explain the terms 'polarisation' and the 'plane of polarisation' in the case of light. (C. U. 1947, '49)

2. Write a short note on Polarisation of light. (C. U. 1951, '54)

3. (*a*) Explain what is meant by polarised light and plane of polarisation. Describe the method of producing plane polarised light by reflection. Explain Brewster's law. (*cf.* C. U. '63 ; 1943, '68)

(*b*) Show that a beam of plane polarised light can be regarded as being composed of two equal and oppositely circularly polarised light. (C. U. 1968)

4. Light wave can be polarised while the sound waves cannot. Why ? (P. U. Suppl. 1955)

5. What do you understand by polarised light ? Explain the action of (*i*) a pile of plates, (*ii*) a tourmaline crystal, in producing plane polarised light. (P. U. Suppl. 1952)

6. Explain what you mean by polarisation of light and distinguish clearly polarised light from ordinary day light.

Describe experiments for polarising ordinary light. (C. U. 1950)

7. Explain what is meant by polarised light, double refraction, plane of polarisation and optic axis (C. U. 1952)

8. Define (*i*) optic axis of a crystal, (*ii*) Principal plane of a crystal, (*iii*) ordinary and extraordinary refractive indices. (*cf.* C. U. '64)

Explain clearly how you will determine the extraordinary refractive index for a calcite crystal. (*cf.* C. U. 1946 ; P. U. 1950)

9. Discuss the significance of the phenomenon of polarisation with regard to the nature of light.

Explain the construction of Nicol prism and show how it can be used as a polariser and as an analyser. (Pat. U. 1952)

10. Explain the double refraction in a calcite crystal. Explain how a Nicol prism is made and how it is used. (Luck. U. 1952)

11. Give Huygens' theory of double refraction in a uniaxial crystal. What are positive and negative crystals ? Name a crystal of each type (*cf.* Del. U. '69 Nag. U. 1953)

12. (*a*) What is meant by optic axis and principal section of a doubly refracting crystal ?

(*b*) Describe a Nicol's prism showing clearly how it is constructed and how it acts. (Del. U. 1969 ; C. U. 1964, '67 ; Utk. U. 1954)

13. Explain the terms polarised light, plane of polarisation, extraordinary ray and optic axis

Describe the construction and action of a Nicol prism. (*cf.* C. U. '66, 1954, 1955)

14. Write a short note on double refraction.

15. (*a*) What do you understand by double refraction ? Describe any method to show that the two refracted rays in double refraction are plane polarised at right angles to each other. State and justify the nature of wave surface in the case of the two rays. (Utk. U 1950)

(*b*) State and explain Brewster's law. Deduce from the law that when a pencil of light falls upon a transparent substance at the polarising angle, the reflected and refracted rays are at right angles Explain the principle of polariscope to detect the plane polarised light (Utk. U. 1970 S)

16. Thin plates are cut from a negative double refracting crystal with the position of the optic axis as described below. Explain, giving diagrams, the nature of refraction observed in each case :—

(*a*) Optic axis inclined to the upper face but lying in the plane of incidence.

(*b*) Optic axis parallel to the upper face and lying in the plane of incidence.

(*c*) Optic axis parallel to the upper face but perpendicular to the plane of incidence.

(*d*) Optic axis perpendicular to the upper face (Bom. U. 1948)

17. Describe with a neat sketch a Nicol's prism and explain its principle of working. (*cf.* Utk. U. '66)

How would you use a pair of Nicol's prisms to produce and detect plane polarised light ? (C. U. 1946 ; *cf.* '49, '52, '53,'57)

18. What are meant by plane polarised, circularly polarised and elliptically polarised light ? How are they produced and how do you distingiush between them ? (Ban. U. 1948 ; Baroda U. 1951)

19. Calculate the thickness of a quarter-wave plate for light of wavelength 6000 Å. U. The refractive index for the ordinary ray is 1·544 and that for the extraordinary ray is 1·553. (Agra U. 1954)

[*Ans.* 0·00167 cm]

20. Describe various methods of producing and detecting a plane polarised light. (Gau. U. 1949)

21. Explain fully how you would produce and detect beams of circularly and elliptically polarised light. (P. U. 1949)

22. (*a*) How would you produce elliptically polarised light ? How would you distinguish it from partially polarised light ?

Calculate the thickness of a quarter-wave plate for light of wavelength 5890 Å. U. ($\mu_e = 1{\cdot}55$; $\mu_o = 1{\cdot}54$). (Luck. U. 1949)

[*Ans.* 0 00147 cm.]

(*b*) What is a half-wave plate ? (C U. 1967)

23. Describe the construction of a Babinet's compensator. Explain how you use it for analysing elliptically polarised light. (Bom. U. 1953)

24. Explain what is meant by a doubly refracting crystal. What is the difference between a negative and a positive crystal ?

A parallel beam of light is incident normally on the plane face of a crystal of quartz. Construct both the refracted wavefronts and the refracted rays when the optic axis of the crystal is (*a*) in the plane of incidence and parallel to the refracting surface, and (*b*) perpendicular to the plane of incidence. (Gau. U. 1957)

25. What do you understand by a quarter-wave plate and a half-wave plate ? (Raj U. 1951)

26. Write short notes on :—

(*a*) Fresnel's rhomb, (*b*) Wollaston prism. (P. U. 1949)

27. What is specific rotation of an optically active substance ? (C.U. 1958)

28. Write short notes on Polarimeters. (C U. 1950)

29. Describe the construction and action of a polarimeter. (Del. U. 1951 ; C.U. 1959)

30 Explain clearly the functions of various parts of Laurent's half-shade polarimeter. (Del. U. 1969 ; P. U. 1948 ; *cf.* Agra U. 1953 ; C.U. '65)

Give Fresnel's explanation of the rotation of the plane of polarisation by an optically active substance. (P. U. 1948)

31. Explain the action of a polarimeter.

Give the construction of a half-shade plate. (Luck. U. 1953)

32. Explain how Fresnel has accounted for the rotation of plane of polarisation. Describe Laurent's half-shade polarimeter. (Ban. U. 1949)

33. (*a*) Describe and explain the action of a half-shade polarimeter.

(*b*) Give Fresnel's explanation of rotatory polarisation of light.
(Bom. U. 1953 ; Agra U. 1955)

34. How would you account for the rotation of the plane of polarisation of a plane polarised light on passage through a doubly refracting crystal ?
(Gau. U. 1949)

35. Explain how a beam of plane polarised light may be regarded as composed of two equal and oppositely circularly polarised light. (C. U. 1941, '49)

Hence explain the rotation of the plane of polarisation of a plane polarised beam on its passage through an active substance.
(C.U. 1941, '49 ; Baroda U. 1951)

CHAPTER V

MISCELLANEOUS OPTICAL PHENOMENA

99. Doppler Effect :—Doppler effect is a well-known phenomenon in acoustics. If there is a change in the relative motion of a source of sound and observer, the apparent frequency of the note as heard by the observer seems to differ from that emitted by the source. It is found that the pitch of the whistle of a train appears to rise when the train approaches the hearer and to fall as the engine recedes from him. [For details see Art. 68, Sound, College Physics, Vol. I]

The principle underlying the phenomenon is equally applicable to all wavemotions and is successfully applied as well in optics. Let us suppose that the observer is at rest at O and the medium has neither any velocity. If λ be the wavelength of the light emitted by the source, then supposing the source S to be stationary, the number of waves emitted by it per second will be equal to the frequency $n = \frac{C}{\lambda}$ where C is the velocity of light. In one second the space traversed by these waves is $C = AB$ in Fig. 103. But if the source moves towards the observer with a velocity V_s, then by the time the first pulse of waves reaches B at the end of

one second, S moves to S' at A'. So the n waves emitted by the source in one second will be confined within the region $AB = C - V_s$. So the original wavelength will be altered to λ' due to the clustering of the waves where $\lambda' = \frac{C - V_s}{n} = \frac{C - V_s}{C}\lambda$ ... ... (1)

Again if the source moves away from the observer, V_s will be negative and relation (1) will take the form,

$$\lambda' = \frac{C + V_s}{C}\lambda \quad \text{... ... (2)}$$

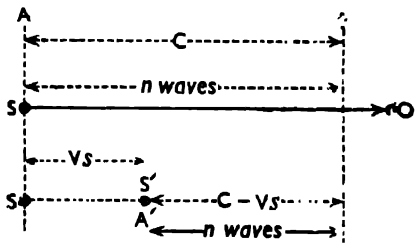


Fig. 103

Relations (1) and (2) are of considerable importance since they have a bearing with many problems in optics and astrophysics. It is evident from the above relations that if the source of light moves away from the observer, the wavelengths of the observed lines in the spectrum become slightly greater and if the source moves towards the observer, the wavelength becomes slightly shorter. In other words, in the above two cases the lines themselves will have a definite but small shift either towards the longer wavelength side or towards the shorter wavelength side. This shift is generally known as the *Doppler shift*. The doppler shift can easily be measured in the laboratory by measuring the wavelength of a particular line in the spectrum of the moving heavenly body and that of the same line from a spectrum produced in the laboratory. Thus the movement of the heavenly body towards or away from the earth can be inferred from the Doppler shift.

By observing this shift of the Fraunhoffer's lines of a solar spectrum taken from the east limb of the sun, it is found that this limb is approaching the earth. Again if the solar spectrum is taken from the west limb of the sun the Doppler shift indicates that this end is receding from the earth. These observations indicate that the *sun is rotating about its own axis*. From observation on the Doppler shift of the Fraunhoffer's lines obtained from the equatorial region of the sun, it is found that here the rotation is faster showing thereby that the sun does not rotate in the same manner as a solid sphere does. By the same method of study, it has been inferred that *there are atmospheric currents on the sun*. The formation of whirls in the neighbourhood of sun-spots have also been verified from a study of the Doppler shift.

The *nature of the Saturn's ring* has been studied with the help of Doppler's principle. The planet Saturn is surrounded by a rotating

ring which was formerly considered to be solid by some observers. Others considered this ring to be made of a huge number of small moons. But from simple considerations of the dynamical laws, it can be shown that if the rings were solid, its outer edge should move faster than the inner edge while if it be a clustering of minute moons, the opposite will be the case. Observing the actual Doppler shifts of the spectral lines obtained from the outer edge as well as from the inner edge of the Saturn's rings, it has been decided that the inner edge moves faster than the outer one and as such the ring itself really consists of a multitude of minute satellites.

By the Doppler effect the rotation of certain pairs of stars have been detected. These stars are known as the *spectroscopic binaries.* Even if the telescope be unable to resolve the two stars of a binary their presence and rotation about each other can be inferred simply from the change in wavelengths of the rays from the approaching and receding components of the binary. Moreover, the shift of the spectral lines obtained from distant spiral nebulæ towards the red end of the spectrum indicates that these nebulæ are receding from us. It has been found that the *greater the distance of the nebula, the greater will be the speed of recession.* A nebula in the constellation Gemini (at about 150,000,000 light-years from the earth) is found to recede with a speed of 15000 miles per sec. Indeed it has been found that all the hundreds of nebulæ that are detectable by high power telescopes are all running away from our solar system at a tremendous speed. These observations corroborate the theory that the universe as a whole is expanding.

When a laboratory source of spectrum is considered, it is found that the atoms emitting the radiation are rapidly moving with respect to the observer due to their thermal agitation. Some of these atoms will be approaching towards while others will be receding from the observer. So the wavelengths of the light emitted will not be strictly monochromatic and will vary over small limits. Thus each of the spectral lines will extend over a small range of wavelengths and will possess a breadth due to this Doppler shift. This is known as the *Doppler broadening of the spectral lines.* This broadening will always be present in a spectral line but its magnitude can be reduced by cooling down the emitting gas to certain extent.

100. Scattering of Light :—The phenomenon of the scattering of light gives rise to many of the colour effects that we see around us. The effect is due to diffraction of light by the scattering particles. When light is incident on a particle whose dimensions are comparable with the wavelength of light, spherical waves will originate and spread out in all directions. The particle robs the incident beam of its energy and scatters it in all directions. This process is known simply as *scattering* or *molecular scattering.* When white light falls on a number of particles of size comparable with the

wavelength, blue and violet components are found to be scattered more than the red. So the smaller the wavelength of the light, the more copiously will it be scattered. This fact can be established from a simple dimensional consideration. It is found that the amplitude A_s of the scattered radiation is proportional directly to the amplitude A of the incident beam. A_s is also proportional to the volume of the scatterer, inversely to the distance r from the scatterer and directly to the wavelength of the incident radiation.

$$\therefore \quad A_s \propto A\frac{v}{r}.\lambda,$$

Here dimension of A_s is L, that of A is L, that of v is L^3 and that of r is L.

The dimensional equation may then be written as,

$$\therefore \quad [L]=\left[\frac{L\,L^3}{L}\right][\lambda^y]\ ;$$

$$\text{or,} \quad [\lambda^y]=[L^{-2}].$$

Since λ also has the dimension of L, $y=-2$.

$$\therefore \quad A_s \propto \frac{1}{\lambda^2}.$$

$$\therefore \quad A_s = K.\frac{Av}{r\lambda^2}.$$

Hence the intensity of the scattered radiation is inversely proportional to the fourth power of λ $\left(i.e.\ I \propto \frac{1}{\lambda^4}\right)$. This is the reason why from small scatterers blue-violet rays are more copiously scattered than the red rays. This copious scattering of blue light by small particles have been experimentally demonstrated by Tyndall and hence the effect is commonly known as the *Tyndall Effect.*

Tyndall effect can be demonstrated by the following very simple experimental arrangement. In a beaker 1 litre of water is taken. To this is added 30 c.c. of about 14 to 15% solution of sulphuric acid. Now 10 to 20 c.c. of a concentrated solution of photographic 'hypo' is added to the contents of the beaker and stirred. Minute sulphur particles are formed in the liquid. If the beaker is illuminated from one side by a strong source of white light, within two or three minutes due to formation of particles of proper size, the transversely scattered rays are found to be blue, whereas the directly transmitted components appear to be rich in orange-red. As the sulphur particles grow in size, more and more light is scattered and

the colour of the scattered light changes from blue to white and the transmitted light becomes less intense.

Simple theoretical considerations of the angular distribution of the intensity of the scattered light show that the intensity is proportional to $(1+\cos^2\theta)$ where θ is the inclination of the scattered rays with the forward direction of the light. Hence the intensity of the scattered light is maximum in the forward and the backward directions and is minimum at right angles to the incident ray. The scattered light is in general partially polarised. At right angles to the incident direction, the scattered light is found to be almost completely plane polarised. In the direction (forward or backward) of the incident light, the scattered light is, however, unpolarised. If there be a larger number of identical scatterers scattering light randomly to the same point, it is found that the total intensity of the scattered light is n times the intensity due to a single scatterer, where n is the total number of scatterers.

The scattered light according to Rayleigh's theory should have the same wavelength as the incident light. In most of the cases of scattering, this is found to be the case and this type of scattering is known as the Rayleigh scattering or *coherent scattering*. According to quantum theory this coherent scattering results from an *elastic collision* between a photon and a molecule. In some other special types of scattering (*e.g.* in fluorescence, Raman Effect etc.), the scattered light is found to undergo a change of wavelength which comes under the category of *incoherent scattering*.

101. Colour of Sky light :—The blue of the sky can be successfully explained from the general theory of scattering. Light from the sun is scattered by the atmospheric particles. The molecules of air and the minute dust particles present in the atmosphere serve as the scatterers. The fluctuation of the density of air arising out of the thermal agitation of the molecules is believed to play an important role in scattering. Since intensity of scattered light is inversely proportional to λ^4, the blue end of the spectrum is scattered much more copiously than the red end. Red light has a wavelength about 1·8 times that of the blue-violet light. Hence the intensity of the scattered blue-violet components is $(1{\cdot}8)^4 = 10$ times that of red light. Whenever we look away from the sun it is the scattered light which reaches us and this is richer in blue-violet components. This richness in the shorter waves gives sky light its characteristic blue colour. If the earth had no atmosphere the sky would have been almost black. About one-fifth of the total illumination on a clear day is due to the sky light. The sky light has a spectral composition entirely different from that of sun light. But on a completely overcast day the former is found to approach the latter when the scattering particles are droplets of water of comparatively larger size. The sky light on a clear day when analysed by a nicol

is found to be partially polarised, the polarisation being maximum when the line of sight is perpendicular to the sun rays.

102 (a). Colours of the sun at rise and set :—The λ^4-law of scattering also accounts for the succession of colours of the sun at sun-rise and sun-set. Close to the surface of the earth there are fine dust, smoke and small water particles. In Fig. 104 these are shown to occupy a dotted belt surrounding the earth. An observer at W sees the mid-afternoon sun, whose rays travel a relatively shorter path of this belt. So only a little violet and blue has been lost due to scattering and the sun appears white. An observer in the region from y to r gets sun rays after they have traversed a gradually increasing path of this belt with the result that due to scattering major portions of the violet and blue rays are scattered. The sun thus appears yellow (at y), then, orange and finally red at sun-set (at r). A similar argument can explain the reddening of the sun at the time of sun-rise.

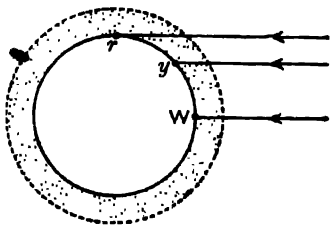


Fig. 104

(b) Blue of the Sea :—The blue colour of the sea has been also explained in the similar way by Sir C. V. Raman. He showed that in this case the water molecules serve as the scatterers. The dimensions of these water molecules are such that they would scatter the blue rays more copiously than the red. So this scattered blue light is the cause of the blue of the sea.

***103. Michelson and Morley's Ether-drift Experiment** :—According to the wave theory of light, the propagation of light waves necessitates the existence of a continuous and all pervading medium, *ether*, which is endowed with definite elasticity and density. As the velocity of light in an optically denser medium is less than that in vacuum, either the elasticity of ether within that medium is less or its density within the medium is greater. Fresnel on the assumption of this density variation, showed that there should be an *ether drag* when the medium itself moves dragging the ether along with it and that the ether outside this moving medium would *drift back* in the opposite directions. He calculated on the above assumption an expression for the *drift velocity* $v_d = v\left(1 - \frac{1}{\mu^2}\right)$ where μ is the refractive index of the medium. Fizeau's experiment on velocity of light in a moving medium seemed to confirm Fresnel's hypothesis, Michelson also, with improved experimental set up, arrived at the same conclusion. But in 1887, Michelson and Morley devised an indirect method for detecting this ether drift. The instrument

used was fundamentally a Michelson interferometer [Fig. 105(*a*)]. A beam of light was sent in two different paths after being divided into two portions at a half-silvered glass plate *G*. These paths were perpendicular to each other. The rays were then reflected from two mirrors M_1 and M_2 and made to interfere after passing through the glass plate *G*. At first the paths of the two rays were respectively perpendicular and parallel to the direction of earth's

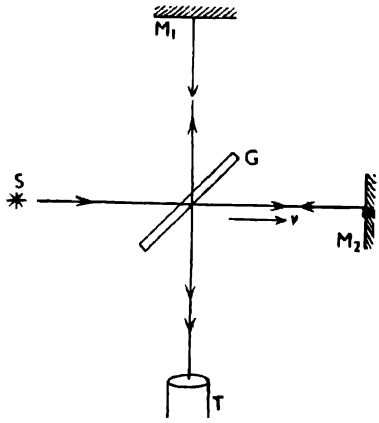


Fig. 105(*a*)

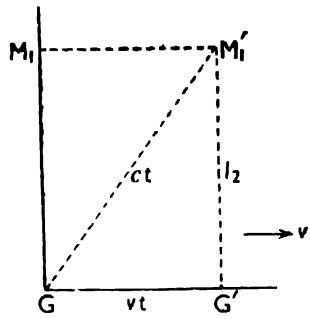


Fig. 105(*b*)

motion round the sun, which may be assumed to be fairly uniform and rectilinear during the time of the experiment. Then the whole apparatus was rotated through 90°. It was expected that a shift of the fringe system should occur. This expectation was based on the possibility of compounding the velocity of light with that of the observer because of a so-called ether drift.

So relative to the observer and the apparatus, the velocity of light in its transit from *G* to M_2 ($GM_2 \parallel$ to Earth's motion) is $c-v$, where v = earth's velocity in the direction shown in Fig. 105(*a*). Again in the return journey from M_2 to *G*, the relative velocity is $c+v$ so that the time taken by light to go from *G* to M_2 and back is,

$$t_{\parallel} = \frac{l_1}{c-v} + \frac{l_1}{c+v} = \frac{2l_1}{c\left(1-\frac{v^2}{c^2}\right)} \quad \ldots \quad \ldots \quad (1)$$

Again during the transit of light from *G* to M_1 ($\perp$ to Earth's motion), if the forward journey requires t secs. the earth by that time moves through vt so that *G* comes to *G'* where $GG' = vt$, [Fig. 105(*b*)]. So M_1 moves to M_1' and the actual path of the ray is $GM_1' = ct$. If $GM_1 = l_2$, we have,

$$c^2t^2 = l_2^2 + v^2t^2 \; ; \text{ or, } \; t^2(c^2-v^2) = l_2^2 \; ; \text{ or, } \; t = \frac{l_2}{c(1-v^2/c^2)^{1/2}}$$

As the return journey from M_1' to newly displaced position of G would require the same time, the total time taken in going from G to M_1 and back, is $t_\perp = \frac{2l_2}{c(1-v^2/c^2)^{1/2}}$.

When G is set exactly 45° to the direction of v and $l_1 = l_2 = l$ and a bright fringe is obtained at the centre of the field, the time difference between the two reflected rays interfering at centre of the field of view is, $T_1 = t_{\parallel} - t_\perp = \frac{2l}{c}\left[\frac{1}{1-\frac{v^2}{c_2}} - \frac{1}{\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}}\right] = \frac{2l}{c}\left[1+\frac{v^2}{c^2} - 1-\frac{v^2}{2c^2}\right] = \frac{l}{c}\cdot\frac{v^2}{c^2}$. If the apparatus be now rotated through 90°, this time difference could be reversed and we get, $T_2 = \frac{l}{c}\frac{v^2}{c^2}$. Therefore the positions of the interference pattern in the two cases (before and after rotation) should be displaced from the position corresponding to $v=0$, on opposite sides by the same amount $\frac{l}{c}\cdot\frac{v^2}{c^2}$. So due to 90°-rotation the fringes should be shifted by an amount corresponding to time interval $\frac{2l}{c}\cdot\frac{v^2}{c^2}$ and the corresponding path difference is (time lag. $\times C$) *i.e.* $\frac{2.lv^2}{c^2}$. If λ be the wavelength of light used, the expected shift is $\frac{2lv^2}{c^2}\Big/\lambda$, which in Michelson-Morley's experiment was expected to be more than 0·37 of a fringe shift. The arrangements made in the experiment ensured the detection of a shift of 0·01 of a fringe even. This was done by increasing the path traversed by each beam sufficiently by multiple reflections from a number of mirrors and taking many other precautionary measures. But no shift (really less than $\frac{1}{30}$ of the calculated) was actually observed.

This null result could only be explained either (1) by supposing the ether to be altogether absent or (2) by supposing that relative to the observer the arm of the interferometer parallel to the direction of earth's motion experiences a contraction to such an amount that the fringe shift is anulled. This second hypothesis was put forward by Fitzgerald and Lorentz. But as the hypothesis seems to be rather artificial, Einstein suggested that the above type of velocity compounding cannot at all take place because it is impossible to detect the absolute velocity of the earth through space. So according to Einstein, absolute velocity is an unpractical proposition as all measurements are made on a moving frame-work of reference. So he formulated the *special theory of relativity* in two

distinct postulates :—(1) The laws of physical phenomena are invariant when stated in terms of either of two reference systems moving at a constant speed relative to each other ; (2) The velocity of light in free space is constant for all observers and is independent of the relative velocity of the source and the observer. With the help of these postulates, Lorentz transformations were deduced and it was found from this transformations that the null result of Michelson-Morley experiment was an inevitable consequence of this theory. The results of earlier experiments carried out Fizeau and others were also explained from this special theory of relativity without reference to any drift or drag of ether.

***104. Raman Effect** :—In 1928, Sir C. V. Raman discovered that when a transparent liquid is irradiated with an intense beam of light, the spectrum of the scattered light contains in addition to the exciting frequencies (ν_0), several weaker lines on both sides of some of the prominent exciting lines. The frequencies of these new lines are given by $\nu_0 \pm \triangle \nu_n$. The effect has been subsequently observed in solids and gases also. The differences in the frequencies (*i.e.* $\pm \triangle \nu_n$) of the exciting lines and the new lines (*i e. Raman lines*), on either side are independent of ν_0 and depend on the nature of the scattering medium alone. These are known as the *Raman shifts*. The Raman lines that are displaced towards the longer wavelength side of the exciting lines are known as the **Stokes lines** and those displaced towards the shorter wavelength side as the **anti-Stokes lines** according to the convention of classical theory of scattering. In Raman effect the anti-Stokes lines are fainter than the Stokes lines. But both these types of Raman lines being very faint, a sufficiently long exposure is needed to photograph them.

The simplest arrangement that can be made to have defined Raman lines with a comparatively shorter exposure is shown in the attached figure (Fig. 106). Here *W* is a horn-shaped glass tube containing the liquid, the horn of which is curved and blackened so that no stray light reflected from this end may reach the other end where observations are made. Such a tube is generally known as *Wood's tube*. *J* is a water jacket surrounding *W*. *M* is a quartz mercury arc serving as the intense source of light. *L* is a big aperture condensing lens. *R* is a plane reflector. L_1 is a second condensing lens to concentrate the scattered light on to the slit *S* of a spectrograph on which is mounted the photographic plate for recording the Raman lines.

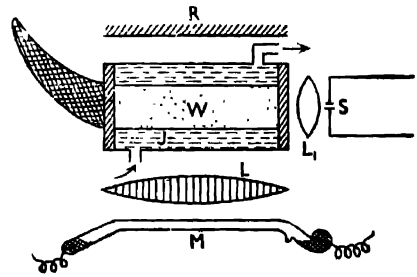


Fig. 106

After an exposure of a few hours the plate is developed and fixed. The wavelengths of the Raman lines can be conveniently measured by a comparator with reference to a standard wavelength-chart. The intensities of the lines can be measured by a microphotometer. Thus Raman shifts can also be measured in terms of frequencies or wave numbers $\left(\frac{1}{\lambda}\right)$ from a knowledge of the frequency of the exciting radiation.

The possibility of the existence of Raman lines had been predicted by Smekal in 1923 from theoretical considerations. According to Smekal, the effect originates as a result of a kind of elastic collision between the photon and the molecule, wherein, due to the exchange of energy, the photon may either be deprived of or be replenished by an amount of energy. In the former case the molecule absorbs energy from the photon and jumps up to a higher energy level and in the latter case the molecule jumps down to a lower energy level and releases the corresponding energy to the photon. Now applying the principle of conservation of energy to the case of an impact of a photon and a molecule, we can write,

$$E_i + \tfrac{1}{2}mv_i^2 + h\nu_o = E_f + \tfrac{1}{2}mv_f^2 + h\nu' \quad \ldots \quad (1)$$

where E_i and E_f are the intrinsic energies (electronic, vibrational and rotational) of the molecule before and after the impact ; v_i and v_f are its velocities before and after the collision ; m is the mass of the molecule ; h is Planck's constant and ν_o and ν' represent the frequencies of the incident and the scattered quanta respectively.

As during the time of the experiment, the temperature of the scattering medium does not change appreciably, $v_i = v_f$, and the eqn. (1) may be written as,

$$E_i + h\nu_o = E_f + h\nu', \text{ or, } h(\nu' - \nu_o) = E_i - E_f ;$$
$$\text{or, } \nu' - \nu_o = (E_i - E_f)/h \quad \ldots \quad (2)$$

When $E_i = E_f$, $\nu' = \nu_o$ and we get the unmodified scattered line ; when $E_i < E_f$, $\nu' < \nu_o$ (*i.e.*, $\lambda' > \lambda_o$) and a Stokes line appears ; and when $E_i > E_f$, $\nu' > \nu_o$ (*i.e.*, $\lambda' < \lambda_o$) and an anti-Stokes line appears.

Putting $\frac{E_i - E_f}{h} = n\nu_m$, or, $\Delta\nu_n$, we have $\nu' = \nu_o \pm n\nu_n$ (3)

Here $\Delta\nu_n = n\nu_m$ represents the Raman frequency shift, ν_m is the characteristic frequency of the molecule and $n = 1, 2, 3, 4$ etc.

Thus the origin of Raman lines and the characteristic Raman shift are explained on the basis of the elementary quantum concept. It must, however, be remembered that a single transition of above type involving a jump of a particular molecule can produce no Raman lines. If a multitude of molecules undergoes a similar transition then and then alone a Raman line of any appreciable intensity will originate.

A more rigorous theory of the effect was subsequently developed by Kramer and Heissenberg on the basis of quantum mechanics.

The intensity of a Raman line is only a few hundredth part of the intensity of its parent line in a liquid and in a gas it is only a few thousandth part. The variations of intensity as we pass from one line to another and from one material to another are of great importance in the study of molecular structure. The Stokes lines are more intense than the anti-Stokes lines. The intensity of a line falls off rapidly with the increase of its separation from the parent line. When the temperature is raised the intensities of lines become more pronounced and the lines move towards the parent line with the rise in temperature. The states of polarisation of different Raman lines are found to be different and they are probably connected with their relative intensities.

Like the band spectra of molecules, Raman spectra may also be electronic, vibrational and rotational in character. The vibrational and rotational Raman spectra closely resemble the near and the far infra-red absorption spectra of molecules. In many cases Raman frequency shifts are actually equal to the infra-red absorption frequencies, or sometimes to the difference of such frequencies. Again many Raman lines are found whose corresponding frequencies are not traceable in the infra-red and certain infra-red frequencies have neither corresponding Raman lines. Moreover, the relative intensities of the Raman lines have practically no correlation with those of the corresponding infra-red lines. So a critical study reveals that as the mode of production and the laws regulating the two types of lines are quite different, a true correspondence cannot exist between a Raman line and an infra-red line.

The characteristics of Raman lines, *e.g.* the frequency shift (characteristics of the scatterer), the close relation with the infra-red absorption frequency, the incoherent nature, the attending anti-Stokes lines and the strongly polarised nature of the lines, *etc.* actually lie in their universality (because exhibited by a variety of substances). The Raman shifts reveal to us a knowledge of the energy levels of molecules just as a band spectrum does. But whereas the band spectra in the infra-red region involve difficulties in measurements, the Raman spectra can be easily studied in the visible or in the ultra-violet regions. Raman effect thus made accurate spectroscopic photography available for the study of the problems of molecular constitution and molecular energy levels. In addition Raman effect supplies another definite proof of the quantum nature of light.

105. Fluorescence :—Some substances irradiated by light may, under suitable circumstances, themselves emit light which persists *so long as the incident radiation is present.* The incident radiation is first absorbed by the flourescing substance and then transformed into a radiation of a newer wavelength. So the phenomenon is a kind of transformation of absorbed radiation. The

emitted radiation is characteristic of the substance and consists of lines or bands generally of longer wavelength side of the exciting radiation. The phenomenon is called *fluorescence* because it was first observed in fluorspar. The phenomenon is exhibited by substances in all states of aggregation, *i.e.* by solids, liquids and gases. The phenomenon is more or less a surface phenomenon and is exhibited by the liquid paraffin, kerosene, aniline dyes (like eosene, fluorescene etc.). Kerosene and paraffin oil when irradiated by sunlight exhibit bluish coloration at the surface. A solution of chlorophyll, the green colouring matter of plant in alcohol, emits a brilliant red colour when exposed to white light in a dark room. When ultra-violet rays fall on aniline dyes the fluorescent effect becomes much more prominent. In fact the remarkable extension of the solar spectrum into the ultra-violet region was first observed by Herschel by throwing the solar spectrum on a piece of turmeric paper when the fluorescent glow was found to extend beyond the violet. In investigating the ultra-violet spectrum fluorescence plays a very important role. Even in the X-ray region the photographs are taken with the help of fluorescent barium-platinocyanide screens.

Sir George Stokes investigated this phenomenon quite thoroughly and framed the famous **Stokes' law** from his observations. This law states that "*whenever the refrangibility of light is changed by fluorescence, it is always lowered and never raised*". Since the lowering of refrangibility indicates an increase in wavelength, Stokes' law implies that in fluorescence the emitted rays will always be of wavelengths greater than that of the exciting radiation.

Most of the *precious stones* and diamond are found to fluoresce conspicuously under an ultra-violet lamp but the imitation stones look much the same as in ordinary light and do not fluoresce. An ordinary *fluorescent lamp* consists of a mercury vapour discharge tube, the inside of which is coated with a fluorescent paint whereby some energy of the ultra-violet radiations from the discharge, which would have been otherwise wasted, is utilised in producing visible radiations. So better illumination is secured at the expense of the same power. Poster paints are now-a-days made so as to contain traces of fluorescent materials. The fluorescent materials produce fluorescence utilising some otherwise lost invisible radiation. Hence a brilliant effect is exhibited by the paint due to greater amount of light scattered from it on the whole.

The main distinctive features of Rayleigh scattering, Raman effect and the fluorescene can be given as follows :—

(1) In Rayleigh scattering and Raman effect light is not absorbed but in fluorescence, incident light is first absorbed and then transformed into the fluorescent radiation.

(2) In Rayleigh scattering the wavelength of light remains unaffected whereas in Raman effect there are changes in wavelengths

both to the longer wavelength as well as to the shorter wavelength side. In fluorescence also there is also a change in wavelength always towards the longer wavelength side.

(3) Whereas Rayleigh scattering can be explained from the classical theory, Raman effect and fluorescence can only be interpreted from the quantum theory.

(4) In Rayleigh scattering there is no critical exciting wavelength. In Raman effect there is a theoretical critical λ in the infrared region and in fluorescence this critical λ may lie either in the visible or in the ultra-violet region.

106. Phosphorescence :—A number of substances being exposed to radiations of short wavelengths are found to emit light even after the exciting source is removed. This phenomenon is known as phosphorescence. We know that fluorescence stops as soon as the exciting source is withdrawn. But phosphorescence may continue for a considerable period after the switching off of the source. In some cases, however, phosphorescence may last only for a fraction of a second when it is hardly worthwhile to distinguish between phosphorescence and fluorescence and the term fluorescence can be used to cover both cases. Sometimes the term *luminescence* is used to designate both fluorescence and phosphorescence.

The term phosphorescence owes its origin to the glow emitted by phosphorus in darkness which actually is a slow oxidation process and has nothing to do with actual phenomenon of phosphorescence. In substances called *phosphors* which are crystalline substances containing traces of metallic impurities phosphorescence is found to occur. Paints containing sulphides of Calcium, Barium and Strontium are good phosphorescent substances. Balmain's paint (mainly calcium sulphide) is found to glow for hours after isolation. But most of the phosphorescent paints require periodic illumination to exhibit a pronounced effect.

Violet and ultra-violet rays are very active in producing phosphorescence. In all cases of phosphorescence, the wavelengths of the emitted light are either greater or at least equal to that of the exciting light. The phosphorescence sometimes become more intense with an exciting radiation of longer wavelength but the duration is proportionately diminished. At very low temperatures some of the ordinary substances are found to exhibit phosphorescence. Feathers, egg shells are found to glow at the temperature of the liquid air.

The surface of the sea and the rotten fishes etc. are also found to glow in the dark. This glow is the result of slow oxidation of minute organism in the sea or in the decaying matter. This type of effect actually belongs to the phenomenon of *bio-luminescence*.

107. Calorescence :—Some substances which can absorb long heat waves and transform them into smaller light waves are called **calorescent** substances. If light from an arc-lamp be passed through a solution of iodine in carbon bisulphide to cut off the light rays, and then the heat radiations be focussed on a thin sheet of platinum, the platinum sheet becomes highly luminous, *i.e.* gives out visible light radiations of smaller wavelengths (first shown by Tyndall).

***108. Compton Effect** :—Arthur H. Compton in 1923 examined the monochromatic *X*-ray scattred by a block of graphite and found that the scattered rays are of two distinct types. He observed that one type of scattered radiations has the same wavelengths as the incident ray (*unmodified line*) and the other type has a slightly greater λ (*modified line*). This phenomenon is known as the *Compton Effect*. He found that the difference of wavelengths between the modified and the unmodified lines depend on the angle of scattering. He explained the effect by assuming the quantum nature of *X*-rays wherein each quantum of radiation should be attended with a definite momentum. When such a quantum or photon directly collides with an electron of the scatterer, the latter recoils absorbing some energy from the incident quantum whereupon the quantum falls in frequency (*i.e.* increases in λ) when scattered. Thus the increase in wavelength of the modified component of the scattered radiation is explained on the assumption of an elastic collision between a particle and a quantum. The wavelength difference between the modified and the unmodified lines as theoretically calculated by Compton is given by,

$$\Delta\lambda = \frac{h}{m_0 C}(1 - \cos\phi) \quad \cdots \quad \cdots \quad (1)$$

where ϕ = angle of scattering, h = Planck's constant, m_0 = rest mass of an electron and C = velocity of light. The relation (1) has been also experimentally verified for different values of ϕ. When $\phi = 90°$, $\Delta\lambda = \frac{h}{m_0 C} = 0{\cdot}024\text{Å}$, which is experimentally supported within the error limits. The presence of recoil electrons has also been demonstrated with the help of Willson's cloud chambers and recently by G. M. Counters. The calculated kinetic energy of this recoil electron from the elastic collision theory of Compton has also been found correct. The ratio of the intensities of the modified and the unmodified lines is found to decrease with the increase of the atomic number of the scattering substances. Compton effect and its correct interpretation from the quantum hypothesis supplied a definite proof (just like the photo-electric effect) of the quantum character of radiation.

***109. Zeeman Effect** :—When a source of line spectrum is subjected to intense magnetic or electric fields, the individual lines are found to be replaced by groups of lines. Faraday anticipated such effects but the apparatus at the time was inadequate to obtain them.

The magnetic effect was first observed by Zeeman in 1896 and the electric effect was discovered in 1913 by Stark.

So the phenomenon in which the spectral lines undergo modifications when the source of the same is placed in a strong magnetic field is known as the *Zeeman effect.* Observation of the modified lines may be made from directions perpendicular or parallel to the lines of force. The effect observed in the perpendicular direction is called the *transverse Zeeman effect* and that seen in the parallel direction is called the *longitudinal Zeeman effect.* In the simplest case, when the effect is observed with a comparatively low resolution instrument, a single spectral line is split up into *three lines* in the *transverse*

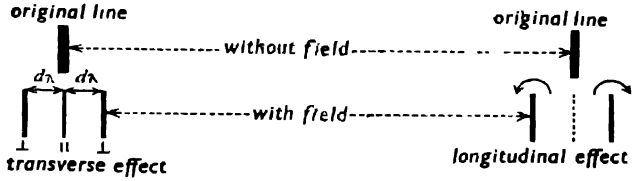


Fig 107

effect and into *two lines* in the *longitudinal effect* as shown in Fig. 107.

In the *transverse effect* all the three components are plane polarised. For the outer two, the direction of vibration is perpendicular to the lines of force and for the central component, it is parallel to the lines of force. The difference of wavelengths between any of the outer components and the central one is the same. It linearly increases with the magnetic field strength and is proportional to the square of the wavelength of the original line, the central line having the same wavelength as the original line. This wavelength difference is given by $d\lambda = \pm \frac{H}{4\pi c} \frac{e}{m} \cdot \lambda^2$ according to the classical electron theory, where H = field strength, c = velocity of light, e = electronic charge in e. m. u., m = mass of electron and λ = wavelength of the original line.

The intensities of the lines are equal and the total intensity of all the lines taken together gives the intensity of the original line.

In the *longitudinal effect* the original line disappears and two equidistant lines appear on two sides of it. Their intensities are equal and the sum of these intensities equals the intensity of the original line. The difference of wavelengths between any of these components and the original line is equal to that in the transverse effect. Both these lines are circularly polarised in the opposite sense—the higher frequency component having a sense of rotation of the magnetising current in the electromagnet.

When the splitting of the spectral line takes place in the above manner, the effect is referred to as the **"normal Zeeman effect"**. But when observed with a high resolution spectrograph, most spectral lines show a more complex pattern which is known as *the anomalous Zeeman effect*. The normal Zeeman effect was originally explained from the classical electron theory. But latter on from the quantum theory of Zeeman effect it became clear that the normal Zeeman effect is only a particular feature of the more general phenomenon of anomalous Zeeman effect. When the field strength is very strong the split pattern of the spectral lines is quite different from the Zeeman pattern and the new effect is known as *Paschen-Back effect.*

Theory of normal Zeeman effect.—To explain the emission of light from atoms Lorentz postulated the existence of virtual oscillators within the atoms. These oscillators are electrons of mass m gm. and of charge e e.m.u. These electrons were supposed to be held to the atoms by elastic forces. When they vibrate about their mean position of rest, radiation will be emitted with a frequency identical with that of the vibrating electrons. The nature of polarisation of the emitted radiation is the same as the nature of motion of the oscillator.

During the emission of an *unpolarised* beam of monochromatic radiation, the virtual oscillators are supposed to vibrate in all possible directions with the same frequency. These vibrations may be resolved in the directions of three mutually perpendicular axes OX, OY, OZ. Let the magnetic field be applied along the Z-axis. In absence of the magnetic field, the three component vibrations have the same amplitude and frequency.

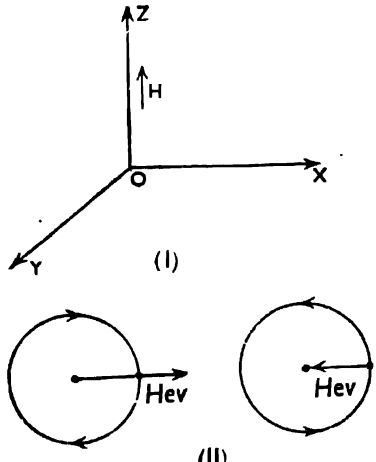


Fig. 107 (*a*)

Let us now consider the Z-vibration, whose equation of motion is,

$$m\frac{d^2z}{dt^2} + fz = 0 \ ;$$

or, $$\frac{d^2z}{dt^2} + \omega_0^2 z = 0,$$

where $\omega_0 = 2n\nu_0 = \sqrt{f/m}$, ν_0 being the natural frequency of the oscillator and f the restoring force per unit displacement.

But even when the magnetic field is applied along the Z-axis, the oscillator still vibrates with a frequency ν_0 and emits light of

the same frequency, which is linearly polarised with its vibrations parallel to the lines of force. So when viewed transversely along a direction perpendicular to the Z-axis, this Z-component appears plane polarised, but when viewed along the magnetic field (*i.e.* Z-axis) no emission of this frequency occurs.

Now considering the vibrations along any of the perpendicular axes (OX and OY), they can be resolved into two equal and opposite circular vibrations [Fig. 107(a) II] executed in a plane normal to the Z-axis. Now since a moving charge is equivalent to a current element, an oscillator with charge e moving in the circular paths will experience a force Hev acting radially outwards when the motion is clockwise and radially inwards when anti-clockise [Fig. 107(a) II], where v is the linear velocity of the oscillator. The directions of force follow directly from Fleming's left-hand rule.

If the radius of the circular orbit, in which the oscillator (or electron) is moving, be r and if F be the centripetal force on the electron, then we have in absence of the field, $F = \frac{mv^2}{r} = m\omega_0^2 r$ ...(1)

where $\omega_0 = 2\pi\nu_0$. When the magnetic field is applied, eqn. (1) will be modified as,

$F \pm Hev = m\omega^2 r$; or, $F \pm He\omega r = m\omega^2 r$ ... ... (2)

where ω is the altered value of ω_0 due to the extra force. From (1) and (2), we have, $\pm He\omega r = mr(\omega^2 - \omega_0^2)$ ... ... (3)

If the altered frequency due to applied field be ν, then $\omega = 2\pi\nu$.

$\therefore$ From (3) $4\pi^2 mr(\nu^2 - \nu_0^2) = \pm He \times 2\pi\nu r$;

or, $\nu^2 - \nu_0^2 = \pm \frac{Hev}{2\pi m}$;

or, $(\nu + \nu_0)(\nu - \nu_0) = \pm \frac{Hev}{2\pi m}$.

As the frequency change is very small, we can put $(\nu + \nu_0) = 2\nu$ and $(\nu - \nu_0) = d\nu$,

$\therefore$ $d\nu = \nu - \nu_0 = \pm \frac{He}{4\pi m}$ ... ... (4)

Again since $\nu = \frac{c}{\lambda}$ and $d\nu = -\frac{c}{\lambda^2} d\lambda$, the corresponding change in wavelength is, $d\lambda = \pm \frac{He\lambda^2}{4\pi mc}$ ... ... (5)

Thus when viewed along the Z-axis (*i.e.* along the lines of force) the linear vibration is absent but two components circularly polarised in opposite sense and of frequencies $\left(\nu_0 + \frac{He}{4\pi m}\right)$ and $\left(\nu_0 - \frac{He}{4\pi m}\right)$ are observed on two sides and at equal distances from the expected central line which is absent in this case (Longitudinal Effect).

Again when viewed along either of the X- and Y-axis (Transverse Effect), the original vibration along Z-axis appears plane polarised and on two sides of this central line appear two other plane polarised lines which arise due to the edge-wise view of the two circular components in the X-Y plane

Experimental Study :—To study Zeeman effect a strong magnetic field is to be applied on the source S emitting the radiation. The field is produced by an electromagnet of pole-pieces P_1 and P_2 shown in Fig. 108, the pole-piece P_2 of which is bored with a central hole. The light emitted by S is then studied from the longitudinal and transverse directions by means of high resolution spectrographs fitted with a constant deviation prism for the isolation of the spectral line under study. The final analysis of the doublet or triplet is made with the help of a Lummer-Gehrcke plate.

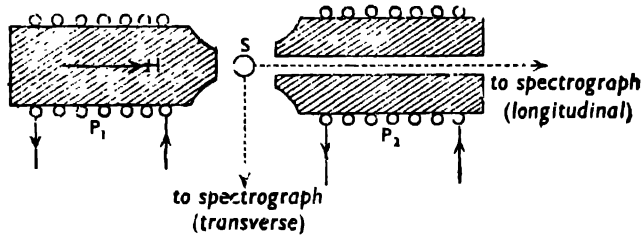


Fig. 108

The measurement of the frequency or the wavelength change in Zeeman effect provides an indirect method of determining the specific change $\frac{e}{m}$ of an electron as is evident from the expressions (4) and (5).

***110. Stark Effect** :—Stark, in 1913, showed that each spectral line emitted by a source becomes a set of lines when an intense electric field is applied. As it is not possible for the gas at the particular pressure of the ordinary discharge tubes to withstand the necessary high fields, Stark had to use very low pressures in the tubes. The positive rays in a high vacuum discharge tube were used as the source. The field applied was of the order of 10^4 to 10^5 volts per cm. In the case of $H\alpha$-line the effect observed was as follows.

In a direction perpendicular to the electric field the $H\alpha$-line is split up into a number of component lines, the central one being coincident with the original line. The other lines are symmetrically distributed on both sides of this line. Some of these lines are polarised perpendicular to the electric field while others are polarised parallel to the field. This is known as the *transverse Stark Effect*. In the *longitudinal effect* $H\alpha$-line splits up into three equispaced lines

which are unpolarised but occupy the same relative positions as the three innermost components of the transverse effect. The phenomenon was successfully explained from the standpoint of the quantum theory by Epstein and Schwarzschild in 1916.

*111. **Faraday Effect** :—When an isotropic transparent medium is subjected to a strong magnetic field, there is a rotation of the plane of vibration of light passing in the direction of the applied field through this medium. This phenomenon is called *Faraday effect.* Unlike ordinary rotatory polarisation as is found in quartz or other optically active substances, the sense of this magnetic rotation of light depends on whether the light travels in the medium in the direction of the lines of force or in the opposite direction. So when a beam of light is reflected back through the medium subjected to a strong magnetic field, the effect is doubled whereas the rotation in an optically active medium is cancelled due to reflection of the beam back along its path. The direction of rotation does not depend on the magnetic nature of the substance. The rotation produced is said to be positive if it is in the same sense as the flow of current producing the field and negative if in the opposite sense. So it is positive if it appears clockwise to the observer looking along the lines of force. This magneto-optical rotation is found to be maximum when light travels along magnetic field and it reduces to zero in the perpendicular direction.

Verdet from experimental results arrived at an empirical relation given by, $\theta = RHl$, where R is a constant known as *Verdet's Constant*, H is the strength of the magnetic field and l is the distance traversed by light in the medium.

So Verdet's constant is equal to the rotation per unit length of the medium per unit magnetic field applied. R is found to be constant for a medium at a particular temperature for a particular monochromatic radiation. It is approximately inversely proportional to the square of the wavelength. The effect has been interpreted from the classical electron theory. But the results are found only to be qualitatively correct. Quantitative agreements are still lacking.

To study the effect experimentally, the iron pole-pieces PP of a large electromagnet are to be bored longitudinally, the two holes being in the same straight line. The transparent material M to be studied is placed between pole-pieces of the electromagnet. A beam of light entering the hole in one of the pole-pieces passes through this material, then enters the second hole in the other pole-piece and finally emerges out. Before entering the first pole the light is made plane polarised by passing it through a polarising nicol. From the polariser the light enters a Laurent's half-shade plate H as shown in Fig. 109 and then proceeds along the axis of the hole in the electromagnet. The emergent beam is finally received on an analysing nicol. Before switching on the current in the electromagnet coil the analysing nicol is so adjusted

that the two halves on the half-shade plate appears equally bright through the analyser. The current is then switched on and the field of view of the analyser becomes unequally bright in the two halves. By rotating this analyser through an angle the matching is again secured. This amount of rotation gives the Faraday rotation. Knowing this rotation, the field strength and the length of the beam through the transparent material used, the Verdet's constant can also be calculated. One centimetre of the heaviest flint glass in a field of 10,000 oersteds gives a rotation of about 150°. For non-magnetic substances the effect is rather small and hence a powerful magnet is needed for study of the effect. Ferromagnets are opaque but when taken in thin films they give enormous rotations. An iron film of 0·02 mm. thickness in a field of several thousand oersteds may produce a rotation of nearly 360°.

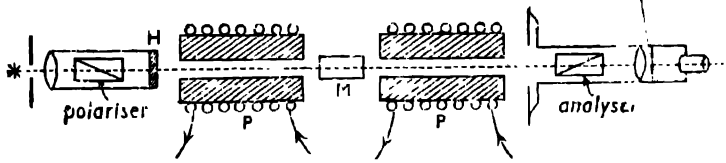


Fig. 109

*112. **Kerr Effect (Electro-optical)** :—Kerr in 1875 observed that when an electric field is established in some isotropic substances, they become doubly refracting with their optic axes parallel to the lines of force, or in other words, some substances, when placed in an electrostatic field, assume the properties of uniaxial crystals with the optic axes parallel to the electric lines of force. This phenomenon is known as *Kerr's electro-optical effect.*

When a parallel monochromatic beam of light is made plane polarised by a polarising nicol and at a distance an analyser is set for extinction the effect can be nicely observed by putting in between the two nicols a glass slab K on the opposite faces of which have been attached two plates of a parallel plate condenser. After setting the analyser A and the polariser P for extinction, the electric field is switched on and the glass slab is found to have acquired the property of a doubly refracting crystal so that the light emerging out of K becomes elliptically polarised. As a consequence the analyser allows a part of the light to pass through it. The sign of ellipticity of the emergent light remains unchanged even if the direction of the field is reserved. The magnitude of the effect is doubled if the light is reflected back on its path through K by a mirror between K and A (Fig. 110).

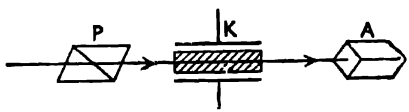


Fig. 110

Kerr observed that some substances like glass put under electric strain acquire the behaviours of a positive uniaxial crystal whereas other substances when similarly strained acquire the properties of a negative uniaxial crystal. This phenomenon was found to be present even in liquids and gases irrespective of whether they are conductors or insulators. Kerr found that due to the doubly refracting property acquired by the electrically strained substance, the phase difference introduced between two components of light (ordinary and extra-ordinary) passing through the so called *Kerr cell* is directly proportional to the thickness of the cell and to the square of the field strength.

Here the phase difference $\phi = 2\pi l\,(\mu_e \sim \mu_o)/\lambda$ where l = thickness of the cell, μ_e = extraordinary refractive index and μ_o = ordinary refractive index.

Again $\phi = KlE^2 = 2\pi BlE^2$, where B is the Kerr constant, which may be expressed from the above two relations as $B = (\mu_e \sim \mu_o)/\lambda E^2$.

In its practical applications in the measurement of velocity of light or in measurement of minute intervals of time, the Kerr cells used generally consists of glass vessels fitted with electrodes, and containing nitrobenzene solution which shows a pronounced Kerr effect.

Attempts were made to explain the effect by Langevin, Born, Raman and Krishnan and many other scientists. But none of the theories succeeded to explain all the aspects of the phenomenon.

The *magneto-optical effect* of Kerr refers to the production of a slightly elliptic polarisation when plane polarised light is reflected from a polished pole of an electromagnet.

113. Anomalous Dispersion :—For transparent media the refractive index (μ) gradually diminishes with the increasing wavelength (λ). The nature of variation of μ with λ is represented by the curve shown in Fig. 111. Cauchy empirically deduced a relation connecting μ and λ which represents the change of μ with λ sufficiently accurately for most substances. Cauchy relation is given as,

$$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} \quad \ldots \quad (1)$$

In many cases only the first two terms are retained and relation (1) is written as,

$$\mu = A + \frac{B}{\lambda^2} \quad \ldots \quad (2)$$

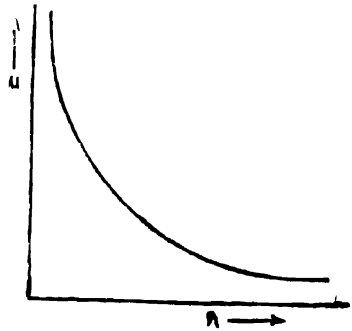


Fig. 111

This also gives the relation between μ and λ quite accurately in most cases.

From (2), the dispersion, $\frac{d\mu}{d\lambda} = -\frac{2B}{\lambda^3}$ (3)

Thus the normal dispersion is proportional to the inverse third power of the wavelength.

So for a transparent medium the refractive index μ and the dispersion $\left(\frac{d\mu}{d\lambda}\right)$ are normally found to decrease as the wavelength λ increases. This behaviour of the medium is known as the **normal dispersion.**

As every transparent substance selectively absorbs light of certain wavelengths either in the ultra-violet, or in the infra-red regions, it has absorption bands in those regions. Near these absorption bands, the refractive index and the dispersion of the substance are found to vary rather anomalously. On the longer wavelength side (or the short frequency side) of an absorption band [A_1 or A_2 in Fig. 112(*a*)], μ is found to increase rapidly as the band is approached. This is shown by the portions *bK* and *en* of the $\mu - \lambda$ curves [Fig. 112(*a*)]. On the shorter wavelength (or higher frequency) side of the absorption bands μ falls rapidly and even goes below unity as the band is approached. This is shown by the portions *a* and *d* of the curves. This behaviour near an absorption band is known as the *anomalous dispersion.* The $\mu - \lambda$ curve of a transparent substance as we expect from Cauchy's relation will be of the form *K b c g h.* But due to the presence of the absorption band A_2 in the position shown, the actual experimental curve follows the path *Kbcd*—the region *bc* of which represents the visible region of the spectrum. So in the visible region, the dispersion of a transparent substance is normal and $\mu - \lambda$ relation is quite in accordance with Cauchy's formula. But as either side of the absorption bands is approached, an apparent discontinuity appears in the curve at the characteristic frequency of absorption. The variation of μ with λ in the neighbourhood of an absorption band may be represented by Sellmeier's formula,

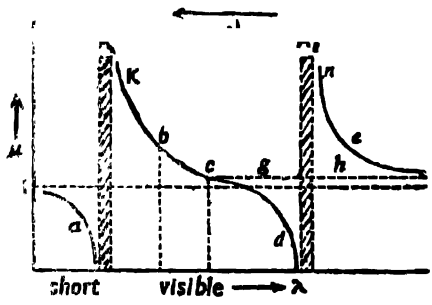


Fig. 112(*a*)

$$\mu^2 = 1 + \Sigma[A_0\lambda^2/(\lambda^2 - \lambda_0^2)] \quad \ldots \quad \ldots \quad (4)$$

where Σ extends over different absorption wavelengths λ_0, A_0 is a constant for a particular λ_0 and λ is the wavelength of the incident

ray. But Sellmeier's formula cannot explain all the aspects of variation of μ with λ. As shown in Fig. 112(*c*), μ does not approach infinity near the absorption band as expected from relation (4), but it actually jumps from a minimum to a maximum value within the band, which fact has been established from the theoretical considerations.

Theory.—Lorentz from the stand-point of electromagnetic theory explained $\mu-\lambda$ variation in all its aspect taking into consideration a damping term proportional to the velocity. According to Lorentz's view, in the vicinity of an absorption band as also within it, dipoles are created by the attending electric field of the electromagnetic waves incident on the transparent substance. As a result of resonance set up between the dipole vibrations and the incident waves, a considerable amount energy is absorbed from the wave and this is responsible for the abnormal $\mu-\lambda$ variation in the absorption region.

If N be the number of displaceable electrons per unit volume of the absorbing medium, e, the electronic charge and x, the displacement suffered, then the polarisation set up in the medium is $P=Nex$, which is the electric moment per unit volume. The electric dipoles execute *S.H.M.* under the action of incident electric field, E. The forces acting on each dipole are (*i*) a frictional force, $b.dx/dt$ and (*ii*) a restoring force, cx where b and c are constants.

So the equation of motion may be put as,

$$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+cx=eE \qquad \ldots \quad \ldots \quad \ldots \quad (5)$$

where m = mass of an electron and E is the instantaneous electric field of the *e.m.* wave. Equation (5) may be rewritten as,

$$\frac{d^2x}{dt^2}+\frac{b}{m}\cdot\frac{dx}{dt}+\frac{c}{m}x=\frac{eE}{m}$$

$$\text{or,}\quad \frac{d^2x}{dt^2}+g_0\frac{dx}{dt}+\omega_0^2x=\frac{eE}{m} \qquad \ldots \quad \ldots \quad \ldots \quad (6)$$

where $g_0=b/m$ and the pulsatance of the free vibration of an electron is $\omega_0=\sqrt{c/m}=2\pi\nu_0$, ν_0 being its natural frequency.

Multiplying throughout by Ne, and replacing Nex by P, we can rewrite eqn. (6) as,

$$\frac{d^2P}{dt^2}+g_0\frac{dP}{dt}+\omega_0^2P=\frac{Ne^2}{m}E \qquad \ldots \quad \ldots \quad (7)$$

Using the operators, $D=d/dt$

and $D^2=d^2/dt^2$ and remembering that E is of the form $E_0e^{i\omega t}$ and hence $D=i\omega$ and $D^2=-\omega^2$, we can write eqn. (7) as,

$$P\left[-\omega^2+ig_0\omega+\omega_0^2\right]=\frac{Ne^2}{m}E$$

$$\text{or,} \quad P=\frac{Ne^2E}{m}\Big/[\omega_0^2-\omega^2+ig_0\omega] \quad \ldots \quad \ldots \quad (8)$$

Since electric displacement, $D=E+4\pi P$, the dielectric constant $K=D/E=1+4\pi P/E$.

But $K=\mu^2$ from electromagnetic theory, (where $\mu=R.I.$ of the medium).

$$\therefore \quad \mu^2=1+\frac{4\pi Ne^2}{m}\Big/[\omega_0^2-\omega^2+ig_0\omega] \quad \ldots \quad \ldots \quad (9)$$

For more than one absorption band,

$$\mu^2=1+\sum\frac{4\pi Ne^2}{m}\Big/(\omega_0^2-\omega^2+ig_0\omega) \quad \cdots \quad (10)$$

The real part of this expression explains all the aspects of the so-called anomalous dispersion—near the absorption band, away from it and within it. The maximum and minimum values of μ can also be obtained, particularly in gases, by differentiating the real part of μ with respect to ω and equating that to zero. These maximum and minimum 'μ' are equispaced with respect to the resonance frequency ($\omega=\omega_0$) as shown in Fig.= 112(c).

When $g_0\to 0$, eqn. (10) reduces to,

$$\mu^2=1+\sum\frac{4\pi Ne^2}{m\times 4\pi^2(\nu_0^2-\nu^2)}=1+\sum\frac{Ne^2}{\pi mc^2}\left[\frac{\lambda^2\lambda_0^2}{\lambda^2-\lambda_0^2}\right]$$

$$=1+\sum A_0\frac{\lambda^2}{\lambda^2-\lambda_0^2}\text{(Sellmeier's form)}$$

$$=1+\sum\frac{A_0'}{\nu_0^2-\nu^2} \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

Again when ω_0 is far away from ω, the eqn. (9) can be reduced to Cauchy's form.

From (11) it follows that so long as the incident frequency ν is fairly different from ν_0 and smaller than it, μ gradually increases as ν increases (or μ increases as λ decreases). This is the behaviour found in normal dispersion. As ν approaches ν_0 in magnitude, the denominator rapidly decreases and hence μ rapidly increases in the immediate vicinity of the absorption band on the longer wavelength side, and ultimately reaches a maximum whence it jumps down to the minimum value near the higher frequency edge of the band.

If ν is greater than ν_0 (or λ less than λ_0), the denominator is negative and μ is less than unity. But as ν increases more and more, the denominator also increases and the negative term on the whole decreases. So μ approaches the unit value gradually.

Thus the so-called anomalous dispersion seems to be a natural consequence near an absorption band according to the general

electromagnetic theory of dispersion. Truly speaking there is no fundamental difference between the normal and the anomalous dispersions. It is only normal when the incident frequency is far

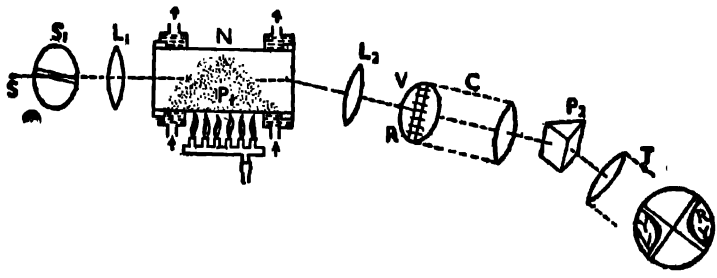


Fig. 112(*b*)

away from an absorption frequency and anomalous in the immediate neighbourhood of such an absorption band.

Anomalous dispersion may be studied with the help to two prisms—one of which is made of the material to be studied and the other of glass. The refracting edges of the prisms used must be mutually perpendicular to each other. The slit should be parallel to the edge of the first prism which is usually horizontal. Usual collimating arrangement is made and a screen is placed to receive the final spectrum. The spectrum formed by first prism containing absorption bands or lines in it is thrown on the vertical slit of a spectrometer, the prism of which has its edge vertical. By means of the telescope objective of this spectrometer, an anomalously dispersed spectrum is obtained on a screen placed in its focal plane. Fig. 112(*b*) shows the arrangement in the case of *Na*-absorption band. Here *Na*-prism is made by evaporating sodium in a cylindrical glass vessel *N* fitted with glass windows on two sides. These glass windows are attached to the cylinder by paraffin wax. Hence cooling arrangements are to be made at the ends as otherwise paraffin wax may melt. Due to evaporation, sodium vapour will be unequally distributed within the vessel, the density in the upper region being less than that at the bottom. So effectively the vapour

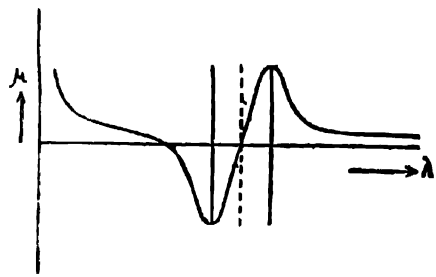


Fig 112(*c*)

behaves as a prism P_1 with its edge horizontal. The anomalously dispersed spectrum will be visible in the focal plane of the telescope as shown in the figure. The spectrum on the green side of the *D*-line will be found to bend down in the spectroscope, which means that the corresponding rays are deviated upwards in passing through the sodium prism, since the spectroscope inverts the image of the slit. The red and orange region is deviated in the opposite direction. In the cases of liquid or solid absorbing materials, the hollow or the solid prisms, as the case may be, are to be used with their refracting edges horizontal.

114. The origin of Line Spectra : Bohr's Theory :—The origin of spectral lines of the different spectral series, particularly of Hydrogen and other hydrogen like atoms (*i.e.* singly ionised Helium, doubly ionised Lithium etc.), was successfully explained by Prof. Niels Bohr in 1913. Balmer's empirical formula for wave-number of a spectral line was also exactly deduced theoretically by Bohr [See Art. 169(*b*), Geom. Optics].

According to the Rutherford-Bohr model, an atom of any element consists of a positively charged massive nucleus round which electrons are moving in different orbits. The mass of the whole atom is supposed to be concentrated in the nucleus, the revolving electrons having negligible masses compared to the nuclear mass. The positive charge on the nucleus is $+Ze$ where Z is the atomic number of the particular atom and e is the electronic charge ($4{\cdot}802 \times 10^{-10}$ c.s.u.) in magnitude whereas the negative charge on an electrons is $-e$. In the case of hydrogen the nuclear mass is nearly 1850 times that of an electron and $Z=1$. The nuclear mass being very very large compared to the electronic mass, the former is supposed to be at rest while the electron revolves round the nucleus. The actual dimensions of the nucleus and the electrons are negligibly small compared to the space occupied by the whole atom. Assuming Newton's laws of motion in the atomic domain, the path of an electron round a nucleus becomes a conic. Assuming this conic to be a circle as a first approximation, the force of attraction between the electron and the nucleus is given by Coulomb's law as,

$F_1 = -\frac{Ze \times e}{r^2} = -\frac{Ze^2}{r^2}$...(1), where r is the radius of the circular orbit.

This electrical force supplies the necessary centripetal force for the orbital motion, which is given by $-\frac{mv^2}{r}$.

$$\therefore \quad \frac{Ze^2}{r^2} = \frac{mv^2}{r} \text{ ; or, } mv^2 = \frac{Ze^2}{r} \qquad \ldots \quad (2)$$

Since the electric potential at any point on the orbit is $\frac{Ze}{r}$, the potential energy of the electron is given by

$$E_p = \frac{Ze}{r} \times (-e) = -\frac{Ze^2}{r}.$$

The kinetic energy of the electron is $E_k = \frac{1}{2}mv^2 = \frac{Ze^2}{2r}$

[From relation (2)]

Hence the total energy of the electron is given by,

$$W = E_p + E_k = -\frac{Ze^2}{2r} \quad \ldots \quad (3)$$

Since the nucleus is stationary, W represents the total energy of the atom excluding the rest energies of the constituent particles. The *negative sign* of W indicates that the energy of the atom decreases as the electron approaches the nucleus.

According to classical electrodynamics, an electron (*i.e.* a charged body) moving in a particular orbit should radiate energy continuously and the orbit gradually would diminish in size and ultimately the electron would fall on the nucleus. This state of affairs should give rise to a continuous spectrum due to the continuous release of energy. But actually discrete line spectra are obtained. So to explain the origin of such line spectra of hydrogen, Bohr introduced two fundamental postulates.

The first postulate assumes that of all possible electron orbits only those are permissible for which the angular momentum of an electron is an integral multiple of $\frac{h}{2\pi}$, where h is Planck's constant and that an electron occupying any of these *quantised orbits*, as they are generally called, does not radiate during its orbital motion. Since the angular momentum of an electron moving with a speed v in a circular orbit of radius r is given by mvr, the orbits permissible by Bohr's first postulate are limited by the relation

$$mvr = \frac{nh}{2\pi}, \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

where n is an integer.

The second postulate states that whenever an electron jumps from one stationary orbit to another of lesser energy, the excess energy is released in quanta of amount $h\nu$ of a monochromatic radiation, ν being the frequency of the emitted radiation. Thus

if W_1 and W_2 respectively represent the energies of the electron before and after the jump, then $h\nu = W_1 - W_2$ ... (5)

The radii of the permissible orbits are found from relations (2) and (4) as follows :—

$$r = \frac{Ze^2}{mv^2} = \frac{Ze^2}{m \times \frac{n^2h^2}{(2\pi mr)^2}} = \frac{4\pi^2 Ze^2 mr^2}{n^2h^2}$$

or, $$r = \frac{n^2h^2}{4\pi^2 Ze^2 m} \quad \ldots \quad \ldots \quad (6)$$

$\therefore$ From (3), $$W = -\frac{Ze^2}{2r} = -\frac{2\pi^2 mZ^2e^4}{n^2h^2} \quad \ldots \quad (7)$$

According to Bohr's second postulate, the frequency of the radiation emitted when an electron jumps from n_1 orbit to n_2 orbit is given by, $h\nu = W_1 - W_2$;

or, $$\nu = \frac{W_1 - W_2}{h} = \frac{2\pi^2 mZ^2e^4}{h^3}\left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right], \quad \ldots \quad (8)$$

where n_1 and n_2 may have any integral values.

Converting ν into wave-number $\bar{\nu}$, we have.

$$\bar{\nu} = \frac{1}{\lambda} = \frac{\nu}{c} = \frac{2\pi^2 mZ^2e^4}{ch^3}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right) \quad \ldots \quad \ldots \quad (9)$$

This is of the same form as Balmer's formula (see, Geom. Optics, Chapt. XI, Art. 169), the Rydberg constant R being $\frac{2\pi^2 mZ^2e^4}{ch^3}$.

Putting the values of the constants m, e, c and h and taking $Z = 1$, R is numerically equal to 109,740 cm.$^{-1}$, which is in excellent agreement with the experimental value of R_H.

According to Bohr's theory the electrons are revolving round the nucleus of the atom in definite orbits that are characterised by the *quantum numbers* $n = 1, 2, 3, 4$, etc. So long as an electron remains in one of these orbits, no energy is radiated, but as soon as it jumps from an outer orbit (of higher energy level) to an inner one (of lower energy level), energy is emitted in the form of radiation—either visible or invisible. Lines of Balmer series originate when electrons jump from an orbit of quantum number higher than 2 to the final orbit of quantum number 2, as shown in Fig. 113.

In addition to this Balmer series, the ultra-violet series of Lymann originate due to the electronic transitions from any of

the outer orbits to the innermost orbit of $n_2 = 1$. Similarly the infra-red series of Paschen, Brackett and Pfund originate respectively due to electron transitions from higher orbits to orbits of quantum numbers $n_2 = 3$, 4 and 5. The formation of different series of hydrogen spectra are shown in Fig. 113. Singly ionised helium and doubly ionised lithium which behave as a single-electron system, are also found to give series spectrum almost of the same general form as the series spectrum of hydrogen.

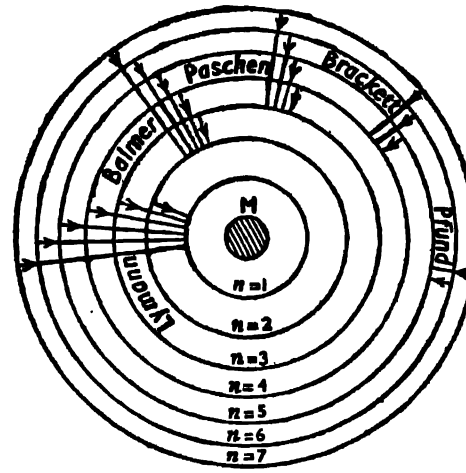


Fig. 113

The simple theory of Bohr has been, however, modified in view of the facts that the nucleus of an atom is not stationary, that the electronic orbits are neither circular, and that the mass of the electron changes with the change of its speed. The theory modified so as to accommodate the above facts actually explains the fine structures of the spectral lines.

114 (a). Correspondence Principle :—

The fact that the general form of the expression, in the earlier quantum theory, for the frequency of emitted radiation is identical with or analogous to that deduced from the classical theory based on Newtonian principles, is known as the *correspondence principle*. The principle consists in the assumption that this correspondence between the classical theory and the quantum theory is universal and is not only limited to frequencies of the emitted rays. Even the corresponding spectral lines are found to have the same state of polarisation.

Long before the introduction of Bohr's model, the frequencies of hydrogen lines were experimentally established by Balmer and others working in the field. Ritz by his combination principle deduced the frequencies of the lines as the difference of two terms. Bohr took a bold step and introduced the quantum concept in explaining the origin of these spectral lines. The assumptions made in the theory were later corroborated by Wilson and others ; and the results obtained from Bohr's theory were almost in agreement

with those yielded by the classical theory earlier. Bohr showed that the quantum physics practically reduces to classical physics for larger quantum number and thus first established the *principle of correspondence.*

As an example, we may consider the following simple case. From the quantum theory we know that the radius of the Bohr orbit of hydrogen atom (for the quantum number, $n=1$) is given by

$$\frac{n^2h^2}{4\pi^2Ze^2m}=\cdot529\times10^{-8}\text{ cm.}$$

This order of atomic radius is also corroborated by the classical calculations from oth.r branches of physics. Moreover, if we consider $n=10{,}000$, it yields $r=5{\cdot}29$ mm, the behaviour of an atom (or particle) of such a large orbital radius being easily describable by the classical mechanics.

115. Correction due to the motion of the nucleus : Discovery of Heavy hydrogen (Deuterium) :—In the above theory of hydrogen spectrum, it has been assumed that the nucleus of the hydrogen atom remains fixed at the centre of the circular electronic orbits. This assumption may be taken to be valid only if the nucleus has an infinitely lar e mass. But as the nuclear mass M and the electronic mass m are both finite, they will rotate about a common centre of mass with the same angular velocity. Due to this motion of the nucleus, the effective mass of the electron in the expression for Rydberg constant should be $\mu=\frac{m}{1+\frac{m}{M}}$ in place of m.

This is known as the **reduced mass** of the electron and can be very easily deduced from simple momentum and energy considerations.

From the above discussions it is clear that even for the same type of atom (having the same atomic number Z), the value of the Rydberg constants should be different for different nuclear masses (*i.e. for different isotopes*). So the wavelengths and wave-numbers should also slightly differ. Hence it is expected that the spectral lines of an element should show the presence of isotopes in their fine structures. It was by this spectroscopic method that presence of the isotopes oxygen was first detected. Three isotopes (O_8^{18}, O_8^{17} and O_8^{16}) of oxygen were identified from the spectral study and the mean atomic weight of oxygen computed on Aston's scale (taking mass of oxygen atom as 16) was found to be 16·0035. This shows a discrepancy of 2·2 parts in 10,000 with its chemical atomic weight. So it was inferred that the atomic weight of hydrogen in Aston's scale should be about 2 parts in 10,000 less than its chemical

atomic weight. It was suggested by Birge and Menzel in 1931 that this discrepancy could be removed by supposing the existence of two isotopes H_1^1 and H_1^2 of hydrogen in the ratio 4500 : 1. In 1932 Urey, Brickwedde and Murphy allowed liquid hydrogen to evaporate mostly whereby the lighter component quickly evaporated and the residue contained a high percentage of the heavier component. This residue was taken in a discharge tube and the lines of Balmer series were photographed using a 21-foot concave grating. Along with normal Balmer lines slightly displaced lines were also detected on the photoplate which were definitely due to the heavy hydrogen isotopes. Thus the presence of *heavy hydrogen* or *deuterium* was definitely established by a purely spectroscopic method.

116. Further modification of Bohr's Theory : Quantum Numbers :—Though the grosser details of hydrogen spectrum could be successfully explained by Bohr's theory, the fine structure of the Balmer lines, as exhibited in high resolution instruments, could not be explained by this theory. So to explain these fine structures Sommerfeld postulated the existence of elliptic orbits. We know that the angular momentum of an electron in a circular orbit can be fully described in terms of its radius because of the symmetrical nature of the orbit. So the condition of quantisation of angular momentum needs only one quantum number n, as given by Sommerfeld-Wilson relation $\int_0 pdq = nh$,

where p and q respectively represent the momentum and positional co-ordinates. This yields the relation, $mvr = \frac{nh}{2\pi}$, as demanded by Bohr's first postulate.

But in the case of an elliptic orbit, the angular momentum is a function of radius vector (r) and the azimuthal angle (θ). So two quantum numbers k and n_r, known as *azimuthal and radial quantum numbers*, are necessary to describe quantisation condition. $(k+n_r)=n$ is called the *principal quantum number*, where k and n_r should have integral values. From wave mechanical considerations, it has been found that instead of k, another quantum number $l=(k-1)$, known as the *reduced azimuthal quantum number* or the orbital quantum number which can take any value between 0 and $(n-1)$ can be used more conveniently for the proper identification of orbits and the orbital angular momentum of an electron is given by $P_l = lh/2\pi$. So the two quantum numbers n and l suffice for the proper identification of the elliptic orbits. The electrons having $l=0$, are called s-electrons by convention, and those having $l=1$, p-electrons $l=2$, d-electrons etc.

In order to explain the multiplet structures of the spectral lines, Goudsmit and Uhlenbeck suggested that the electrons are also spinning about their own axes. The spin angular momentum due to this revolution is also quantised and the third quantum number known as the *spin quantum number* arises due to this quantisation. This is designated by s which may have values $\pm \frac{1}{2}$ because of the two possible directions of orientation. The spin angular momentum of the electron is given by $p_s = sh/2\pi$. Another quantum number, known as the *total angular quantum number* (j), denotes the numerical value of the vector sum of l and s such that the to al angular momentum of an electron is given by $p_j = jh/2\pi = (l+s)h/2\pi$.

Over and above these four quantum numbers, three more magnetic quantum numbers should be considered when the atom is placed in a magnetic field. *Orbital magnetic quantum number* (m_l) represents the projection of the orbital moments vector l in the field direction, m_l may have any one of $(2l+1)$ values within the range $-l$ to $+l$. *Magnetic spin quantum number* (m_s) represents the projection of spin vector s in the field direction, m_s shall have also values $\pm \frac{1}{2}$ according to the direction of spin. *Total magnetic quantum number* (m_j) is the projection of j in the field direction and can have any one of the $(2j+1)$ values from $-j$ to $+j$ excepting zero.

Thus the movement and position of an electron in its orbit are controlled by these seven quantum numbers. It has also been shown that in course of the movement of the electron in an elliptic orbit, its velocity changes considerably—being increased when it approaches the nucleus at one of the foci. Hence a relativistic correction term for the mass is to be introduced. By introducing this correction Sommerfeld showed that the path of the electron round a nucleus should be of rosette form. This correction automatically accounts for the hyperfine structure of the spectral lines.

Pauli's Exclusion Principle and Periodic Classification.—The introduction of the quantum numbers was quickly followed by the enunciation by Pauli of a principle for the assignment of these quantum numbers to the electrons in an atom. This principle is known as Pauli's *exclusion principle.* It states that no two electrons in an atom can have the same set of values for the four quantum numbers n, l, m_l, m_s. According to this principle if we assign a particular value to the principal quantum number n, then there will be only a definite possible values which can be assigned to other quantum numbers and hence the number of electrons possessing this particular value of n is fixed. For each value of n, we assign a shell. When $n=1$, the shell is known as K-shell ; when $n=2$, the shell is called L-shell. Similarly M and N-shells refer to principal quantum numbers $n=3$ and $n=4$. It can be shown from Pauli's principle that the number of electrons in a particular shell is given by $2n^2$ where n refers to the principal

quantum number of the shell. This can be shown from the following considerations. When $n=1$, l can have only one value zero and so $m_l=0$ and $m_s=+\frac{1}{2}$ and $-\frac{1}{2}$. So the total number of electrons in the K-shell is 2. Again when $n=2$, l can have two values 0 and 1 corresponding to two subshells L_I and L_{II}. For L_I subshell as $l=0$, $m_l=0$ and $m_s=\pm\frac{1}{2}$, there can only be two electrons in it. For L_{II} subshell as $l=1$, $m_l=(2l+1)=3$ and for each m_l, $m_s=\pm\frac{1}{2}$. So, in all, in this subshell there will be $2(2l+1)=2\times3=6$ electrons. This relation $2(2l+1)$ is true for all subshells. So in L-shell total number of electrons is $2+6=8=2\times2^2$ (2 is the principal quantum number of the L-shell). When M-shell is considered $n=3$, $l=0$, 1, 2 and the subshells are M_I, M_{II}, M_{III}. For M_I subshell, $l=0$, and the number of electrons $2(2l+1)=2$; for M_{II} subshell, $l=1$, and number of electrons$=2(2l+1)=6$; for M_{III} subshell, $l=2$, and the electron number$=2(2l+1)=10$. So in all in the M-shell there are $2+6+10=18=2\times3^2$ electrons, 3 here being the principal quantum number of the shell. So we find that if n be the principal quantum number characterising a prticular shell, it will have, according to Pauli's exclusion principle, only $2n^2$ electrons. Hence, K-shell has only 2 electrons moving in different orbits in it. L-shell 8 electrons ; M-shell 18 electrons and so on. It is with the help of this electron distribution in different shells that the periodic classification of different elements have been logically and successfully explained avoiding all the anomalies that crept in the original periodic table of Mendeleev. This is so because all chemical behaviours of an element are controlled by the orbital electrons in the atom. The positions of these electrons are systematised and controlled by the exclusion principle of Pauli as has been shown above. They are distributed in definite shells and subshells and each of these shells or subshells, when complete, indicates one of the *stablest configurations* of the electrons. The nearer is the shell to the nucleus, the lesser will be the energy state of the corresponding electrons and by Minimum Potential energy theorem, the more and more stable become the orbits.

In explaining the periodic classification of the elements, hydrogen having only one orbital electron is taken as the starting element.

This electron evidently occupies the innermost orbit having $n=1$, $l=0$ and $m_s=$ either $+\frac{1}{2}$ or $-\frac{1}{2}$. In the case of helium, $n=1$, $l=0$ and $m_s=+\frac{1}{2}$ and $-\frac{1}{2}$ for the two electrons. Thus *K*-shell is complete with *He* and this fact accounts for the stability of Helium-configuration. On this Helium-configuration as the core, let us add electrons one by one and get elements of gradually increasing atomic numbers like *Li*, *Be*, *Bo* etc. In Lithium there is only one electron in L_1-subshell and hence this element has a tendency to part with this electron and to have the stable *He*-configuration. This is why Lithium is *electro-positive* and *monovalent*. As an element is more and more away from a stable configuration, it gradually becomes less and less electro-positive and increases in valency till half the shell is completed. Any more electron means the nearness of the element to the next stable configuration and the element becomes predominantly *electro-negative*. The state of affairs continues upto the next stable Neon-configuration ($Z=10$). The element just before *Ne* is Fluorine which lacks only one electron to have the stable *Ne*-configuration. So it tries to take up one electron readily and is highly *electro-negative and monovalent*. These general aspects in the change of properties recur after completion of each stable configuration, like *He*, *Ne*, Argon, Krypton, Xenon, and Radon. The elements preceding them are *electro-negative* and those following them are *electro-positive*, *the case of hydrogen being a solitary exception.*

A number of anomalies, however, occur in these electron configurations as in the cases of Potassium, Calcium, and some other elements of higher atomic numbers. In these cases, as found from spectroscopic data, the outermost electrons occupy orbits of a new shell without completing the preceding subshells. Potassium is an example of such an anomaly. For $K(Z=19)$ electron configuration may be written as $[K_2L_{2+6}M_{2+6+-}N_1]$. Since electrons in each subshell are supposed to be moving in ellipsoids of varying eccentricities, it is probable that N_I subshell has a lower energy state than the M_{II} subshell and as a consequence the electron first fills up the more stable orbits and the above anomaly disappears. This interpretation has a strong spectroscopic support in its favour. Similarly other anomalous cases may be also interpreted.

Thus the periodic classification is accounted for from the fundamental atomic concept with the help of exclusion principle.

Example

Calculate the radius of the first Bohr orbit of a hydrogen atom. [*Given :— Planck's constant* $= 6{\cdot}624 \times 10^{-27}$ *erg-sec., electronic charge* $= 4{\cdot}8 \times 10^{-10}$ *e.s.u. and mass of an electron* $= 9{\cdot}1 \times 10^{-28}$ *gm.* (*I. A. S. 1956*)

Ans. $r = \frac{n^2 h^2}{4\pi^2 Ze^2 m}$; for hydrogen $Z = 1$ and for the first Bohr orbit $n = 1$.

$\therefore$ The radius of the first Bohr orbit $= r_1 = \frac{(6.624 \times 10^{-27})^2}{4 \times (3{\cdot}14)^2 \times (4{\cdot}8 \times 10^{-10})^2 \times 9{\cdot}1 \times 10^{-28}}$

$= 0{\cdot}529 \times 10^{-8}$ cm. $= 0{\cdot}529$ Å. U.

Questions (Chapter V)

1. What is Doppler Principle ?

Show how the observed frequency of waves will differ from the frequency at which they are emitted if the source is approaching a moving observer.

A spectroscopic examination of light from a certain star shows that the apparent wavelength of a certain spectral line is 5001 Å. U., whereas the observed wavelength of the same line produced by a terrestrial source is 5000 Å. U. In what direction and at what speed do these figures suggest that the star is moving relative to the earth ?

[*Ans.* Away from the earth ; speed $= 6 \times 10^6$ cm./sec.]

2. What is Doppler effect ? Describe and explain the formulae for the apparent frequency due to it.

3. Write a short note on the blue of the sky.

4. Write short notes on Doppler effect.
Cite two astrophysical applications of the effect.

5. Write short notes on :—
Fluorescence and Phosphorescence.

6. Briefly describe the following :—
(*a*) Zeeman effect ; (*b*) Raman effect ; (*c*) Faraday effect ; (*d*) Kerr's electro-optical effect.

APPENDIX

[*Physical Optics, Art. 47(a)*] **Holography** :—*Holography* is a new method introduced in microscopy by Gabor in 1948. Here light from a point source *S* (Fig. 1) falls on the small object *O* under study at a small distance from it. Only a small portion of light is diffracted from the object. On a photo-plate at *AB*, these diffracted waves from *O* are superposed on a strong coherent background of the primary waves from *S*. This diffraction pattern formed on *AB* is known as the *hologram*. Since the primary wave is uniform and much stronger than the diffracted secondary waves, the intensity variations in the pattern correspond to the phase variations in the secondary waves. When this photo-plate is developed by reversal, the transmission of light at any point is determined by the phase of the diffracted wave. If this developed plate is now placed at *AB* and illuminated by *S* alone, the light transmitted by the hologram contains, as a major component, the wavelets reconstructing the original wave. Now looking through *AB* towards *S*, the object will appear to be placed at *O*. Gabor has shown that the unwanted components of the light transmitted through the plate can be almost completely removed and only the reconstructed waves can be made to form the virtual object at *O*. If the object be three dimensional, the three different planes can be seen by simply adjusting the focus of the observing lens.

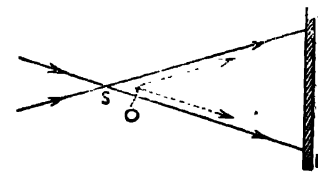


Fig. 1

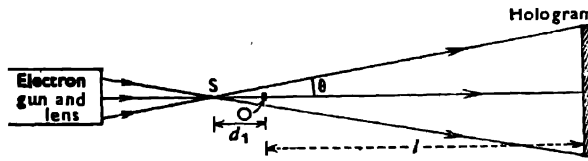


Fig. 2. Analysis with electron beam.

The method is important because of the fact that light of one wavelength (λ_1) can be used in the analyser to get the hologram and

13. What characteristics must a material have so as to serve as an efficient light pipe ?

14. Account for the following :

(a) The twinkling of stars.

(b) A soot-covered brass ball appears silvery when completely dipped in water.

(c) Diamond sparkles more than a similarly cut glass piece.

(d) An ordinary opaque piece of white paper becomes translucent when a drop of oil is poured on it.

15. If you place an iron ball on a stout glass rod fully immersed in glycerol, the observer sees that the ball is floating on the liquid. How do you explain it ?

16. How in the cold regions, you can occasionally see the inverted ships or boats hanging in air ?

17. Justify the following :

(a) The sun is seen a little before it is actually on the horizon in the morning.

(b) The sun appears red at sun rise or sun set.

18. Draw a neat diagram of any type of a constant deviation prism.

19. How can you deviate a ray through 90° and 180° with the help of a prism ? What type of prism should they be ?

20. (a) How with a suitable prism can you arrange for the inversion of an image ?

(b) If the critical angle for glass to air be θ_c, what should be the angle of no emergence from a glass prism at all possible angles of incidence ?

(c) Name a method which you may conveniently use to determine the refractive index of a liquid available in a small quantity only. Details need not be given.

21. What do you mean by the following terms with reference to a lens ?

(a) Principal axis, (b) First and second principal foci, (c) Focal length, (d) Lens aperture.

22. State whether the following sentences are correct or wrong :

(a) When a search-light sends out a parallel-beam of light, it experiences a force associated with the emission.

(b) In an unsymetrical thin lens the positions of a point object on the axis and its image are reversible.

(c) The longitudinal magnification of a lens is equal to its lateral magnification.

23. Three small circular pieces of glass are mounted on stands. One of them is plane and the other two are concave and convex lenses. Without the help of a lensometer how would you distinguish them ?

24. Why a convex lens is converging and a concave lens is diverging in nature ? Answer briefly.

25. One convex lens and a concave lens are arranged so that,

(a) both the lenses are in contact.

(b) each of them are separated by a distance.

Can you call both the systems as an equivalent lens in the same sense ? Justify your answer.

26. Answer the following:

(a) Can the optical path between two points can ever be less than the geometrical path ?

(b) Does the focal length of a lens depend on the medium in which the lens is immersed ?

(c) Under what conditions would the lateral magnification for lenses and mirrors become infinite ?

(d) On what factors does the dispersive power of a prism depend ?

27. Fill up the gaps in the following sentences :

(a) In the — — position of a prism the incident and the emergent rays are —— disposed.

(b) When an object is placed in front of a convex lens, a real image is cast on a screen on the other side of the lens only when the distance between the screen and the object is not — than — times the focal length of the lens.

(c) If refracting angle of a prism be equal to or greater than— the critical angle for the prism material and air, there will be no emergence for any angle of incidence.

28. In the displacement method of finding the focal length of a convex lens, how the magnified and minified images formed on the screen related to the original object in respect of their sizes ?

29. What do you mean by the principal points of an optical system ? State their importance in ascertaining the images formed by a lens system. Why they are called unit points ?

30. Briefly state why in air the nodal points of a lens-system coincide with its unit points.

31. Mention the names of different types of Seidel aberrations. Why they are called monochromatic ?

32. What do you mean by :

(a) The shape factor of a lens ?

(b) A crossed lens ?

(c) An achromatic lens system ?

33. Does the aperture of a lens in anyway affect the spherical aberration of a lens ?

34. We use in most of the practical cases plano-convex lenses as lenses almost corrected for spherical aberration. Discuss briefly.

35. What are the aplanatic points in the case of transparent sphere ? Show that these aplanatic points are also free from comatic errors.

36. (a) How do you remove astigmatic errors in a lens ?

(b) Name the types of distortion that occur in a concave and a convex lens respectively.

37. Answer the following :

(a) Why in the eyepiece of a telescope or a compound microscope two lenses are required ?

(b) What do you mean by the f-number of a camera lens ?

(c) On what does the depth of field of a camera lens depend ?

38. With two lenses of proper shape and refractive indices achromatisation can be duly effected when they are (a) in contact ; (b) separated by a distance. Tick the correct answer.

39. Correct the following statements wherever necessary :

(a) An observer sees a beautiful rainbow near the western horizon at 5 P.M.

(b) The primary rainbow has its upper edge violet and the lower edge red whereas a secondary bow has its upper edge red and the bottom edge violet.

(c) The secondary bow is formed inside the primary bow.

(d) The space between the two rainbows appears somewhat brighter than the rest of the sky.

40. What do you mean by the supernumerary bow ? In what parts of a rainbow these are formed ?

41. What changes would you observe :

(a) If you approach the rainbow ?

(b) If you rise up to a high altitude ?

42. For a reading glass will it be more convenient to place the lens (a) very close to the eye ? (b) very close to the object to be viewed ? Justify your answer.

43. Why the image seen by a reading glass does not suffer from chromatic effect ? Explain briefly.

44. Write short notes on :

(a) Visual angle, (b) Least distance of distinct vision, (c) Field of view, (d) Aperture and (e) Stop as applied to optical instruments.

45. Fill up the blanks :

(a) The phenomenon of resolution of a composite pencil of light into its components of different wavelengths is known as - and the band of component wavelengths is called the

(b) The spherical aberration of a convergent lens for objects beyond its focus is ——— and that for a divergent lens is ———.

(c) The accommodation is that property of the eye-lens by which its effective focal length is ——— ——— to view distant or near objects.

46. Why do you call Huygen's eyepiece a negative eyepiece and Ramsden's eyepiece a positive one ?

47. Inspite of the fact that neither the spherical nor the chromatic aberration is completely removed in a Ramsden's eyepiece, why this type of eyepiece is preferred in most of the optical measuring instruments ?

48. Why in an oil immersion objective of a microscope cedarwood oil is used ? What are its functions here ? Has it anything to do with the limit of resolution of the instrument ?

49. What are the types of eyepiece you would prefer :

(a) in a travelling microscope ?
(b) in a biological microscope ?
(c) in a telescope attached to a cathetometer ?
(d) in an astronomical telescope ?

50. Write a few lines on each of the following :

(a) Ultramicroscope ; (b) stereoscopic vision ; (c) relative aperture ; (d) speed of a lens ; (e) portrait lens.

51. Why in the construction of a binocular, Galilean telescopes are preferred to terrestrial ones ? Give reasons.

52. So far as the construction of the giant telescopes is concerned, what are the advantages of the reflecting telescopes over the refracting ones ?

53. What is a telephoto lens ? What is its importance ?

54. (a) Give an idea of the resolving power of a human eye.

(b) Write a short note on binocular vision.

55. (a) Mention a method of determining the velocity of light that

appears to measure the phase velocity. Justify your view. (b) What is a light year ?

56. (a) For determining the velocity of light in a liquid medium, what method would you prefer and why ?

(b) Does the speed of light in vacuum depend on (i) the frequency, (ii) the intensity, (iii) wavelength ?

58. Why accurate determination of velocity of light is important in Physics ? Discuss. Give its present accepted value.

59. Define, (i) luminous intensity, (ii) luminous efficiency, (iii) luminous flux and (iv) Illuminance.

60. Light is incident normally on a screen one metre away from it. If the intensity of the source is increased 5 times, the screen is moved 2 metres away from the source and is inclined so that the angle of incidence is 60°, then the intensity is altered by a factor :

(a) 10, (b) 5, (c) 0.5, (d) 0.625.

Tick the correct answer.

61. Physically explain why with help of a large telescope, a star can be seen even in day time.

62. Write a short note on the modern form of the standard of luminous intensity (Temperature standard).
How is candela related to the intensity of this standard ?

63. Name the modern units of (a) luminous flux, (b) illumination and (c) illuminance or brightness.

64. Why the Planckian Standard is preferred to other standards of luminous intensity ? Give reasons.

65. What are actinic rays ? Why they are so called ?

66. Describe the ideal conditions for an experimental set up necessary for the production a pure spectrum. Why do you set the prism in the minimum deviation position ?

67. Briefly describe how by Schuster's method you can focus a spectrometer for parallel rays.

68. Complete the following sentences :

(a) The continuous spectrum is emitted by ————.

(b) Bright line spectra are emitted by ————.

(c) Band spectra arise due to ————.

(d) Dark line spectra are due to

69. Give a brief account of the ultra-violet spectroscopy of the quartz region only.

70. Name the different methods that you would take help of in studying infra-red region.

71. Distinguish between the mixing of coloured rays and the mixing of coloured pigments.

72. The eye is most sensitive to the yellow-green colour. Why then are the danger signals arranged in red ?

B. PHYSICAL OPTICS

1. Why Newton's particulate theory failed to explain the phenomenon of reflection and refraction of light ? Briefly explain.

2. (a) How Huygen's theory successfully explained reflection and refraction ?

   (b) Why it was then necessary to consider light as an electromagnetic wave ?

3. The electromagnetic waves in a hypothetical medium may be somehow conceived : but how can we conceive the energy flux to be propagated as waves in a vacuum ? Clarify the idea briefly.

4. What do you mean by a wave-front and a ray ?

5. Name sources from which you can get the following :

   (a) A plane wave-front.

   (b) A spherical wave-front.

   (c) A cylindrical wave-front.

6. Explain briefly the phenomenon of total internal reflection from the wave theory of light.

7. State the conditions of substained interference.

8. Give two examples of each of the following :

   (a) Interference by Division of amplitude.

   (c) Interference by Division of wave-front.

9. Do you consider Young's experiment on superposition of light waves to be fundamentally an interference or a diffraction phenomenon ? Justify your answer.

10. Distinguish between Newtonian and Haidinger fringes. Give an example of the latter.

11. Fill up the gaps in the following sentences :

   (i) The intensity distribution in an interference pattern follows the ——— —— law.

   (ii) At the positions of the destructive interference, the energy of the rays is found to ———.

(iii) At the positions of the constructive interference, the energy of the rays is found to ———.

12. Fresnel's biprism experiment is done with the help of a linear source and the biprism. So the fringes you get may be a single slit diffraction pattern. Do you agree ? If not give reasons for your answer.

13. In a Biprism experiment,
    (a) Is there any influence of the single slit diffraction pattern in the system of fringes ?
    (b) When the separation between the slit and the biprism is increased what happens to the system of fringes ?
    (c) How from the experimental observations, the angle of the biprism can be found ?
    (d) What happens to the fringe system if the whole arrangements are immersed in water ?

14. Why in the measurement of the separation of two virtual sources in the biprism experiment, the source and the observing eyepiece are to be separated by a distance more than a definite minimum ? What is this minimum value ?

15. If the source of light be white, why do you get a limited number of fringes with a biprism ?

16. How do you get achromatic fringes with a white source of light ?

17. Why due to reflection from denser medium on to a rarer one, there is phase change of π in the reflected component ?

18. What do you mean by localisation of fringes ? Give an example of localised fringes.

19. Distinguish between the fringes of equal width and the fringes of equal inclination.

20. In a Newton's ring experiment is the central spot, as seen by reflection, dark or bright ? Explain.

21. Very small changes in the angle of incidence do not change interference conditions much in thin films. Why ?

22. Can you measure with the help of Newton's ring arrangement
    (a) The refractive index of a liquid ?
    (b) The radius of curvature of a lens ?
    (c) Wavelength of a monochromatic light ?

23. Why an extended source is needed for the Newton's ring experiment ?

24. What type of interferometer is needed for a quicker measurement of the thickness of a thin film :

(a) A Michelson Interferometer, or.

(b) A Jamin Interferometer ?

25. Why do the scientists prefer to standardise a metre in terms of wavelengths of light ?

26. Fabry Perot interferometer produces fringes which are :

(a) straight, (b) circular, (c) hyperbolic.
Tick the correct answer.

27. What type of fringes are produced in Lummer-Gehrcke interferometer : (a) Newtonian or (b) Haidinger ?

Tick the correct answer and state why it is so.

28. On what factors do the Resolving Powers of the following instruments depend :

(a) Fabry-Perot's interferometer ?

(b) Lummer-Gehrcke interferometer ?

29. Is a phenomenon similar to beat in sound, possible in light ? Justify your answer.

30. (a) What is the order of the area covered by a half period zone of yellow light ?

(b) What is a zone plate ? State its importance.

31. In a single slit diffraction pattern we get patches of dark and illuminated regions whereas in a double-slit or a grating we get sharp lines. Why this is so ?

32. How does diffraction help you to interpret the rectilinear propagation of light ? Explain without mathematics.

33. Distinguish between Fresnel and Fraunhoffer diffraction phenomena.

34. Will there be any difference between a single slit Fresnel and Fraunhoffer patterns ? Briefly describe.

35. What are absent spectra and ghost lines ? Why do they arise ?

36. In a single slit diffraction spectrum (Fraunhoffer) :

(a) Why is the intensity distribution so irregular ?

(b) What is the intensity of the first maximum in relation to that of the central maximum ?

37. Draw out the similarities and dissimilarities of interference and diffraction phenomena.

38. Why do radio-waves diffract round buildings although the light-waves do not ?

39. Correct or justify the following :
    (a) A person holds a single slit in front of his eye and looks at a distant light source in the form of a long heated filament. He sees a Fraunhoffer pattern.
    (b) If the whole apparatus of a single slit Fraunhoffer arrangement is immersed in water, the fringes are found to be thinner.

40. In a single slit Fraunhoffer pattern, what do you expect if : (i) the wavelength of the source is increased and (ii) the slit width is increased ?

41. Do the diffraction effects occur for virtual images, as well as for real images ?

42. Discuss the statement that "a diffraction grating may also be called an interference grating".

43. Why in a grating : (a) the rulings are closely spaces ? (b) Why the number of rulings are large ?

44. Show that for a given wavelength and for a given angle of diffraction, the resolving power of a grating depends only on the width of the grating.

45. In Fraunhoffer's diffraction in a circular aperture, what is the diameter of the first Airy's disc ? What is its importance in optical instruments ?

46. Explain briefly what is meant by Rayleigh's criterior of resolution.

47. Write down expressions for the following :
    (a) Resolving power of a prism.
    (b) Limit of resolution of a telescope.
    (c) Limit of resolution of a microscope.
    (d) Resolving power of a diffraction grating.

48. In an interference pattern all the bright bands have equal intensity. But in a Fresnel biprism pattern, a definite variation of intensity is found. Why it is so ?

49. What is a Rowland's circle ? Why it is so important in a concave grating experiment ?

50. Correct the wrong sentences and tick the correct ones :
    (a) In an interference pattern, the minimum intensity positions are not completely dark whereas in a diffraction pattern they are perfectly so.
    (b) The diffraction fringes are never equispaced but the interference fringes are always so.
    (c) The intensity of a grating spectrum is far superior to that of a prism spectrum.

(d) The echelon grating is only suitable for the study of the fine structure or hyperfine structure of spectral lines.

(e) For a particular number of lines per cm, the grating spectrum does not depend on the nature of the transparent material of the grating.

51. (a) Does the Extra-ordinary wave in doubly refractive crystals always travel with a speed of c/μ_e, c=vel. of light in vacuum, μ_e = R.I. for E-ray ?
(b) What are the distinguishing features of a positive and a negative crystal ?

52. What are meant by the following :
(a) Plane of polarisation.
(b) Plane of vibration.
(c) Brewster's law.
(d) Malus' Cosine Square law.

53. Write short notes on :
(a) Pile of plates.
(b) Nicol prism.
(c) Rochon prism.
(d) Wollaston prism.

54. Explain the terms *optic axis* and principal axis of a double refracting crystal.

55. What is quarter-wave plate ? How do you identify its optic axis ?

56. What would be the action of a half-wave plate on (a) a plane polarised light, (b) circularly polarised light and (c) unpolarised light ?

57. Which of the following instruments would you prefer to analyse different types of polarised light :
(a) Fresnel's rhomb ?
(b) Babinet's compensator ?

58. Can a plane polarised beam of light be represented as the sum of two circularly polarised beams of opposite rotations ? If the phase of one of the components (circular) be changed, how would the situation alter ?

59. How would you experimentally identify (a) plane polarised light, (b) unpolarised light and (c) circularly polarised light ?

60. When observing a clear sky through a polarising sheet, one finds that the intensity varies by a factor of two on rotating the sheet. This does not happen when the sky is cloudy. Explain the observations.

61. How rings and brushes provide a handy method for testing the sign of a crystal ? Briefly explain.

62. Define specific rotatory power and molecular rotatory power with reference to optically active solutions.

63. Mention a few important applications of polarimeters.

64. Cite two astronomical observations which can be explained by Doppler effect.

65. Write short notes on :

(i) Tyndall effect, (ii) Blue of the sea, (iii) Colour of the sun at the sunset, (iv) Fluorescence and (v) Phosphorescence.

66. (a) What do you mean by Raman effect ?

(b) Are the Stokes lines and the anti-Stokes lines therein equidistant from the exciting line ?

(c) Are these Stokes and anti-Stokes lines of identical intensity ?

(d) Are Raman spectra related to the infra-red spectra ?

67. Why ether as the medium of propagation of the electromagnetic waves has been ultimately discarded ? Briefly explain.

68. Does Compton scattering in any way resemble Raman scattering ?

69. Give a brief review of normal Zeeman effect.

If a high resolution instrument is used for observation, what kind of change would you observe ?

70. Write short notes on :

(i) Kerr's Electro-optical Effect.

(ii) Correspondence principle.

(iii) Quantum number.

(iv) Pauli's exclusion principle.

71. Why dispersion near an absorption band is said to be anomalous ? Is there any anomaly in its ? Justify your answer briefly.

72. Discuss the assumptions made by Bohr in connection with the hydrogen atom problem.

73. Name the different series of spectra of hydrogen and state which of them are in what position of the general electromagnetic spectral order.

74. What do you mean by quantisation of angular momentum ? How do you support this quantisation view ?

75. In Bohr's theory of hydrogen atom, it is found that the potential energy is negative and greater in magnitude than the Kinetic energy. What is the implication of this fact ?

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